22.01 Fall 2016, Problem Set 8 Solutions

December 19, 2016

Complete all the assigned problems, and do make sure to show your intermediate work.

Part I
Skill-Building Questions (50 points)

1 Conceptual Questions (3 points each)

1. Define the major short-term biological effects due to intense gamma radiation exposure, and explain their origins.
   During an intense gamma ray exposure, the following major things will happen: (1) Very sensitive cells (like stem cells) will die, leaking their fluids into the intercellular spaces, (2) the cascade of radiolysis products will be produced, causing much oxidative damage to the tissue. The follow-on effects take four phases, as described below:

Four Phases of Acute Radiation

- **Prodromal phase**
  - Initial symptoms of exposure, 1-3 days after or sooner
- **Latent phase**
  - Apparent recovery from prodromal symptoms, but lab tests show serious changes in blood & lymph
- **Manifest illness phase**
  - Specific signs of each syndrome appear depending on the dose
- **Final phase**
  - Recovery or death, takes months to years

Acute Radiation Symptoms

M. M. Garau, A. L. Calduch, E. C. Lopez. *Rep. Practical Oncology and Radiotherapy*, 16:123 (2011).

Table 1 – Signs and symptoms of prodromal phase.[4]

Signs and symptoms	Mild (1–2 Gy)	Moderate (2–4 Gy)	Severe (4–6 Gy)	Very severe (6–8 Gy)	Lethal (>8 Gy)
Vomiting	≥2 h after	1–2 h after	<1 h after	<30 min after	<10 min after
Onset	exposure	exposure	exposure	exposure	exposure
% of incidence	10–50	70–90	100	100	100
Diarrhea	None	None	Mild	Heavy	Heavy
Onset			3–8 h	1–3 h	Within min
% of incidence			<10	>10	100
Headache	Slight	Mild	Moderate	Severe	Severe
Onset			4–24 h	3–4 h	1–2 h
% of incidence			50	80	80–90
Consciousness	Unaffected	Unaffected	Unaffected	May be altered	Unconsciousnes
Onset					s/min
% of incidence					100 at >50 Gy
Body temperature	Normal	Increased	Fever	High fever	High fever
Onset		1–3 h	1–2 h	<1 h	<1 h
% of incidence		10–80	80–100	100	100

Dose correlates quite well to onset time and severity of symptoms

Acute Radiation Symptoms

M. M. Garau, A. L. Calduch, E. C. Lopez. *Rep. Practical Oncology and Radiotherapy*, 16:123 (2011).

Table 2 – Signs and symptoms of latent phase.[4]

Signs and symptoms	Mild (1–2 Gy)	Moderate (2–4 Gy)	Severe (4–6 Gy)	Very severe (6–8 Gy)	Lethal (>8 Gy)
Latency period	21–35 days	18–28 days	8–18 days	≤7 days	None
Lymphocytes G/L (days 3–6)	0.8–1.5	0.5–0.8	0.3–0.5	0.1–0.3	0.0–0.1
Granulocytes G/L	>2.0	1.5–2.0	1.0–1.5	≤0.5	≤0.1
Diarrhea	None	None	Rare	Appears on days 6–9	Appears on days 4–5
Depilation	None	Moderate, beginning on day 15 or later	Moderate, beginning on day 11–21	Complete earlier than day 11	Complete earlier than day 10

Table 3 – Signs and symptoms of critical phase.[4]

Signs and symptoms	Mild (1–2 Gy)	Moderate (2–4 Gy)	Severe (4–6 Gy)	Very severe (6–8 Gy)	Lethal (>8 Gy)
Onset of symptoms	>30 days	18–28 days	8–18 days	<7 days	<3 days
Clinical manifestations	Fatigue, weakness	Fever, infections, weakness, depilation	High fever, infections, bleeding, depilation	High fever, diarrhea, vomiting, dizziness, desorientation, hypotension	High fever, diarrhea, unconsciousness
Lymphocytes G/L (days 3–6)	0.8–1.5	0.5–0.8	0.3–0.5	0.1–0.3	0.0–0.1
Platelets G/L	60–100	30–60	25–35	15–25	<20
% of incidence	10–25	25–40	40–80	60–80	80–100
Lethality	0%	0–50%	20–70%	50–100%	100%
Onset time		6–8 week	4–8 week	1–2 week	1–2 week

22.01 – Intro to Ionizing Radiation Chem/Bio Effects Radiation, Slide 38

The loss of hair stems directly from killing the hair follicles, the gastrointestinal syndrome (diarrhea, lack of nutritional uptake) follows from the killing of stem cells in the villi of the intestine, the general infections and sepsis comes from killing the bone marrow which produces white blood cells, ditto for hemophilia and platelets, and the jury is still out on the vomiting (though it may have to do with chemical signals sent out during intestinal cell death).

