

22.01 Fall 2016, Quiz 2 Study Sheet

November 17, 2016

Familiarize yourself with all these concepts from an *intuitive* and a *quantitative* point of view. Keep in mind that the material we covered in class will be particularly relevant. ***When in doubt, refer to the pictures of blackboards from class*** so you know what is most important. ***If you can understand all the material on the blackboards, then you are ready.***

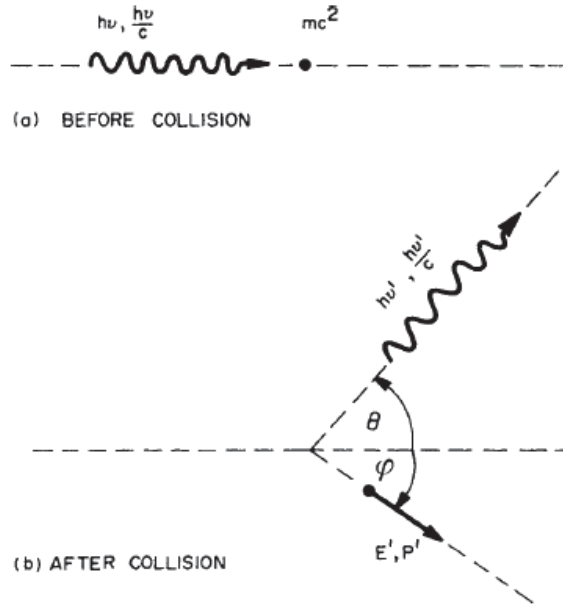
General Tips for Success on the Exam

1. The exam will have a few short answer problems, which can be answered with one sentence, one equation, and/or one graph. It will also have two long-answer problems with deeper intuition and analytical calculations.
2. Any calculations required will be order of magnitude only, no calculator is required.
3. You may bring ***one double-sided 8.5" x 11" sheet of paper*** with anything you want printed/written on it.
 - (a) No other materials are allowed, including notes, calculators, phones, anything.
4. Don't worry if you can't finish all the problems. Generous partial credit will be given for explaining your methodology, or completing parts of the problems.

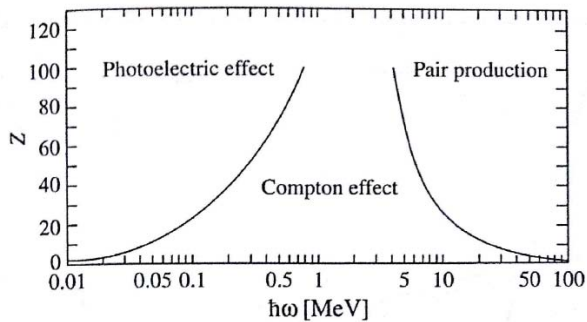
1 Photon Interactions with Matter

- Three main mechanisms of interaction

- Photoelectric Effect - *Ejection of an electron by photon absorption, and transfer of most of the energy to the electron*
- Compton Scattering - *Scattering of a photon by an electron, causing ionization and energy transfer to the electron*



- Pair Production - *Spontaneous creation of a positron-electron pair from a photon interacting with the Coulomb field of the nucleus*
- **Know where each mechanism is most dominant, and why from their cross sections:**



- Mechanisms and Energetics

- Photoelectric Effect

- * Photon is absorbed by an electron, causes an ejection
- * $T_{e^-} = \hbar\omega - E_{binding}$
- * Usually an inner shell electron
 - Level 1 electrons about 80% of the time
- * $\sigma_{\tau} \propto Z^5 \left(\frac{1}{\alpha}\right)^{\frac{7}{2}}$
 - Strongly more likely with increasing atomic number

