1 Problem Set 6 Solutions

1. The non-relativistic Bethe-Block formula is:

$$-\frac{dE}{dx} = \frac{4\pi e^4 z^2}{m_e v^2}(nZ) ln \frac{2m_e v^2}{I} \tag{1}$$

Here, $z = 1$ for an incident proton and Z is the target particle. $n = \frac{\rho N_A}{A}$. We also know that $E = 1/2 m_e v^2$. Recognize that,

$$ln \frac{4E_e}{I} = ln \frac{4E_p m_e}{I m_p} \tag{2}$$

Putting everything together we get,

$$-\frac{dE}{dx} = \frac{4\pi e^4}{(\hbar c)^2} \frac{(\hbar c)^2}{2E_p} \frac{m_p}{m_e} \rho N_A \frac{Z}{A} ln \frac{4E_p m_e}{I m_p} \tag{3}$$

Plug in all of the known numbers to get,

$$-\frac{dE}{dx} = \frac{143.6}{E_p} \rho \frac{Z}{A} ln \frac{E}{459I} \tag{4}$$

Eliminate ρ by denoting $\xi = \rho x$,

$$-\frac{dE}{d\xi} = \frac{0.144}{E} \frac{Z}{A} ln \frac{E}{459I} \tag{5}$$

By substituting the appropriate numbers for Al into our result, we can numerically calculate the curve in the figure. When we compare this curve to Marmier and Sheldon, the difference becomes most apparent when the proton energy exceeds 10MeV. In this case, we must treat the proton as a relativistic particle.

For an α-particle, the result can be modified to read,

$$-\frac{dE}{d\xi} = \frac{2.368}{E} \frac{Z}{A} ln \frac{E}{1840I} \tag{6}$$

Obtain the necessary numbers for air and silicon from the given table to get the following plot.

2. If you assume that the energy of the incoming particle is much larger than the ionization energy I, then

$$\frac{dE}{dx} = -\frac{k_1}{E_0} \tag{7}$$

Integrate,

$$\int_{E_0}^{E_1} EdE = \int_0^x -k_1 dx \tag{8}$$

$$x = \frac{E_0^2 - E_1^2}{2k_1} \tag{9}$$

$$E_1^2 = E_0^2 - 2k_1 x \tag{10}$$

Here E_1 is the energy at the interface between the two materials, and E_0 is the incoming energy.k_1 is the constant of the first material. Similarly, we can find,

$$E_1^2 = E_2^2 + 2k_2 x \tag{11}$$

Here E_2 is the exit energy. We can combine the two results to eliminate E_1 to obtain,

$$E_2^2 = E_0^2 - 2(k_1 + k_2)x \tag{12}$$

We now have an expression for E_2 in terms of E_0. Because k_1 and k_2 are interchangable, so are the two materials.

3. The energy loss of a heavy charged particle is,

$$-\frac{dE}{dx} = \frac{4\pi e^4 z^2}{m_e v^2} nZ[ln\frac{2m_e v^2}{I} - ln(1 - \frac{v^2}{c^2}) - \frac{v^2}{c^2}] \tag{13}$$

If we let $a = \frac{4\pi e^4 z^2}{m_e} nZ$ and $b = \frac{2m_e}{I}$ then,

$$-\frac{dE}{dx} = f(v) = \frac{a}{v^2}[ln(bv^2) - ln(1 - \frac{v^2}{c^2}) - \frac{v^2}{c^2}] \tag{14}$$

Differentiate with respect to v,

$$\frac{df}{dv} = \frac{2a}{v^3}[-ln(bv^2) + ln(1 - \frac{v^2}{c^2}) + 1 + \frac{v^2}{c^2 - v^2}] \tag{15}$$

If we take the non-relativistic limit,

$$\frac{df}{dv} = \frac{2a}{v^3}[-ln(bv^2) + 1] \tag{16}$$

Set the above result equal to zero to get,

$$ln(bv^2) = 1 \tag{17}$$

$$v^2 = \frac{2.72}{b} = \frac{1.36I}{m_e} \tag{18}$$

$$T = \frac{1}{2}m_0v^2 = 0.68\frac{m_0}{m_e}I \tag{19}$$

4. For each atom in the material,

$$-\frac{dE}{dx}_1 = \frac{4\pi e^4 z^2}{m_e v^2}(n_1 Z_1)B_{e1} \tag{20}$$

$$-\frac{dE}{dx}_2 = \frac{4\pi e^4 z^2}{m_e v^2}(n_2 z_2)B_{e2} \tag{21}$$

n_1 and n_2 are the number densities of atoms 1 and 2,

$$n_1 = \frac{f_1 \rho N_A}{f_1 A_1 + f_2 A_2} \tag{22}$$

$$n_2 = \frac{f_2 \rho N_A}{f_1 A_1 + f_2 A_2} \tag{23}$$

Combine the two equations above to obtain,

$$-\frac{dE}{dx} = \frac{4\pi e^4 z^2}{m_e v^2} n(f_1 Z_1 B_{e1} + f_2 Z_2 B_{e2}) \tag{24}$$

where $n = \frac{\rho N_A}{f_1 A_1 + f_2 A_2}$

$$-\frac{dE}{dx} = \frac{4\pi e^4 z^2}{m_e v^2}\frac{\rho N_A}{17}[1(7)(4) + 3(1)(5.6)] = 0.145 Mev/cm \tag{25}$$

5. The graph is as follows,

Graph deleted.

The units given in the formula are g/cm^2. Therefore, we need the densities of Al and air.

$$\rho_{Al} = 2.7 g/cm^3 \tag{26}$$

$$\rho_{air} = 1.16E^{-3} g/cm^3 \tag{27}$$

The table looks like the following:

Table deleted.

6.

$$\epsilon\epsilon' = cos\beta sin\xi \tag{28}$$

$$\frac{d\sigma}{d\Omega} = \frac{r_e^2}{4}(\frac{v'}{v})^2(\frac{v}{v'} + \frac{v'}{v} + 4cos^2\beta sin^2\xi - 2) \tag{29}$$

$$= \frac{r_e^2}{4}(\frac{v'}{v})^2(\frac{v}{v'} + \frac{v'}{v} + 4sin^2\xi) + \frac{r_e^2}{4}(\frac{v'}{v})^2(\frac{v}{v'} + \frac{v'}{v} - 2) \tag{30}$$

$$= \frac{r_e^2}{2}(\frac{v'}{v})^2(\frac{v}{v'} + \frac{v'}{v} - 2cos^2\xi) \tag{31}$$

$$= \frac{r_e^2}{2}(\frac{v'}{v})^2(\frac{v}{v'} + \frac{v'}{v} - 2sin^2\theta cos^2\eta) \tag{32}$$

We can resolve the incident radiation into two orthogonally polarized components, each carrying one-half the incident intensity (Evans, Atomic Nucleus, pg.682). Choose the orientation such that one lies perpendicular to the scattering plane $\eta = 90$ and the other lies parallel $\eta = 0$.

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}(\frac{d\sigma}{d\Omega})_{\eta=90} + \frac{1}{2}(\frac{d\sigma}{d\Omega})_{\eta=0} \tag{33}$$

$$= \frac{r_e^2}{2}(\frac{v'}{v})^2(\frac{v}{v'} + \frac{v'}{v} - 2sin^2\theta) \tag{34}$$

For the second part, use the fact that,

$$\frac{v}{v'} = 1 + \epsilon(1 - cos\theta) \tag{35}$$

to solve the problem.