2. Starting from the entry of a quantum of ionizing radiation into the body, explain, step by step, the most likely mechanism to induce mutations in a cell.
It is much more likely that indirect DNA damage will cause cell mutation, as there is much more water surrounding the DNA than actual DNA itself. First, the physical ionization of a water molecule would take place, followed by the chain of radiolysis products being produced. The G-Values tell how many of each oxidative species will remain after about a microsecond, which is the timescale between when the species are created, and they diffuse far enough away that they cease to inter-react. Then the oxidative species may encounter (diffuse to) DNA, where they can cause an ionization and a kink in the chain, changing a gene to either cause lack of cell reproduction or triggering cancer.

3. List some of the other, non-radioactive sources of the free radicals responsible for DNA damage and eventual cell mutations. Where do they come from?
Many of the same free radicals produced from radiation are present from the foods we eat, as well as oxidative metabolic by-products of the cell's own metabolism. So far, research shows that they are not unique to any particular source, including radiation.

4. You are given four cookies containing dangerous levels of a high-activity isotope. Each has the same half life, the same concentration, and therefore the same activity. One emits alpha particles, one emits betas, one emits gammas, and one emits neutrons, all at the same energies. You must put one in your pocket, hold one in your hand, eat one, and give one to a "friend." What do you do, and why? **Use your knowledge of stopping power, range, and relative biological effectiveness to explain. First of all, eat the gamma cookie, as its RBE is very low, and its range is extremely high,**

so most gammas will travel right through you. Give the neutron cookie to the "friend," as its range is decently high (˜cm) in water-based lifeforms, though it causes tons of damage. Put the alpha cookie in your pocket, as your pocket will block all the alpha particles, and hold the beta cookie out at arms length, to minimize the dose through the dead layer of skin.

5. Explain why cancerous tumors are relatively resistant to radiation compared to normal cells, making radiation therapy more difficult.
   There are two supposed reasons for this: (1) In the center of a tumor the cells are not dividing much and the environment is quite hypoxic, meaning that the cells aren't as susceptible to oxidative mutation, and that radiation is less effective in a reducing environment (the same is true in reactors!). (2) You only have to mutate one normal cell to induce a tumor, but you have to mutate/destroy *all* cancerous cells to stop it.

2 Analytical Questions (25 points)

For these questions, read the following article on The Demon Core accident, and this article detailing the resulting radiation effects suffered by the two workers exposed.

1. (5 points) Explain, step by step, what caused the high exposure to radiation.
   First, it should be noted that Daghlian's lack of adherence to basic safety protocols is what caused this to happen! Physically speaking, the plutonium sphere was a sub-critical mass on its own, but completely surrounded by WC (tungsten carbide), it could become supercritical. That is exactly what happened, Daghlian's slipping of the brick caused the neutron non-leakage terms in the six-factor formula to increase, raising the criticality of the sphere. Most of the neutrons from fission were reflected, and any alphas or betas didn't make it out of the sphere, so the dose incurred by Daghlian was mostly from gammas produced by fission and x-rays from ionization of the Pu and WC, and subsequent relaxation of excited electron states. Daghlian's quick thinking in moving the brick away demonstrated knowledge of ways to make the core more sub-critical.
   The next incident occurred when a similar experiment was being tried, but this time by using a screwdriver to tweak the amount of neutron leakage of the core by increasing or decreasing the physical space from which neutrons could escape without being reflected back in. When the screwdriver holding up the beryllium half-sphere held by Slotin slipped, the assembly went critical, and Slotin's removal of the top hemisphere stopped the reaction from continuing. The flash of blue light observed in Slotin's eyes would have been Cherenkov radiation, from a multitude of particles traveling faster than the speed of light in water.