- Strongly less likely with increasing photon energy
- Compton Scattering
 - * Photon gets scattered by an electron, leaves at a lower energy and at a different angle
 - * Key relationships: See Equations 10.6 - 10.8 in Yip (also 8.13 - 8.24 in Turner)
 - Equation 10.6 - Shows the *increase* in photon wavelength

$$\Delta\lambda = \frac{h}{m_e c} (1 - \cos\theta) \quad (1)$$

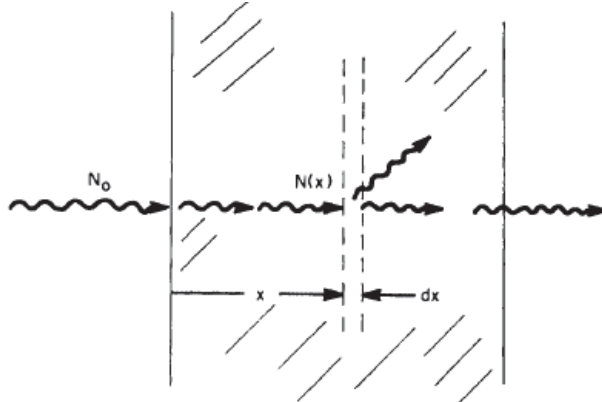
- Equation 10.8 - Gives the energy of the Compton electron

$$T = h\nu - h\nu' = h\nu \frac{1 - \cos\theta}{\frac{m_e c^2}{h\nu} + 1 - \cos\theta} \quad (2)$$

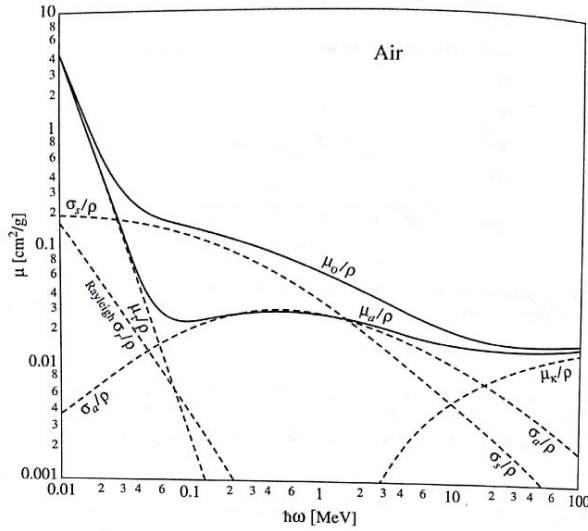
- * σ_C : See Equation 8.28 (Klein-Nishina cross section)
 - **Know what this means in terms of forward scattering bias**
- * It is the Compton *electrons* that are counted in detectors, along with any other ionized electrons from any process
- Pair Production
 - * Photon above 1.022MeV gets near the Coulomb field of the nucleus
 - * Spontaneously changes into an electron/positron pair
 - * Electron energy is counted by an ionization cascade
 - * Positron moves some distance in a material, annihilates with another electron
 - * Gives off two 511keV gammas in opposite directions
 - * Single escape peak - One gets out of the detector, one is reabsorbed (medium-sized detectors)
 - * Double escape peak - Both get out of the detector (small-sized detectors)
 - * No escape peaks - All photons get absorbed (large detectors)
 - * 511keV peak - Pair production happened somewhere *outside* the detector, then a 511keV photon enters the detector and undergoes photoelectric or Compton scattering
- See the Yip book, Equations 10.43 - 10.45 for cross section comparisons.
- Mass attenuation coefficients - density-normalized photon interaction probabilities. **Note that an attenuation coefficient (μ) is a macroscopic cross section! Check the units!!! They're both in cm^{-1} :**

$$I = I_0 e^{-\left(\frac{\mu}{\rho}\right)\rho x} = I_0 e^{-\Sigma x} \quad (3)$$

- Attenuation coefficients (μ) and mass attenuation coefficients ($\frac{\mu}{\rho}$) denote removal of photons from a tightly collimated beam by any means:

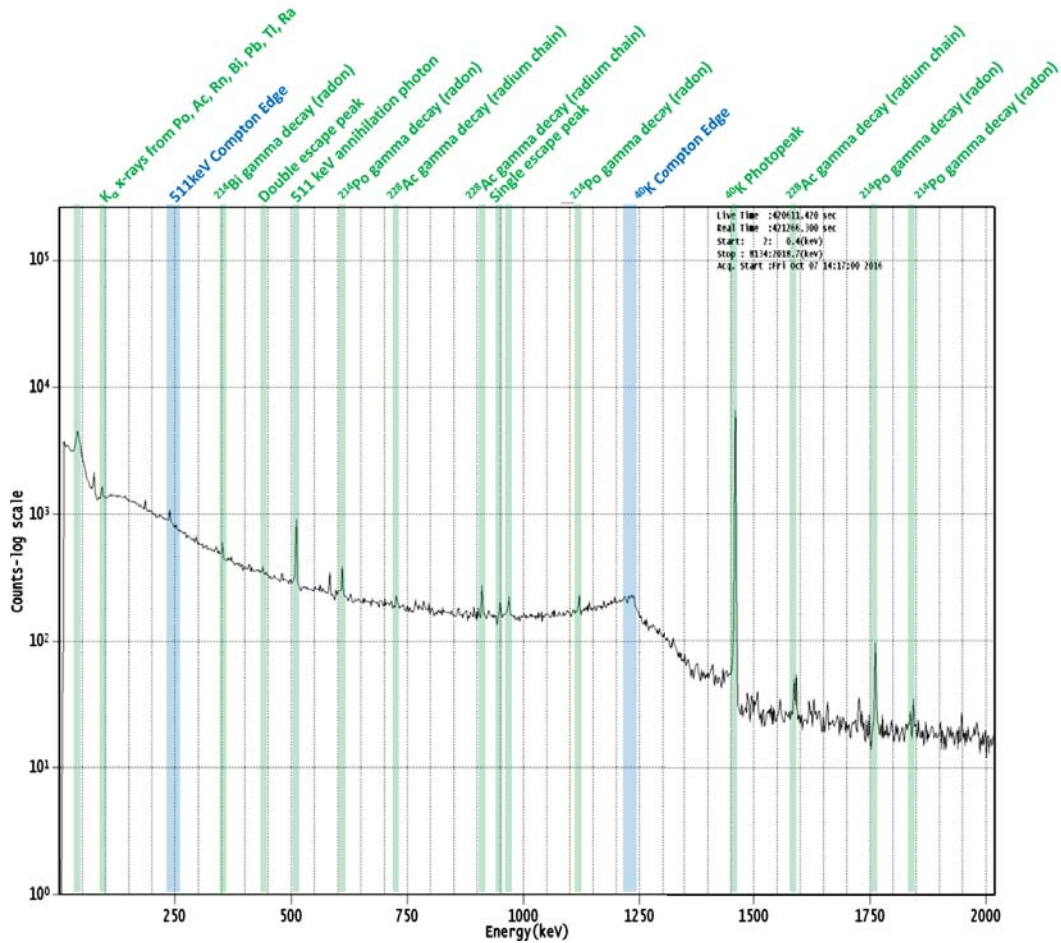


- Remember that the total mass attenuation coefficient is the sum of those from the different interactions:



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- **Know how to interpret photon spectra from detectors** - see our banana spectrum for an example. If we gave you a spectrum like this, you should be able to guess where each peak comes from, and which ones may be missing:



- Understand why different elements have differing mass attenuation coefficients from

their cross sections. In other words, be able to explain all the differences between elements in this graph:

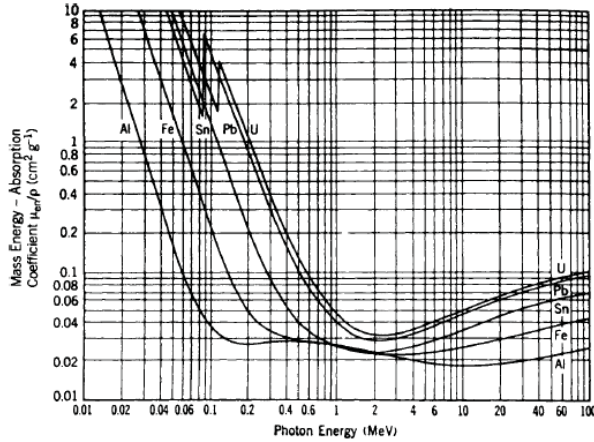


Fig. 8.11 Mass energy-absorption coefficients for various elements. [Reprinted with permission from K. Z. Morgan and J. E. Turner, eds., *Principles of Radiation Protection*, Wiley, New York (1967). Copyright 1967 by John Wiley & Sons.]

