Math and Feature Models of Assemblies

- Start of series of 5 classes on math/CAD models
  - basic matrix representations and Feature-based Design
  - constraint
  - variation
  - assembly sequence analysis
  - Datum Flow Chain

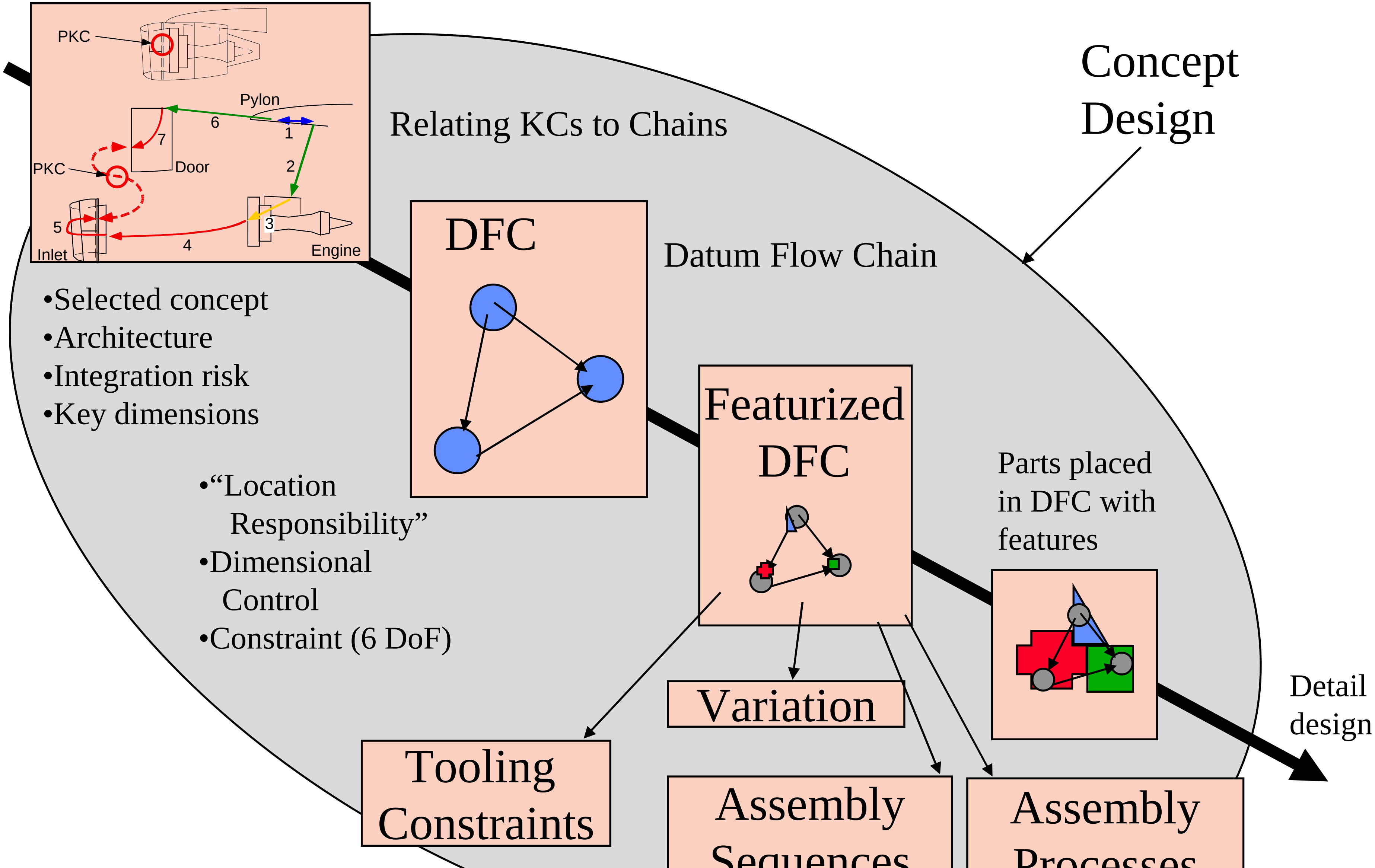
Objectives of Assembly Modeling

- Provide a computer environment that permits top-down design of assemblies with a persistent database that captures the assembly as an assembly
- Should link to geometry creation (CAD, rule-generated)
- Should permit specification of Key Characteristics, constraints on location, datums and locators, and variation analysis for KCs using the assembly model
- Should permit assembly planning, vendor interfaces, ramp-up, and production support

Top-down and Bottom-up Design

- Top-down defines an assembly in this order:
  - major customer deliverables
  - chains of delivery through possible parts
  - main part mates and necessary features
  - detailed part geometry
- Bottom-up defines
  - the same things but in the reverse order
  - requires having some idea of final assembly layout first
- Top-down used to be the only way before CAD
- CAD seems to encourage bottom-up