2. (15 points) Calculate or find the total energy, dose, and equivalent dose absorbed by both Daghlian and Hemmerly in (a) Roentgen, (b) Rad, (c) Gray, (d) Rem, (e) Sieverts.
   Roentgen
   (Source: second article): Daghlian: 590 Roentgens, Hemmerly: 14.45 Roentgens. However, an answer of "Roentgens are inapplicable" is also valid, as Roentgens specifically measure the number of ionizations produced *in air*. It is relatively difficult to translate this directly into a soft tissue equivalent dose.
   Rad
   (Source: first article): Daghlian: 330 Rad, Hemmerly: 8.1 Rad
   Rem
   (Source: first article): Assume that gammas have a quality factor of 1, while the neutrons from fission were fast (1MeV) with a quality factor of 20. This gives effective doses of Daghlian: 4,110 Rem, Hemmerly: 160,1 Rem
   Sieverts
   (Source: first article): Assume that gammas have a quality factor of 1, while the neutrons from fission were fast (1MeV) with a quality factor of 20. This gives effective doses of

Daghlian: 41.1 Sv, Hemmerly: 1.601 Sv
Gray
(Source: first article): Daghlian: 3.3 Gy, Hemmerly: 0.081 Gy

3. (5 points) If the men's equivalent doses had been due to fast neutrons instead of gamma and x-rays, what would the dose absorbed in Gray been for each person?
If the full doses had been in neutrons, rather than a mix of neutrons and gammas, things would have been even worse for Daghlian in terms of equivalent dose. However, the energy in Roentgens would have been the same, as the dose in Roentgens is just a measure of the charge created (ionizations) per kg of weight, it does not take into account the relative effectiveness of each type of radiation. Were this question to be asked in terms of Sieverts, Daghlian would have have 66 Sv of fast neutron radiation, while Hemmerly would have had 1.62 Gy, a negligible increase.

3 Radiation in the Environment (5 points each)

1. List the five largest *natural* sources of background radiation from living in Cambridge, MA, and what percent of your yearly background dose they comprise. Give citations where appropriate.
The first largest source is always radon inhalation, and Cambridge, MA is in the highest of the three EPA zones at >4pCi/L of air[1]. It is estimated at about 2mSv per year, or a little over 50% of your yearly dose.
The next largest source is almost always internal radiation, from the K-40, C-14, and other isotopes in the body. It comprises about 0.4mSv per year, or around 11% of your yearly dose.
The third largest source is likely cosmic radiation, consisting mostly of the high energy gamma rays produced by neutral pion decay resulting from protons striking the upper atmosphere, and showers of particles from muon production in the atmosphere. It constitutes about 0.27mSv per year, or around 8% of your yearly dose.
The fourth largest source is radiation from the building materials around us, like wood (which contains much K-40), and brick & granite, which can contain plenty of uranium & radium, especially if sourced locally (Conway granite from New Hampshire is particularly radioactive). It constitutes about the same dose as cosmic radiation.
The fifth source doesn't occur for everyone. If you're a smoker (technically a natural source, as it's just the dust on the surfaces of tobacco leaves), your background dose per year can be as high as 100x[2] the normal background dose in Boston. If the person is a smoker, then the above sources get shifted down by one level in importance.

2. Estimate and explain the increase in background dose per hour from flying from Boston to Japan, over the North Pole. Use the BOS-NRT flight as your reference case, taking into account any increase in radiation from (1) altitude and (2) flying close to the north pole.
A direct flight from Boston to Japan takes 14 hours when traveling East to West. The Federal Aviation Administration (FAA) actually has this nifty radiation calculator[3] for exactly this purpose! Assuming the flight is from BOS (Logan International) to NRT (Narita International), and it takes 30 minutes each to reach and descend from a cruising altitude of 38,000 feet, a dose of 0.09239mSv will be incurred, or about 1/3rd of the normal *yearly* cosmic radiation dose!

[1] http://www.epa.gov/radon/find-information-about-local-radon-zones-and-radon-programs#stateradon
[2] http://www.rmeswi.com/36.html
[3] http://jag.cami.jccbi.gov/cariprofile.asp

Part II
Noodle Scratchers (50 points)

4 Radiation Resistance and Fevers (20 points, open-ended)

For these questions, you will calculate a few parameters related to radiation resistance and sensitivity by changing someone's body temperature. We will focus on two free radicals produced by radiation: hydrogen peroxide (H_2O_2) and the uncharged hydroxide group (OH). The first liberates free oxygen in water, while the other tears electrons from other molecules to form the more stable OH^- hydroxide ion.

