22.02 Intro to Applied Nuclear Physics

Mid-Term Exam

Thursday March 17, 2011 Name: . . . .

Problem 1: Short Questions *24 points*

These short questions require only ***short answers*** *(but even for yes/no questions give a brief explanation)*

1) What information about a quantum system can you obtain from the wavefunction?

2) If we measure the kinetic energy of a quantum particle and *immediately* after we measure its momentum, is the result of the second measurement random?

3) What does the Coulomb term in the Semi-empirical mass formula describe?

4) A particle is in the quantum state, $\psi(y) = Ae^{-i\pi y}$.
a) What are the possible results of a momentum measurement?
b) What are the probabilities of each possible momentum measurement?
c) What physical situation is represented by this quantum state?

5) When is the wavefunction describing a quantum system an energy eigenfunction?

6) Which one of the following statements (if any) is correct, based on the properties of the angular momentum and its eigenfunctions?
a) A particle is in the angular momentum eigenstate, $\psi_{l,m_z}(\vartheta, \varphi) = |l{=}3, m_z{=}{-4}\rangle$.
b) A particle is in the angular momentum eigenstate, $\psi_{l,m_x,m_z}(\vartheta, \varphi) = |l{=}4, m_x{=}3, m_z{=}{-2}\rangle$.
c) A particle is in the angular momentum eigenstate, $\psi_{l,m_x}(\vartheta, \varphi) = |l{=}4, m_x{=}3\rangle$.

7) When is a quantum system "bound"? Give a condition in terms of the system energy E and potential energy V.

8) Is the Q-value of a nuclear reaction (such as alpha-decay) the only factor that determines if the reaction does happen spontaneously?

Problem 2: Rotations and angular momentum *26 points*

a) Consider *classical* rotations in a 3D Euclidean space. We define $R_{\vec{n}}(\vartheta)$ the operator describing a rotation around the axis $\vec{n}$ by an angle ϑ. Do the operators $R_z(\vartheta)$ and $R_z(\varphi)$ commute? Do the operators $R_x(\vartheta)$ and $R_y(\varphi)$ commute? *(a yes/no answer is enough)*

b) Now we consider rotations in quantum mechanics. We write rotations as the operators $\hat{R}_{\vec{n}}(\vartheta)$.
For small angles ϑ we can write these rotations using the *angular momentum* operator as $\hat{R}_{\vec{n}}(\vartheta) = 1 - i\frac{\vartheta}{\hbar}\hat{L}_n$ (for example $\hat{R}_x(\vartheta) = 1 - i\frac{\vartheta}{\hbar}\hat{L}_x$). Do rotations in quantum mechanics commute?

c) Calculate the difference between making first a rotation $R_y(\varphi)$ followed by a rotation $R_x(\vartheta)$ and making first $R_x(\vartheta)$ and then $R_y(\varphi)$. Can you express this difference as a rotation?

d) A quantum system is in a state ψ such that it is left unchanged by a rotation along x: $\hat{R}_x(\vartheta)\psi = \psi$.
Is ψ an eigenfunction of $\hat{L}_x$?

e) We studied in class that the eigenvalues of the angular momentum operator along x, $\hat{L}_x$, are $\hbar m_x$ with integers $m_x = -l, -l+1, \dots, l$. Consider a quantum state $\psi = \frac{1}{\sqrt{6}}\varphi_{-2} + \frac{1}{2}\varphi_0 + \sqrt{\frac{7}{12}}\varphi_1$, where φ_m is the normalized eigenfunction of $\hat{L}_x$ corresponding to the eigenvalue $\hbar m_x$.
What is the probability of finding $\hat{L}_x = 0$ in a measurement? What is $\left\langle \hat{L}_x \right\rangle$?

Problem 3: Radioactive decay by proton emission *30 points*

Useful quantities: Proton mass, $m_p c^2 = 938.272$ *MeV;* $\hbar c = 197$*MeV fm;* $\frac{e^2}{\hbar c} = \frac{1}{137}$*;* $c = 3 \times 10^8$*m/s;* $R_0 = 1.2$*fm.*

a) Consider the isotope Europium-131 ($^{131}_{63}$Eu), with mass 121919.966 MeV. Given its A and Z numbers, do you expect this isotope to be stable?

A possible decay channel for $^{131}_{63}$Eu is proton emission. We want to analyze this decay mode following the same theory we saw for alpha decay and in particular *estimate* the half-life of $^{131}_{63}$Eu. The following questions will guide you through the estimation.