Sketch of Top-Down Assembly Design



Goals of this Class

- Review basic math that relates adjacent coordinate frames
- Model assemblies as chains of frames
- Attach these frames to "mating features"
- Introduce feature based design for assembly

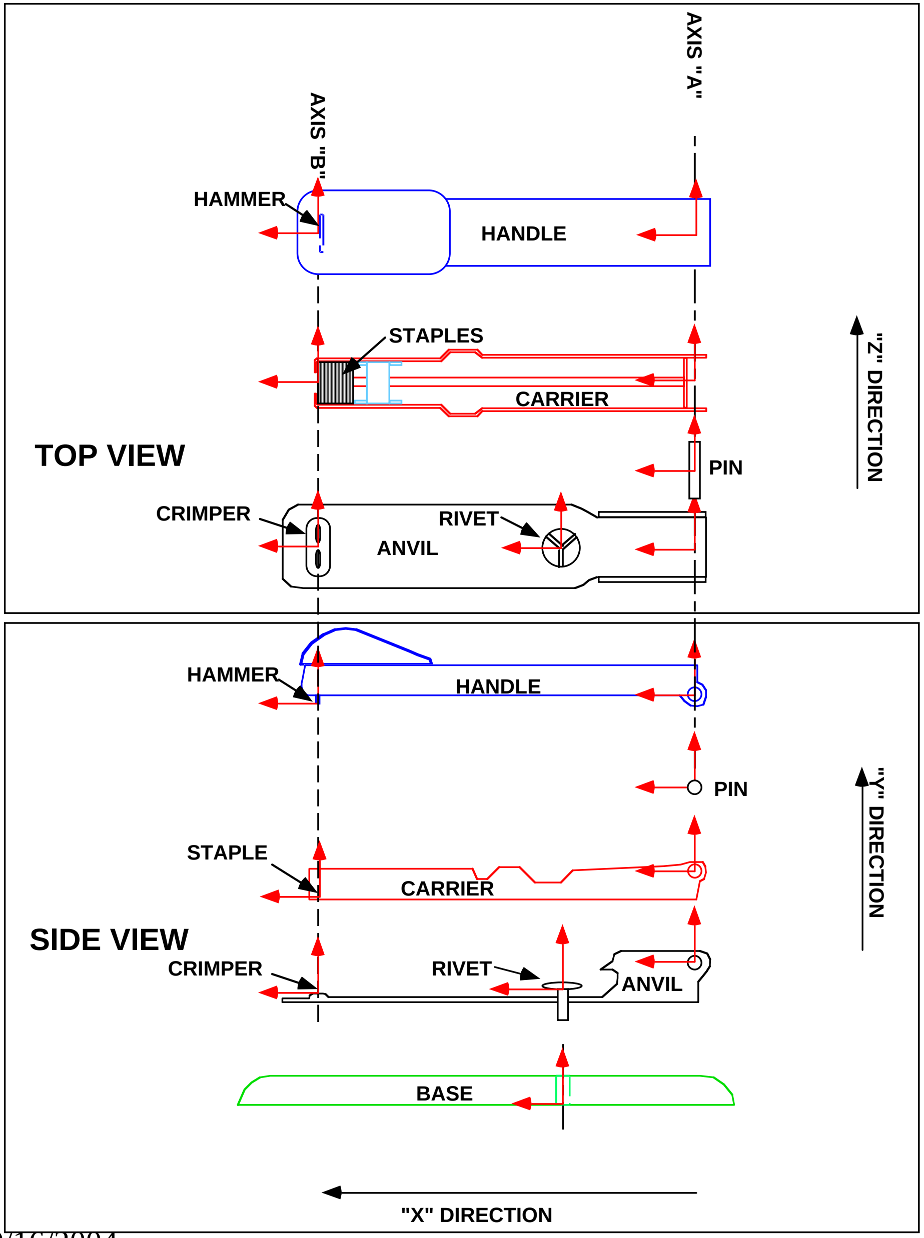
Basic Math

- Uses 4x4 matrices to relate adjacent frames
- Permits chaining together of parts
  - same math is used to describe robots
- The matrix contains a rotational part and a translational part
- The matrix is designed to translate first and then rotate so that rotation does not change position of new frame
- This matrix is a subset of a more general projection matrix that includes perspective

History of this Representation

- Basic to Kinematics (Denavit & Hartenberg)
- Used to model assemblies in 1970s:
  - S N Simunovic Master's Thesis, MIT, 1972
  - Edinburgh University AI Lab robot assembly 1976
- Used by CAD researchers
  - Steve Coons, 1960's
  - Gossard and others, 1980s
- Used by CAD systems to locate surfaces wrt each other

Stapler


AXIS "B"
AXIS "A"
HAMMER
HANDLE
STAPLES
CARRIER
TOP VIEW
PIN
CRIMPER
RIVET
ANVIL
"Z" DIRECTION
HAMMER
HANDLE
PIN
STAPLE
CARRIER
SIDE VIEW
CRIMPER
RIVET
ANVIL
BASE
"Y" DIRECTION
"X" DIRECTION