1. Standard diffusion of species in liquids and solids follows the well known Arrhenius law:

$$D(T) = D_0 e^{\frac{-E_A}{kT}} \tag{1}$$

where $D(T)$ is the diffusion coefficient in $\left[\frac{m^2}{sec}\right]$, D_0 is the diffusion prefactor, E_A is the activation energy in eV for the species to move, k is Boltzmann's constant, and T is the temperature in Kelvin. Using the data for hydrogen peroxide from this article (see p. 558) and this article for hydroxide ion data (see Table 1), find the values of D_0 and E_A for each molecule's diffusion.

The following data were used for this calculation:

	D at 25C $\left(\frac{m^2}{s}\right)$	**D at 37C** $\left(\frac{m^2}{s}\right)$	**D at 40C** $\left(\frac{m^2}{s}\right)$
(OH)	$4.56 \cdot 10^{-9}$		$5.54 \cdot 10^{-9}$
(H_2O_2)	$1.43 \cdot 10^{-9}$	$1.83 \cdot 10^{-9}$	

These can be solved using a set of simultaneous equations:

$$D(T_1) = D_0 e^{\frac{-E_A}{kT_1}}; \qquad D(T_2) = D_0 e^{\frac{-E_A}{kT_2}} \tag{2}$$

Divide the two equations by each other to get:

$$\frac{D(T_1) = \cancel{D_0} e^{\frac{-E_A}{kT_1}}}{D(T_2) = \cancel{D_0} e^{\frac{-E_A}{kT_2}}} \tag{3}$$

Take the natural log of both sides:

$$ln\left(\frac{D(T_1)}{D(T_2)}\right) = E_A\left(\frac{1}{kT_2} - \frac{1}{kT_1}\right); \quad \frac{ln\left(\frac{D(T_1)}{D(T_2)}\right)}{\left(\frac{1}{kT_2} - \frac{1}{kT_1}\right)} = E_A \tag{4}$$

Using this equation, we can find the values of E_A, and plug into either equation to get values of D_0. The following data were obtained:

	D at 25C $\left(\frac{m^2}{s}\right)$	**D at 37C** $\left(\frac{m^2}{s}\right)$	**D at 40C** $\left(\frac{m^2}{s}\right)$	D_0 $\left(\frac{m^2}{s}\right)$	E_A (eV)
(OH)	$4.56 \cdot 10^{-9}$		$5.54 \cdot 10^{-9}$	$2.62 \cdot 10^{-7}$	**0.104**
(H_2O_2)	$1.43 \cdot 10^{-9}$	$1.83 \cdot 10^{-9}$		$5.12 \cdot 10^{-8}$	**0.089**

2. Calculate the distance that one of each molecule will travel in 10^{-6} seconds at body temperature ($37C$), or about the time that intracascade reactions stop.

For OH-, we need to calculate the diffusion coefficient at 37C using this Arrhenius relation, which we find to be $5.34 \cdot 10^{-9} \frac{\text{m}^2}{\text{sec}}$. The value for ($H_2O_2$) was found in the paper. Using the following equation from the notes:

$$\frac{\lambda^2}{6\tau} = D \tag{5}$$

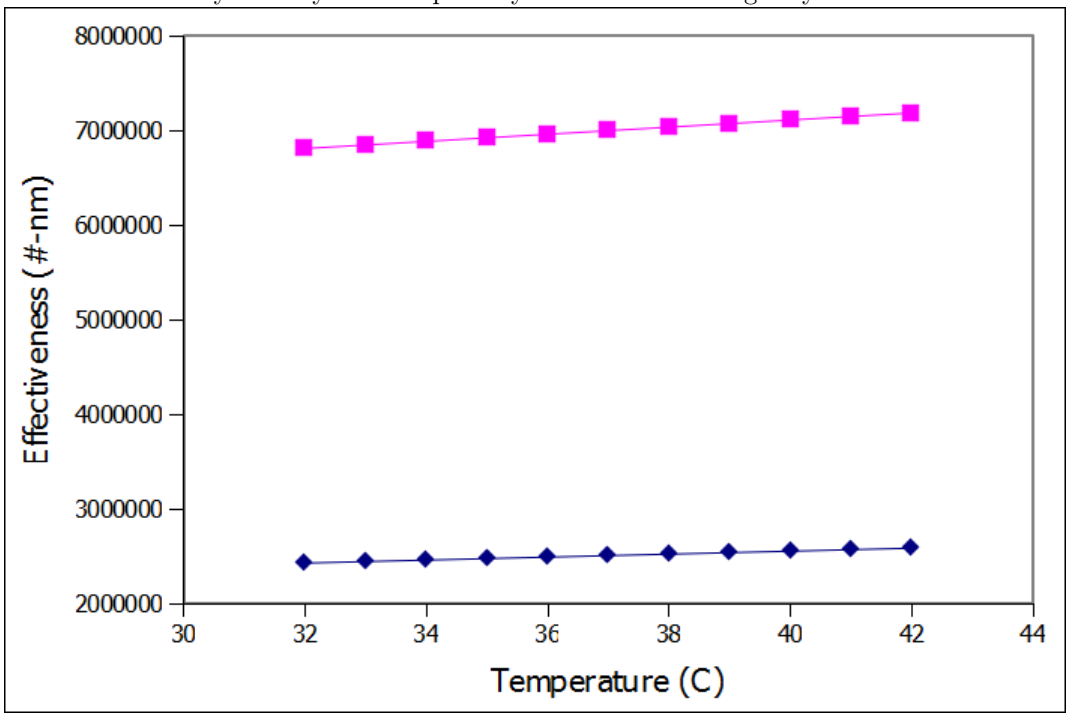
which relates the average distance traveled by a particle in a time τ to its diffusion coefficient, we find the following data:

	D, 25C $\left(\frac{m^2}{s}\right)$	**D, 37C** $\left(\frac{m^2}{s}\right)$	**D, 40C** $\left(\frac{m^2}{s}\right)$	D_0 $\left(\frac{m^2}{s}\right)$	E_A (eV)	λ, $\tau = 10^{-6}s$, $37C$
(OH)	$4.56 \cdot 10^{-9}$	$5.34 \cdot 10^{-9}$	$5.54 \cdot 10^{-9}$	$2.62 \cdot 10^{-7}$	**0.104**	**179 nm**
(H_2O_2)	$1.43 \cdot 10^{-9}$	$1.83 \cdot 10^{-9}$		$5.12 \cdot 10^{-8}$	**0.089**	**105 nm**

3. Suppose that someone has ingested al alpha emitter, perhaps by smoking, which releases 4MeV alpha particles. Which of the two molecules do you expect to do more damage to DNA? You should consider both the amount of each produced, as well as how far they can travel. Develop an expression for the "damage effectiveness" of each of these ions, based on your calculations.
Looking up the G-values for alpha particles from the notes, we get values of 0.35 (OH) per 100eV of energy for 4 MeV α, while the value for (H_2O_2) is 1.64. We can define a simple measure of damage effectiveness, or just the G-value, times the particle energy in units of 100 eV, times the distance each particle will travel as a measure of the average "reach" of each particle. By doing so, we get the following table:

	D, 25C $\left(\frac{m^2}{s}\right)$	**D, 37C**	**D, 40C**	D_0	E_A (eV)	λ	**G**	**#-nm**
(OH)	$4.56 \cdot 10^{-9}$	$5.34 \cdot 10^{-9}$	$5.54 \cdot 10^{-9}$	$2.62 \cdot 10^{-7}$	**0.104**	**179 nm**	**0.35**	**2,506,000**
(H_2O_2)	$1.43 \cdot 10^{-9}$	$1.83 \cdot 10^{-9}$		$5.12 \cdot 10^{-8}$	**0.089**	**105 nm**	**1.64**	**6,888,000**

4. Graph this "damage effectiveness" as a function of temperature from 32-42C (the range of body temperatures that won't ensure death). Does your answer to (4.3) change with changing temperature? What does this say about your susceptibility to radiation damage if you have a fever of 40C?



Yes, the answer does indeed change with temperature, suggesting that increasing one's body temperature may make one more susceptible to radiation damage of DNA by free radical diffusion.

5 Spooning (30 points, answer given)

Estimate the additional dose incurred by spooning (see Figure 1) compared to sleeping alone. Use your data from your EHS full body count in your answer.

Courtesy of Val D'Aquila on Flickr. License CC BY-NC-SA.