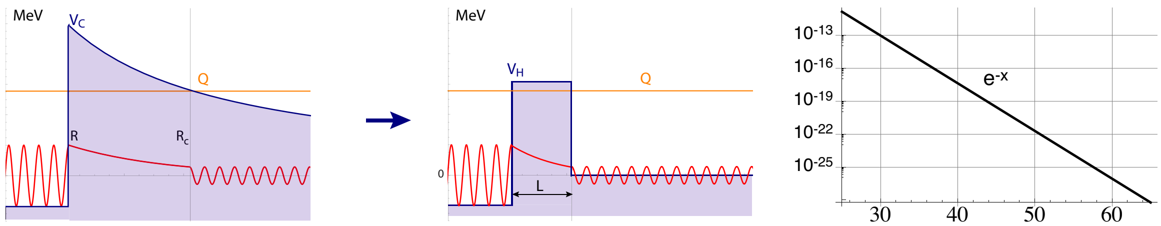
b) The mass of Samarium-130 is 120980.755 MeV. What is the Q-value for the reaction $^{131}_{63}\text{Eu} \rightarrow {}^{130}_{62}\text{Sa} + {}^{1}_{1}\text{H}$?

c) Calculate the frequency $f = \frac{v}{R}$ for the proton to be at the edge of the Coulomb potential. Here R is the Samarium radius and v the proton speed when taking Q as the (classical) kinetic energy.

d) What is the Coulomb potential at the distance R, $V_C(R)$? (this is the potential barrier height).
What is the distance R_c at which the Coulomb potential is equal to the Q-value?

e) To estimate the tunneling probability we replace the Coulomb barrier with a rectangular barrier of height $V_H = V_C(R)/2$ and length $L = (R_c - R)/2$ (see figure). What is the tunneling probability?

f) Finally, give the decay rate λ and the half-life for the proton emission decay of $^{131}_{63}$Eu.

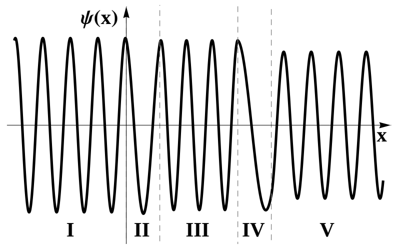


Problem 4: Match the potential *20 points*

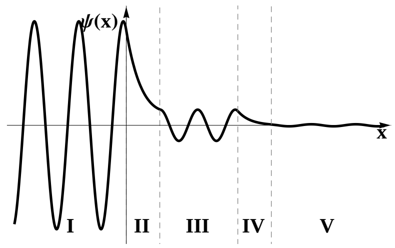
A quantum system in a 1D geometry is subjected to the potential energy as in the figures on the right, with 5 regions of different potential height.

Match the 1D energy eigenfunctions on the left with the correct energy (if any) depicted on the right. Provide a **brief** explanation of the reasoning that lead you to each of your matchings.

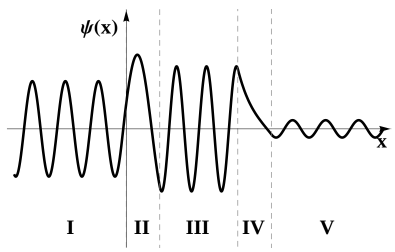
(Notice: here I plot the real part of the eigenfunction).


ψ(x)
x
I
II
III
IV
V

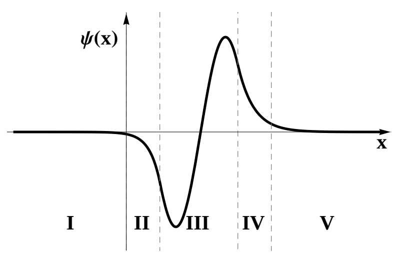
A


ψ(x)
x
I
II
III
IV
V

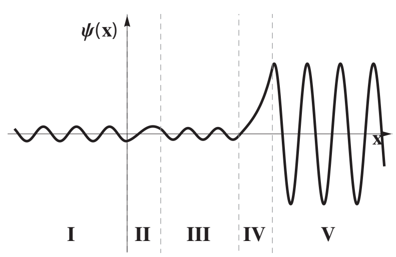
B


ψ(x)
x
I
II
III
IV
V

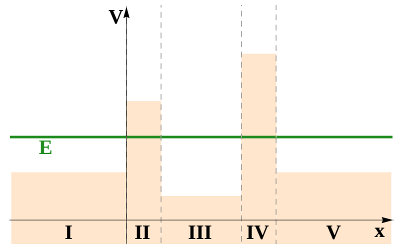
C


ψ(x)
x
I
II
III
IV
V

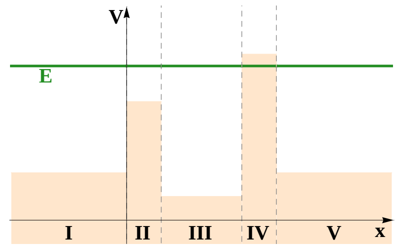
D


ψ(x)
x
I
II
III
IV
V

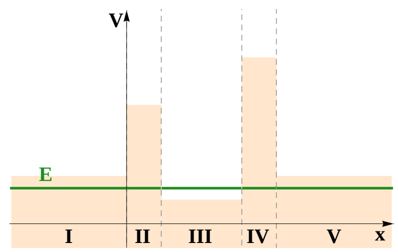
E


V
E
I
II
III
IV
V
x

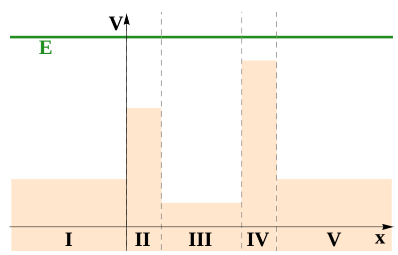
1


V
E
I
II
III
IV
V
x

2


V
E
I
II
III
IV
V
x

3


V
E
I
II
III
IV
V
x

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