Stapler Frames and KCs

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Source:
Figure 3-5 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Frames and Chains

- By following the arrows, you can travel from frame to frame
- On the previous slide, the anvil was chosen as the origin part, and the anvil-pin joint on the anvil was chosen as the location of the origin frame.
- All arrows go out from the origin frame
- You can travel from one end of a KC to the other by moving from frame to frame along the arrows, sometimes in arrow direction and sometimes in reverse

Basic Translation and Rotation

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Source:
Figure 3-6 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Translate first, then rotate

Basic 4x4 Transform

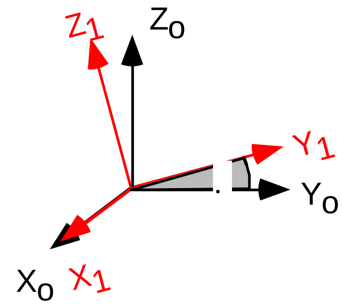
$$T = \begin{bmatrix} R & p \\ 0^T & 1 \end{bmatrix} \qquad T^{-1} = \begin{bmatrix} R^T & -R^T p \\ 0^T & 1 \end{bmatrix}$$

$$T = \left[\begin{array}{ccc|c} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

All the information about location (position and orientation) is inside the matrix

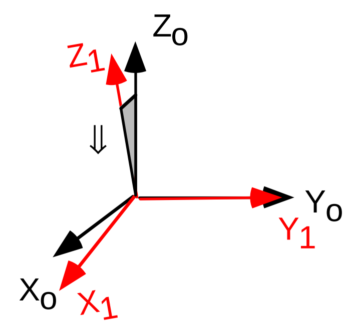
Basic Rotation Matrices

$$Rot(x,\theta)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos\theta & -\sin\theta & 0\\ 0 & \sin\theta & \cos\theta & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

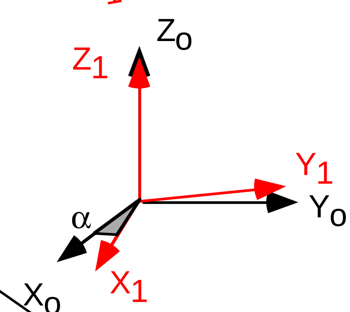


θ is in radians

$$Rot(y,\phi)=\begin{bmatrix}\cos\phi & 0 & \sin\phi & 0\\ 0 & 1 & 0 & 0\\ -\sin\phi & 0 & \cos\phi & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$



$$Rot(z,\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0 & 0\\ \sin\alpha & \cos\alpha & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$



No translation

Basic Translation Matrix

No rotation

$$trans(x,y,z) = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

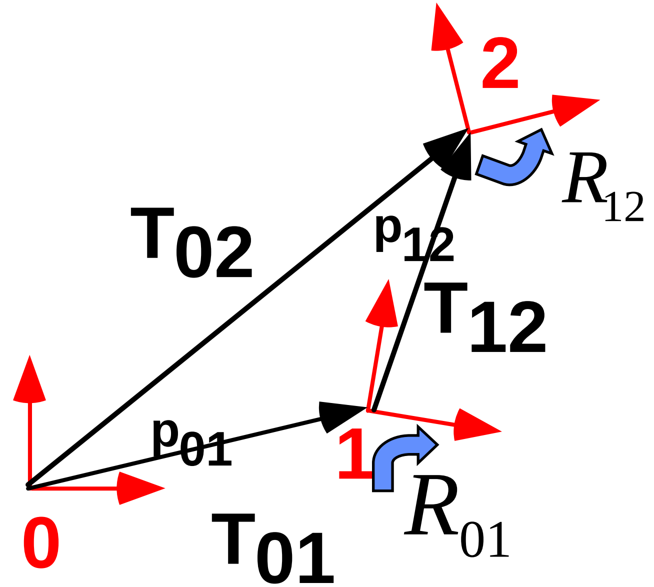
This and the three basic rotation matrices
are matlab .m files on MIT Server that you can use

Composite Transforms

$$T_{02} = T_{01}\,T_{12}$$

$$T_{02} = \begin{bmatrix} R_{01} & p_{01} \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} R_{12} & p_{12} \\ 0^T & 1 \end{bmatrix} =$$

$$\begin{bmatrix} R_{01}R_{12} & R_{01}\,p_{12}+p_{01} \\ 0^T & 1 \end{bmatrix}$$



T_{01} locates frame 1 in frame 0 coordinates
T_{12} locates frame 2 in frame 1 coordinates
T_{02} locates frame 2 in frame 0 coordinates

Transform Order is Important

$$T_{AB} = T_A T_B \cdot \quad T_{BA} = T_B T_A$$

More About Transform Order

T_{i+1} works on new axes created by T_i

$$T_{1-n} = T_{01} T_{12} \cdots T_{(n-1)-n}$$

Nominal Mating of Parts

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Source:
Figure 3-17 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Example

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Source:
Figure 3-21 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development.* New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