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- Ion/Electron Interactions

- Four methods: elastic & inelastic collisions, with other electrons and with nuclei
 - * Elastic collisions with electrons are the main mechanism of energy loss
- Derived using the hollow cylinder approach (see Turner pp. 117-119) as a Coulomb force balance & integration
- $p_{e^-} = \int F_y(t) dt = m \int a(t) dt = mv$
- Stopping power formula: $-\frac{dT}{dx} = \frac{4\pi k_0^2 N Z_1^2 Z_2 e_c^4}{m_e v^2} \ln\left(\frac{2m_e v^2}{I}\right) = \frac{4\pi k_0^2 N Z_1^2 Z_2 e_c^4}{E_i} \frac{M}{m_e} \ln\left(\frac{\gamma_e E_i}{I}\right)$; $\gamma_s = \frac{4m_e M}{(M+m_e)^2}$
- Relativistic correction: $-\frac{dT}{dx} = \frac{4\pi k_0^2 N Z_1^2 Z_2 e_c^4}{m_e v^2} \left[\ln\left(\frac{2m_e v^2}{I}\right) - \ln(1-\beta^2) - \beta^2 \right]$
- Stopping power curve shape: follows 1/E times ln(E), see & understand Figure 11.4
- Number of ion pairs produced: $i = \frac{1}{W} \left(-\frac{dT}{dx}\right)$
- Cross section comparison - see Equations 11.18 - 11.21 (p. 233)
- Know when ionization loss is high (low energy) and radiation loss is high (very high energy). Also note why ionization stopping power decreases at very low energies (charge neutraliation) and nuclear stopping power increases:

$$-\frac{dT}{dx}_{ioniz.} = \frac{4\pi N k_0^2 z^2 Z e^4}{m_e c^2 \beta^2} \ln \left[\frac{2m_e c^2 \beta^2}{I(1-\beta^2)} - \beta^2 \right]; \quad k_0 = 8.99 \cdot 10^9 \frac{N - m^2}{C^2} \quad (4)$$

$$\frac{-\left(\frac{dT}{dx}\right)_{ioniz.}}{-\left(\frac{dT}{dx}\right)_{nucl.}} = \frac{2M}{m_e Z} \frac{\ln\left(\frac{\gamma_e E_i}{I}\right)}{\ln\left(\frac{\gamma_e E_i}{E_d}\right)}; \quad \gamma = \frac{4mM}{(m+M)^2} \quad (5)$$

$$\frac{-\left(\frac{dT}{dx}\right)_{rad.}}{-\left(\frac{dT}{dx}\right)_{ioniz.}} = \left(\frac{m_e}{M}\right)^2 \frac{Z E_i}{1400 m_e c^2} \quad (6)$$

- Range: When the ions come to rest, and all energy is lost to stopping power:

$$Range = \int_0^{E_i} -\frac{dT}{dx}^{-1} \propto E_i^2 \quad (7)$$

- Neutron interactions

- No charge, almost all nuclear elastic collisions, absorption, fission, capture
 - * Never forget the Q-equation! See Yip p. 143 to refresh your memory
 - * Assumptions:
 - Q=0 (elastic scattering)
 - M1=M3=1 amu, M2=M4=A amu
 - A neutron can lose at most a fraction of $(1 - \alpha)$ of its energy in an elastic collision, $\alpha = \left(\frac{A-1}{A+1}\right)^2$
 - We assume that scattering is isotropic in angle, this is less true with increasing energy
- Inelastic Scattering
 - * One neutron goes in, *compound nucleus* formed, different neutron comes out
 - * See p. 244 for energy level diagram, p. 246 for diagram with resonance with nuclear energy levels
 - * Often $Q < 0$, cross section is zero until $E=Q$
- Fission
 - * Also a compound nucleus formation, but this time the “liquid drop” splits into two uneven sized droplets
 - * First, the neutron rich fission products shed 1-2 neutrons each
 - * Then, beta decay reduces their asymmetry further
 - * See p. 255 (Figure 12.11) for a time diagram of what happens