Figure 1: Spooning

Answer: *About 0.15μSv*

Solution: Let us first assume two rectangularly prismatic puppies. From our EHS full body count data, we can clearly see only one peak of significance, and it belongs to the gamma ray from ^{40}K. Here was mine:

From this, we know that only gamma rays are responsible, and they have energies of 1.460 MeV. We next make the following assumptions in solving this problem:

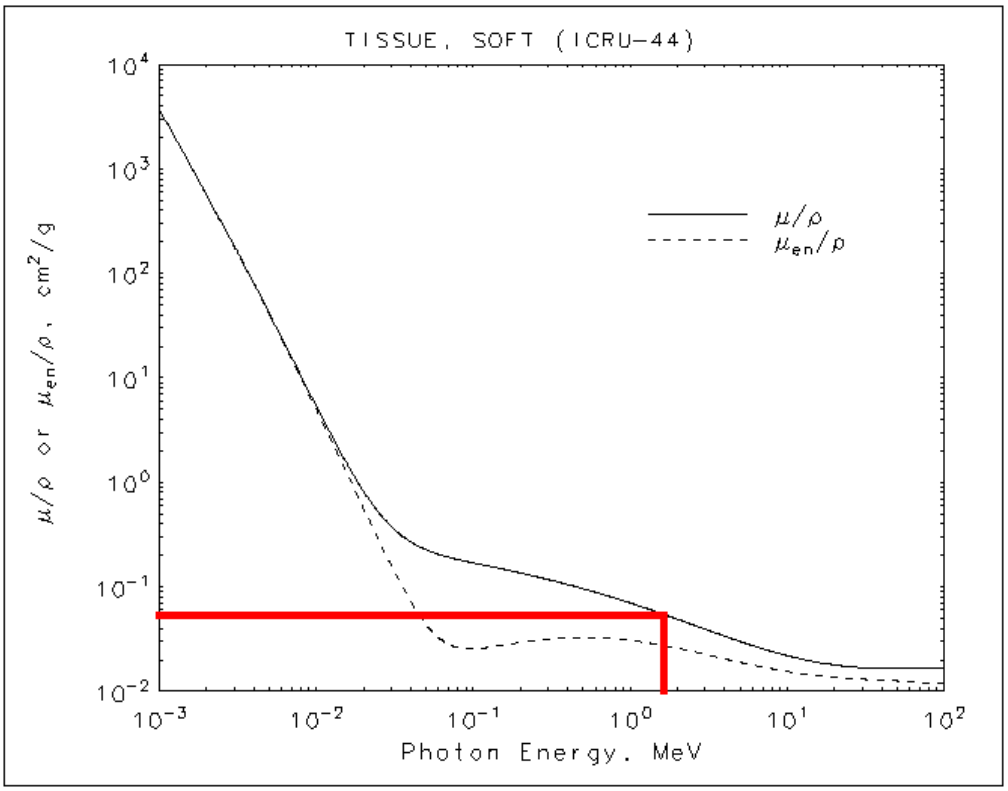
- Each slice of dog emits radiation isotropically, and the dogs are close enough together (touching) that 2π steradians worth of solid angle from each slice of dog 1 is subtended by dog 2.
- The mass attenuation coefficient of dog is approximated by that of standard soft tissue, and that each dog has a density of 1 $\frac{g}{cm^3}$, making $\left(\frac{\mu}{\rho}\right) = \mu$. This simplifies our calculations considerably.
- If Compton scattering occurs in dog 2, then the Compton scattered photon will be assumed to get absorbed in dog 2. This means that any gamma ray that interacts in dog 2 deposits all its energy. This gives us a slight overestimation of the dose to dog 2, which is OK.

In setting up the problem, let's think about the mechanisms of gamma ray interactions, and the physical processes that cause dog 2 to incur dose from dog 1:

- The energy absorbed in Gy *is equal to* the radiation dose in Sv, because gamma rays have a relative biological effectiveness (tissue quality factor) of unity, and gamma rays have a radiation quality factor of unity. This means that $Sv = (Q_{tissue})(Q_{radiation})\,Gy = (1)(1)\,Gy = Gy$.
- Exactly half the photons leaving dog 1 *move towards* dog 2, but ***self-shielding*** attenuates some of these photons before they can reach dog 2.
- Some of the photons that reach dog 2 will ***leak out the other side***, causing no radiation dose.
- The most important equation for us to use is that of exponential photon attenuation: $I(x) = I_0 e^{-\left(\frac{\mu}{\rho}\right)_{dog}\rho_{dog}x}$

Using the NIST table of x-ray attenuation coefficients[4], we find that at an energy of 1.460 MeV, (just to keep things in SI units):