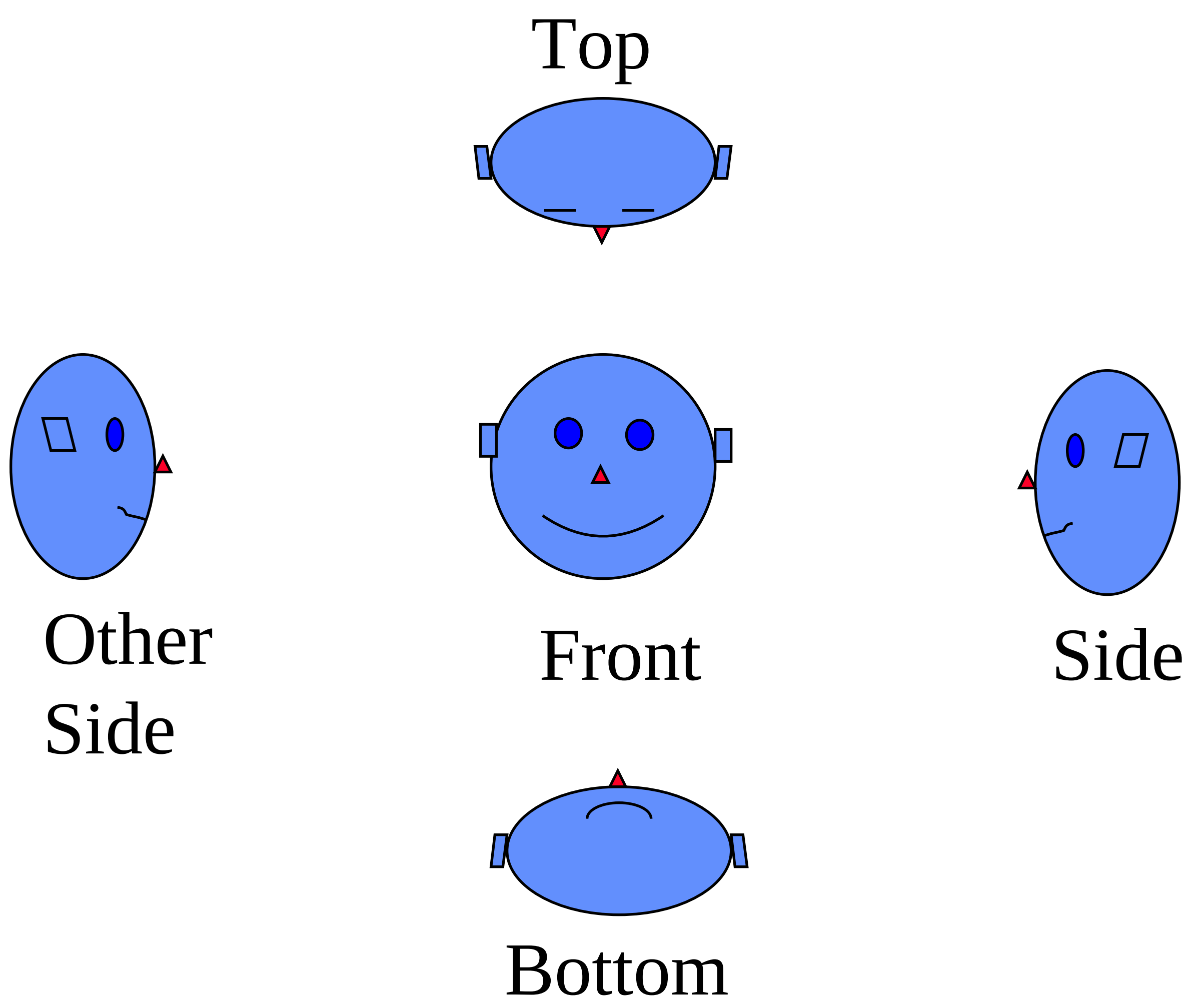
$$>> T_{AB} = trans(3,0,4)$$

$$T_{AB} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Coordinate Frames

MATLAB(TM) Code

Front, Top, and Side Views



Another Example

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Source:
Figure 3-22 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development.* New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