- Neutron Transport

- Balance of neutron gains & losses from a location (dV), in a certain energy range (dE), going in a certain solid angle direction (dΩ)
- Gains
 - * Fission
 - * External sources
 - * Scattering down into our energy group
- Losses
 - * Scattering down out of our energy group
 - * Absorption
 - * Leakage
- Terminology
 - * $\phi(E)$: Angular flux, $\Phi(E)$: Total flux
 - * $\Sigma_t = \Sigma_a + \Sigma_s$ (total = absorption + fission)
 - * $\Sigma_a = \Sigma_f + \Sigma_\gamma$ (absorption = fission + capture)
 - * $\Sigma_s = \Sigma_{s,el} + \Sigma_{s,non-el}$ (scattering = elastic + inelastic)
 - * E is our energy, E' is some other energy
 - * Ω is our angle, Ω' is some other angle
- Equational form:

$$\frac{dn(\mathbf{r}, E, \boldsymbol{\Omega}, t)}{dt} = \frac{\nu\chi(E)}{4\pi} \int_V \int_E \int_{\Omega'} d^3r dE' d\Omega' \Sigma_f(E') \phi(\mathbf{r}, E', \boldsymbol{\Omega}', t) \quad (8)$$

$$\begin{aligned}
& +S_0(\mathbf{r}, E, \boldsymbol{\Omega}, t) + \int_V \int_E \int_{\Omega'} d^3r dE d\Omega \Sigma_s(E') \phi(\mathbf{r}, E, \boldsymbol{\Omega}', t) F(E' \rightarrow E, \Omega' \rightarrow \Omega) \\
& - \int_V d^3r dE d\Omega \Sigma_t(E') \phi(\mathbf{r}, E, \boldsymbol{\Omega}, t) - \int_V d^3r dE d\Omega \boldsymbol{\Omega} \cdot \nabla \phi(\mathbf{r}, E, \boldsymbol{\Omega}, t)
\end{aligned}$$

- Terms: change = fission + external + scattering in - all collisions - flow outwards
- Simplifications:

* make everything isotropic (eliminate angular dependence)

$$\begin{aligned}
\frac{d\Phi(\mathbf{r}, E, t)}{dt} &= \nu\chi(E) \int_V \int_E d^3r dE' \Sigma_f(E') \Phi(\mathbf{r}, E', t) \\
& +S_0(\mathbf{r}, E, t) + \int_V \int_E d^3r dE \Sigma_s(E') \Phi(\mathbf{r}, E, t) F(E' \rightarrow E) \\
& - \int_V d^3r dE \Sigma_t(E') \Phi(\mathbf{r}, E, t) - \int_S d^3r dE (J \cdot \hat{n})
\end{aligned} \tag{9}$$

* Steady state, use divergence theorem to change surface integral to volume integral:

$$\begin{aligned}
0 &= \nu\chi(E) \int_V \int_E d^3r dE' \Sigma_f(E') \Phi(\mathbf{r}, E') \\
& +S_0(\mathbf{r}, E) + \int_V \int_E d^3r dE \Sigma_s(E') \Phi(\mathbf{r}, E) F(E' \rightarrow E) \\
& - \int_V d^3r dE \Sigma_t(E') \Phi(\mathbf{r}, E) - \int_V d^3r dE \nabla (J \cdot \hat{n})
\end{aligned} \tag{10}$$

* Homogeneous reactor, any volume element is the same (eliminate r-dependence)

$$\begin{aligned}
0 &= \nu\chi(E) \int_E dE' \Sigma_f(E') \Phi(E') \\
& +S_0(E) + \int_E dE \Sigma_s(E') \Phi(E) F(E' \rightarrow E) \\
& - \int_E dE \Sigma_t(E') \Phi(E) - \int_E dE \nabla J(E)
\end{aligned} \tag{11}$$

- Big simplification: Assume all neutrons are in one energy group, homogeneous (perfectly mixed) reactor with one average material, forget the external source

Combine ($\Sigma_s - \Sigma_t = -\Sigma_a$)

Substitute Fick's law for gradient of current: $\nabla J = -D\nabla^2\Phi$; $J = -D\nabla\Phi$ (Note that diffusion happens *down* a concentration gradient, hence the minus sign):

$$0 = \nu\bar{\Sigma}_f\Phi - \bar{\Sigma}_a\Phi + \bar{D}\nabla^2\Phi \tag{12}$$