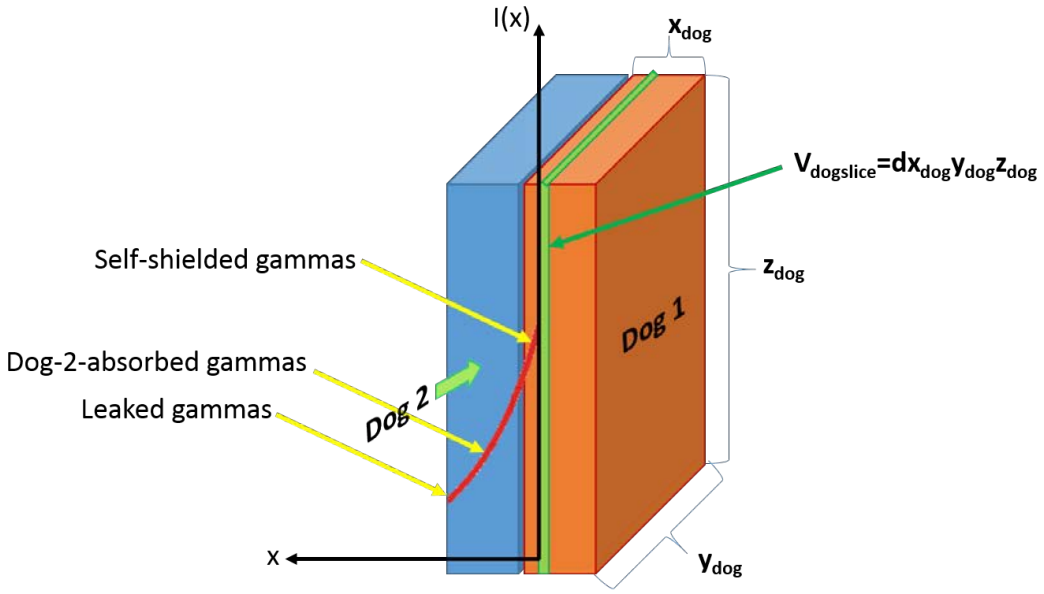
$\mu_{tissue} = \mu_{dog} = 0.05\,cm^{-1}$



Public domain, from U.S. NIST.

This simplifies our expression for exponential attenuation to $I(x) = I_0 e^{-5x}$. Next, let's make a visual description of the situation:

[4] http://physics.nist.gov/PhysRefData/XrayMassCoef/ComTab/tissue.html



Here we note three major features:

- Some gammas are *self-shielded*
- Many photons are *absorbed* by dog 2
- Some photons *leak*

Now let's put this all into math. If we choose an arbitrary slice of Dog 1, at any particular position x_i where $0 < x_i < x_{dog}$, we can write the total number of gammas absorbed by Dog 1 as follows:

$$\gamma_{absorbed} = \gamma_{total} - \gamma_{self-shielded} - \gamma_{leakage} \tag{6}$$

We can write the rate of gammas absorbed in Dog 2 from each slice of Dog 1 with a simpler expression, plugging in the starting and ending positions of Dog 1 relative to x_i:

$$\gamma_{rate-absorbed,slice} = I\left(x_i\right) - I\left(x_i + x_{dog}\right) = I_0\left[e^{-5x_i} - e^{-5(x_i+x_{dog})}\right] \tag{7}$$

Next we multiply by the volume of each differential dog slice, and by the solid angle of 2π steradians to change from intensity (I_0) to source specific gamma activity ($A_{specific}$):

$$\gamma_{absorbed,slice} = \left(\frac{2\pi}{4\pi}\right) V_{slice} A_{specific}\left[e^{-5x_i} - e^{-5(x_i+x_{dog})}\right] = 0.5 y_{dog} z_{dog} I_0\left[e^{-5x_i} - e^{-5(x_i+x_{dog})}\right] dx_i \tag{8}$$

Finally, we integrate over every differential dog slice to get the total gamma absorption rate by Dog 2:

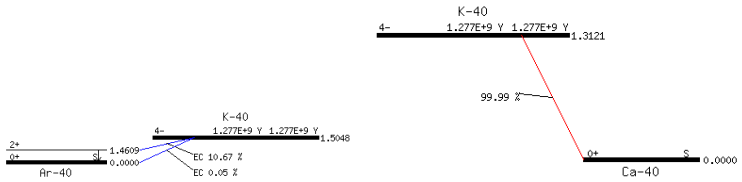
$$\gamma_{absorbed} = \int_0^{x_{dog}} 0.5 y_{dog} z_{dog} A_{specific}\left[e^{-5x_i} - e^{-5(x_i+x_{dog})}\right] dx_i \tag{9}$$

A quick bit of dimensional analysis shows that we have a specific activity $\left(\frac{\gamma}{m^3}\right)$ times a volume (dx*y*z) times a bunch of dimensionless exponentials, indeed yielding a gamma absorption rate. This is a good sign.