$$>> T_{AC} = T_{AB}\, roty\,(dtr\,(90))$$

$$T_{AC} = \begin{bmatrix} 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
function degtorad = dtr(theta)
% Converts degrees to radians
degtorad=theta*pi/180;
```

Example Feature on Part

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Source:
Figure 3-23 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development.* New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

$$>> T_{AD} = trans(3,2,4)\,roty(dtr(90))$$

$$T_{AD} = \begin{bmatrix} 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 2 \\ -1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Feature on Second Part

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Source:
Figure 3-24 in [Whitney 2004] Whitney, D. E.
Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development.
New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

$$>> T_{EF} = trans(6,0,1)$$

$$T_{EF} = \begin{Bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{Bmatrix}$$

Assembling These Parts

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Source:
Figure 3-25 in [Whitney 2004] Whitney, D. E.
Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development.
New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

$$>> T_{DE} = rotz\ (dtr\,(180))$$

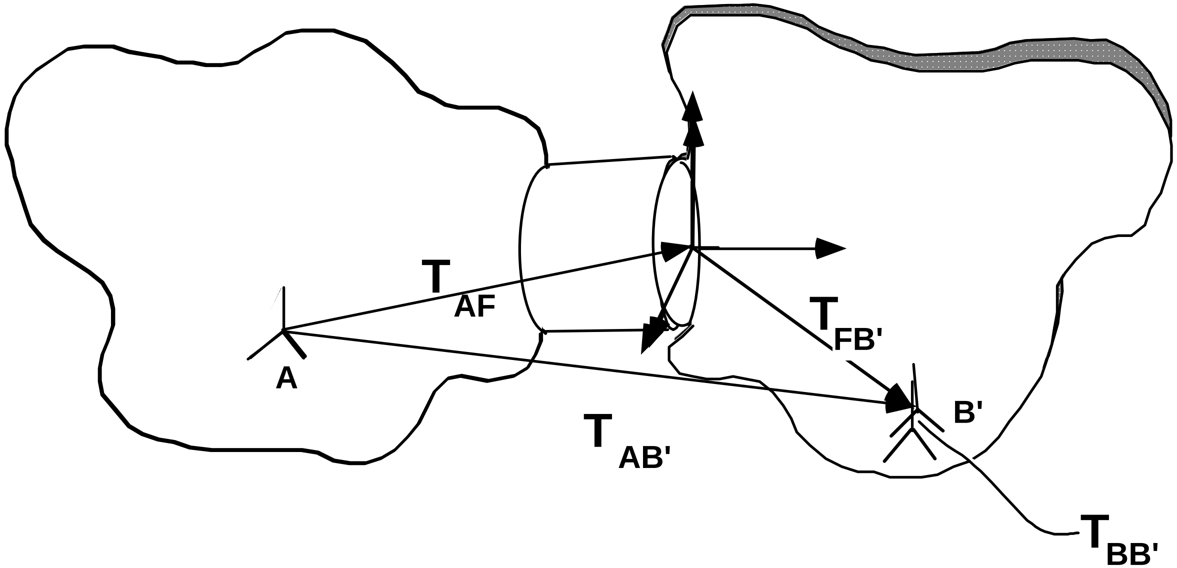
$$T_{DE} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$>> T_{AF} = T_{AD} T_{DE} T_{EF}$$

$$T_{AF} = \begin{bmatrix} 0 & 0 & 1 & 4 \\ 0 & -1 & 0 & 2 \\ 1 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4x4_examples copy

Varied Part Location Due to Tolerances



The varied location of Part B can be calculated from the nominal location of Part A. This process can be chained to Part C, etc., including errors on Part B. It uses the same math as the nominal model.

Equations for Connective Models

Nominal

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Source:
Figure 3-19 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Varied

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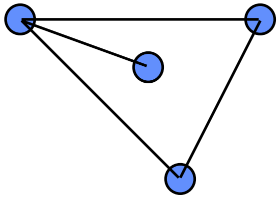
