Homework Assignment #2

22.105 Electromagnetic Interactions

Fall 2005

Distributed: Tuesday, September 26, 2005

Due: Thursday, October 5, 2005

The purpose of this problem is to determine how the electric field behaves around a noncircular conducting electrode. The geometry is idealized to simplify the calculations but the essential physics is maintained.

Consider a thin hollow perfectly conducting grounded cylinder with an elliptical cross section. The surface of the ellipse is given by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

with $b > a$. A line charge of magnitude λ coul/m is placed on the axis $x = 0, y = 0$.

a. Calculate the radial electric field, E_0, at $x = 0, y = b$ for the circular reference case $b = a$.

b. Now assume that $b \neq a$. Try to calculate $E_r(x = 0, y = b)$ using separation of variables in cylindrical (r, θ) coordinates and find the maximum value of b/a for which the expansion converges.

c. Calculate $E_r(x = 0, y = b)$ for arbitrary b/a using the Green's function procedure described in class. In particular evaluate and plot $E_r(x = 0, y = b)/E_0$ as a function of $\varepsilon = R_c/(ab)^{1/2}$. Here, R_c is the radius of curvature at the tip of the cylinder and normalizing it to $(ab)^{1/2}$ is equivalent to considering a sequence of electrodes with fixed cross sectional area but varying elongation. Note 1: A convenient angle v with which to parameterize the surface is defined as follows, $x = a\cos v, y = b\sin v$. Note 2: There is a considerable savings in algebra and computation by immediately focusing Green's theorem on the observation point $x = 0, y = b$.