Let's assume typical dog dimensions of $x_{dog} \approx 0.2$m, $y_{dog} \approx 0.3m$, $z_{dog} \approx 1.5m$. Then we can look up the normal activity of potassium in the human body, either from Slide 62 of the class notes on Dose &

Dosimetry, or from outside references[5] to be about 4.4kBq. We also note from the decay diagram of ^{40}K that about one out of every ten disintegrations (Bq) results in a gamma ray, as an isomeric transition from electron capture (EC):

- Atomic Percent Abundance: 0.0117%
- Spin: 4-
- Half life: 1.277E+9 years
- Mode of decay: Beta to Ca-40
  - Branch ratio: 89.28 %
  - Decay energy: 1.311 MeV
- Mode of decay: Electron capture to Ar-40
  - Branch ratio: 10.72 %
  - Decay energy: 1.505 MeV



Using our estimated dog volume, that would yield a specific activity of about 4,889$\frac{\gamma}{m^3}$. We substitute this specific gamma activity in for A_{specific}:

$$\gamma_{absorbed} = \int_0^{0.2} 0.225\,(4,889)\left[e^{-5x_i} - e^{-5(x_i+0.2)}\right] dx_i \tag{10}$$

$$\gamma_{absorbed} = 1100 \int_0^{0.2} \left[e^{-5x_i} - e^{-5(x_i+0.2)}\right] dx_i \tag{11}$$

$$\gamma_{absorbed} = 1100 \int_0^{0.2} \left[e^{-5x_i} - e^{-5x_i-1}\right] dx_i \tag{12}$$

$$\gamma_{absorbed} = 1100 \int_0^{0.2} \left[e^{-5x_i} - e^{-1}e^{-5x_i}\right] dx_i \tag{13}$$

$$\gamma_{absorbed} = 1100 \int_0^{0.2} \left[e^{-5x_i} - 0.368e^{-5x_i}\right] dx_i \tag{14}$$

$$\gamma_{absorbed} = 1100 \int_0^{0.2} 0.632e^{-5x_i} dx_i \tag{15}$$

$$\gamma_{absorbed} = 695.2 \int_0^{0.2} e^{-5x_i} dx_i \tag{16}$$

$$\gamma_{absorbed} = 695.2\left[\left[-0.2e^{-5x_i}\right]_{0.2} - \left[-0.2e^{-5x_i}\right]_0\right] \tag{17}$$

$$\gamma_{absorbed} = 695.2\left[-0.2e^{-1} - \left(-0.2e^{-0}\right)\right] \tag{18}$$

$$\gamma_{absorbed} = 695.2\left[0.2e^{-0} - 0.2e^{-1}\right] \tag{19}$$

$$\gamma_{absorbed} = 695.2\left[0.2 - 0.074\right] \tag{20}$$

$$\gamma_{absorbed} = 695.2\left[0.126\right] \tag{21}$$

$$\gamma_{absorbed} = 87.6\frac{\gamma}{sec} \tag{22}$$

Now, assuming one typically sleeps for 8 hours (28,800 seconds), and noting that each gamma ray has an energy of 1.460 MeV, we can calculate the energy absorption in $\frac{J}{kg}$, or Gy:

$$\frac{\left(87.6\frac{\gamma}{\cancel{sec}}\right)(28,800\,\cancel{sec})\left(1.460\frac{\cancel{MeV}}{\gamma}\right)\left(1.6\cdot 10^{-13}\frac{J}{\cancel{MeV}}\right)}{\left(0.2*0.3*1.5\cancel{m^3}\right)\left(1,000\frac{kg}{\cancel{m^3}}\right)} = 6.5nGy = 6.5nSv \tag{23}$$

This is about twenty times lower than the normal answer of $0.15\mu Sv$.

[5] http://www.physics.isu.edu/radinf/natural.htm

22.01 Introduction to Nuclear Engineering and Ionizing Radiation

Fall 2016