Source:
Figure 3-20 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

A Hierarchy of Assembly Models

- Bill of materials - lists the parts in no particular order
- Structured BOM - aka drawing tree - groups parts by sub-assembly
- Liaison graph - (Bourjault) parts are dots, joints are lines



- Ordered liaison graph - the lines have arrows
- Attributed liaison graph - the lines have constraint or feature information
- Ordered-attributed liaison graph (Datum Flow Chain)

Assembly Types Classified Technically

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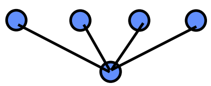
Source:
Figure 3-1 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

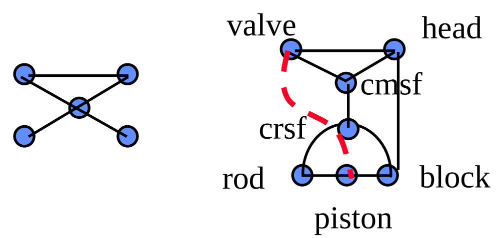
“Assembly Types” Classified by Liaison Diagram Form

- Hub and spokes



- Loop

- Network



- Stack

Inside a Car Engine

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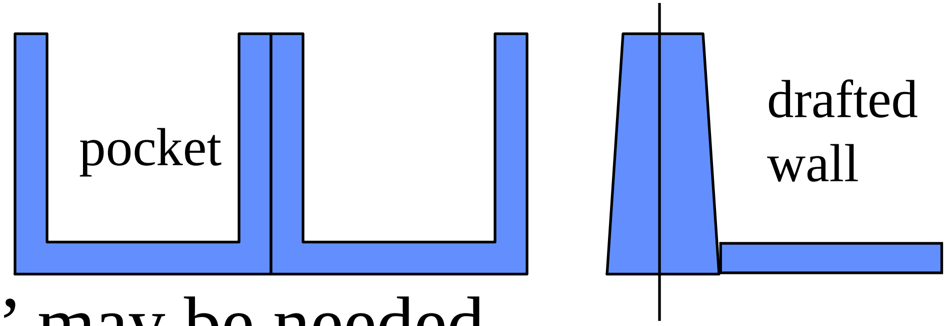
Source:
Figure 5-2 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Feature-based Design

- Seeks to rise above geometry and capture intent
- First efforts in machined features
  - slots
  - pockets
  - holes
- Features look different depending on how they are made
  - drafted walls if cast
  - pockets if cut



- “Feature recognition” may be needed

Assembly Features

Each feature has nominal geometry and a reference coordinate frame expressed as a 4x4 matrix. It also has a variety of other attributes as needed for its type.

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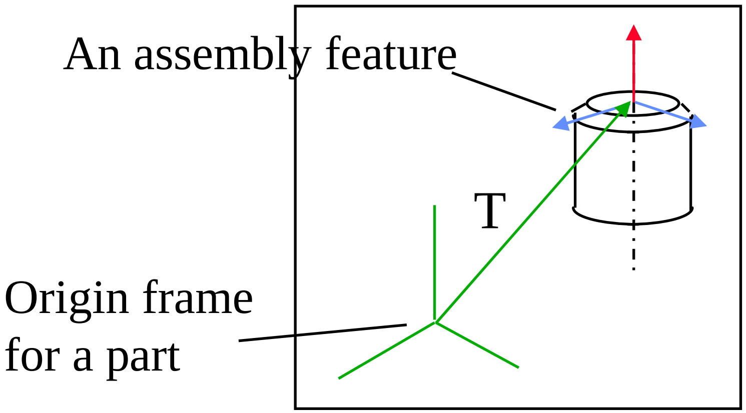
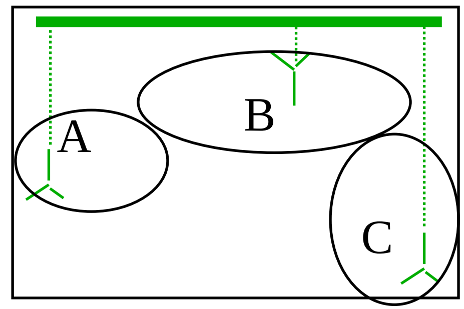
Source:
Figure 3-12 in [Whitney 2004] Whitney, D. E.
Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development.
New York, NY: Oxford University Press, 2004.
ISBN: 0195157826.

“Feature recognition” may not be needed.

Story: Feature recognition of Philips Head screws

Assembly Features and Assembly Data Models

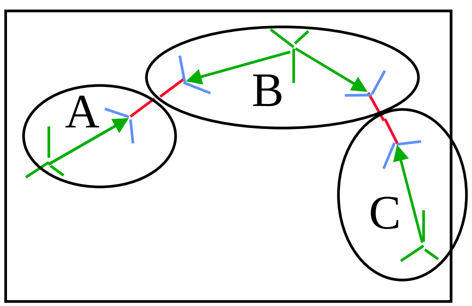




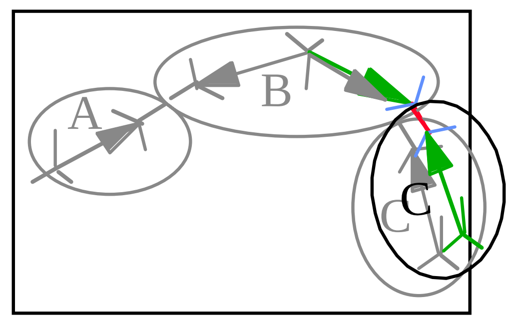
Validity of this model depends on shapes of parts being correct