- where each cross section for each *isotope* and for each *reaction* needs to be taken as a *flux-averaged value*:

$$\bar{\Sigma} = \frac{\int_0^{E_{max}} \Sigma(E) \Phi(E) dE}{\int_0^{E_{max}} \Phi(E) dE} \tag{13}$$

- New equation: gains = losses, introduce k_{eff} as the ratio of gains to losses:

$$\bar{\Sigma}_a \Phi - \nabla \bar{D} \cdot \nabla \Phi = \frac{\bar{\nu} \bar{\Sigma}_f \Phi}{k_{eff}} \quad (14)$$

- Solution takes form of $\frac{-\nabla^2 \Phi}{\Phi} = \text{constants}$; *sin & cos*
- Eliminate sin due to symmetry concerns, take solution to be $\Phi = A \cos Bx$ for a 1D infinite slab reactor
- Solution is $\frac{-\nabla^2 \Phi}{\Phi} = B^2$; B is the “buckling”
 - * Material buckling & criticality:

$$\Sigma_a \Phi - \nabla D \cdot \nabla \Phi = \frac{\nu \Sigma_f \Phi}{k_{eff}} \implies \Sigma_a \Phi - DB^2 \Phi = \frac{\nu \Sigma_f \Phi}{k_{eff}} \implies k_{eff} = \frac{\nu \Sigma_f}{\Sigma_a + DB^2} \quad (15)$$

- * k_{eff} is a balance between neutron production & destruction
- * If $k_{eff} = 1$, the reactor is critical (perfect balance between production & loss)
- * Then, using Equation 14, we can give a criticality condition based on the materials in a reactor:

$$B_{material}^2 = \frac{\nu \Sigma_f - \Sigma_a}{D} \quad (16)$$

- * and its geometry:

$$\frac{-\nabla^2 \Phi}{\Phi} = B_{geometry}^2; \quad \Phi = A \cos(B_g x) \quad (17)$$

- * The reactor is perfectly critical when

$$B_g^2 = B_m^2 \quad (18)$$

- Writing criticality conditions for multigroup neutronics problems: For each neutron group, write down the gains & losses. Example for the fast/thermal/ultracold neutronics problem (three energy groups):

$$\frac{1}{k_{eff}} [\bar{\nu}_{fa} \bar{\Sigma}_{f,fa} \phi_{fa} + \bar{\nu}_{th} \bar{\Sigma}_{f,th} \phi_{th} + \bar{\nu}_{uc} \bar{\Sigma}_{f,uc} \phi_{uc}] = [\bar{\Sigma}_{a,fa} + \bar{\Sigma}_{s,fa \rightarrow th} + \bar{\Sigma}_{s,fa \rightarrow uc}] \phi_{fa} - \bar{D}_{fa} B_g^2 \phi_{fa} \quad (19)$$

$$\bar{\Sigma}_{s,fa \rightarrow th} \phi_{fa} + \bar{\Sigma}_{s,uc \rightarrow th} \phi_{uc} = [\bar{\Sigma}_{a,th} + \bar{\Sigma}_{s,th \rightarrow uc}] \phi_{th} - \bar{D}_{th} B_g^2 \phi_{th} \quad (20)$$

$$\bar{\Sigma}_{s,fa \rightarrow uc} \phi_{fa} + \bar{\Sigma}_{s,th \rightarrow uc} \phi_{th} = [\bar{\Sigma}_{a,uc} + \bar{\Sigma}_{s,uc \rightarrow th}] \phi_{uc} - \bar{D}_{uc} B_g^2 \phi_{uc} \quad (21)$$

- Note that k_{eff} is placed only on the gain terms of neutrons for the *whole system*, as scattering only represents a change of energy.
- We also considered *upscattering* here, which is not usually a physical process that we care about.

- Neutronics Transients

- A small fraction β of neutrons are born *delayed*, with lifetimes of 0.2-54 seconds. $\beta = 0.0064$ for U-235.
- These neutrons are what stop reactors from going *prompt-supercritical*, reducing reactor periods (time it takes for power to increase by e) from 0.001s to 100s.
- See the blackboard from Nov. 10 for all important notes regarding flux time-dependence

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Fall 2016

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