Typical CAD Model
Based on World Coordinates

Validity of this model does not depend on part shapes being correct



An Assembly Model
Based on Connecting
Assembly Features



Assembly model with variation based on error in feature location in part B

A Really Bad Example

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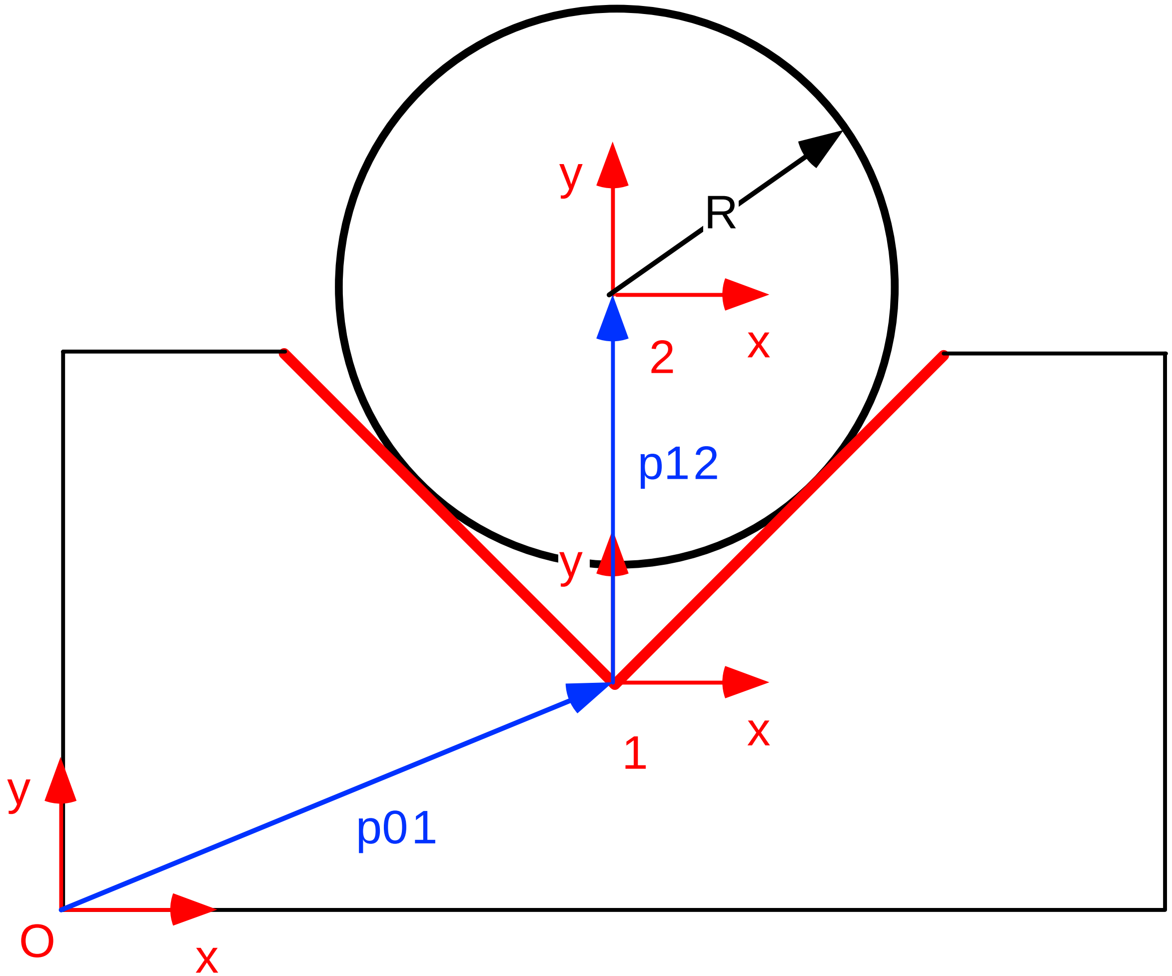
Source:
Figure 3-43 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

A Better Way, Based on Features and Frames

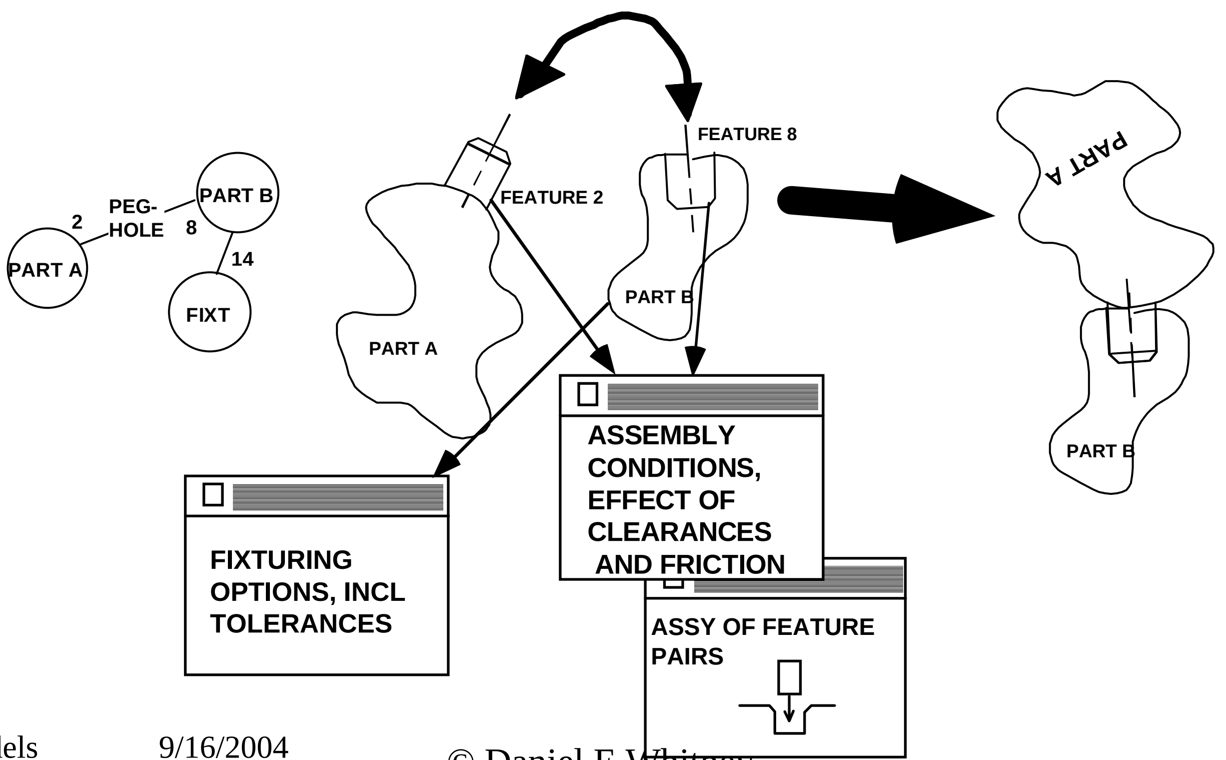


What The Different Models Do

- World coordinate model is good for drawing pictures of the nominal arrangement; can find interferences based on errors in the nominal but can’t help you find out why they happened
- Chained model is good for capturing relational information and design intent, and can trace effects of variation from the nominal; won’t necessarily find interferences because it forces things to be “assembled”; can help you find out why things don’t fit

Assembly Modeling in CAD

Parts can be defined with mating features on them.
Features can be mated directly.
An assembly database builds up automatically.
Assembly knowledge can be accessed.



Information in Assembly Models

- What parts mate to what parts
- What features define the mates and where they are on the parts
- What interfaces must be controlled, plus a formal way of describing them
- Constraints and rule-checking
  - about assembly in the small
  - about assembly intent in terms of features
  - about assembly in the large, including alternate parts
- It is a completely abstract and general model based on connectivity
- Geometry is an attribute of the parts

Example Assembly Data Model

DECLARED ASSEMBLY FEATURE

ON PART_____ (text)

TYPE NAME_____SPECIFIC NAME_____

LOCATION ON PART _____ (4x4)
LOCAL ESCAPE DIRECTION____
(DEFAULT: Z AXIS)
TOLERANCES

GEOMETRY_____
PARAMETERS________
TOLERANCES_____

OPTIONAL: FEATURES IT CAN MATE TO
CONSTRAINTS

MATED TO FEATURE___ ON PART ___
CASE... (other parts in other circumstances)

Seeker Head

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Source:
Figure 3-28 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Seeker Liaison Diagram

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Source:
Figure 3-29 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Parts and Features List

PART	PART NAME	FEATURE	FEATURE NAME	FEATURE CLASS
A	BASE	1	BEARING BORE	(CHAMFERED) BORE
		2	TRUNNION BORE	BORE
		3	TRUNNION BORE	BORE
		4	BEARING BORE	(CHAMFERED) BORE
B	OUTER GIMBAL	1	BEARING BORE	(CHAMFERED) BORE
		2	TRUNNION BORE	BORE
		3	TRUNNION BORE	BORE
		4	BEARING BORE	(CHAMFERED) BORE
		5	RET. SCREW HOLE	THREADED BORE
		6	TRUNNION	(CHAMFERED) PIN
		7	TRUNNION	(CHAMFERED) PIN
		8	RET. SCREW HOLE	THREADED BORE
C	INNER GIMBAL	1	RET. SCREW HOLE	THREADED BORE
		2	TRUNNION	(CHAMFERED) PIN
		3	TRUNNION	(CHAMFERED) PIN
		4	RET. SCREW HOLE	THREADED BORE
D	OUTER BEARING	1	BORE	(CHAMFERED) BORE
		2	OUTER DIAMETER	(CHAMFERED) PIN
		3	INNER RACE FACE	PLANE
E	RETAINING SCREW	1	THREAD	THREADED PIN
		2	HEAD	PLANE
F	OUTER BEARING	1	BORE	(CHAMFERED) BORE
		2	OUTER DIAMETER	(CHAMFERED) PIN
		3	INNER RACE FACE	PLANE
G	RETAINING SCREW	1	THREAD	THREADED PIN
		2	HEAD	PLANE
H	INNER BEARING	1	BORE	(CHAMFERED) BORE
		2	OUTER DIAMETER	(CHAMFERED) PIN
		3	INNER RACE FACE	PLANE
I	RETAINING SCREW	1	THREAD	THREADED PIN
		2	HEAD	PLANE
J	INNER BEARING	1	BORE	(CHAMFERED) BORE
		2	OUTER DIAMETER	(CHAMFERED) PIN
		3	INNER RACE FACE	PLANE
K	RETAINING SCREW	1	THREAD	THREADED PIN
		2	HEAD	PLANE

Liaison Diagram with Features

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Source:
Figure 3-30 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Feature-based Design Video

- Made at Draper in 1990
- Illustrates a bottom-up approach
- First demo of integrated design of assembly tools hooked to a CAD system
  - parts designed with mating features
  - parts joined by connecting the features
  - liaison diagram constructed automatically
  - assembly data model passed to CAE routines for assembly sequence, assembly system design, and economic analysis