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Introduction to Manufacturing Systems

Single-part-type, multiple stage systems

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Setups

- *Setup:* A setup change occurs when it costs more to make a Type j part after making a Type i part than after making a Type j part.

- Examples:
  - ⋆ Tool change (when making holes)
  - ⋆ Die change (when making sheet metal parts)
  - ⋆ Paint color change
  - ⋆ Replacement of reels of components, when populating printed circuit cards

Setups
Costs

- Setup costs can include
  - ⋆ Money costs, especially in labor. Also materials.
  - ⋆ Time, in loss of capacity and delay.

- Some machines create scrap while being adjusted during a setup change.

- Setups motivate *lots* or *batches:* a set of parts that are processed without interruption by setups.

Setups

Costs

- *Problem:*
  - ⋆ Large lots lead to large inventories and long lead times.
  - ⋆ Small lots lead to frequent setup changes.

- Reduction of setup time has been a very important trend in modern manufacturing.

Setups
Flexibility

- *Flexibility:* a widely-used term whose meaning diminishes as you look at it more closely. *(This may be the definition of a buzzword.)*

- *Flexibility:* the ability to make many different things — ie, to operate on many different parts.

- *Agility* is also sometimes used.

Setups
Flexibility

Which is more flexible?

- A machine that can hold 6 different cutting tools, and can change from one to another with zero setup time.
- A machine that can hold 25 different cutting tools, and requires a 30-second setup time.

Which is more flexible?

- A final assembly line that can produce all variations of 6 models of cars, and can produce 100 cars per day.
- A final assembly line that can produce all variations of 1 model at 800 cars per day?

Setups

Setup Machines vs Batch Machines

- A machine has setups when there are costs or delays associated with changing part types. Machines that require setups may make one or many parts at a time.

- A machine operates on batches of size n if it operates on up to n parts simultaneously each time it does an operation. Batch machines may or may not operate on different part types. If they do, they may or may not require setup changes. (Also, the batches may or may not be homogeneous.)

  - ⋆ *Examples:* Ovens and chemical chamber operations in semiconductor manufacturing; chemical processing of liquids.

Capacity

- Consider a machine that does operations on k part types. Assume the change-over time is 0 and that the machine is perfectly reliable.
- The time to do an operation on a type i part is τ_i.
- We make U_i type i parts. If we have to make them in time T, we can do it if

$$\sum_{i=1}^{k} U_i \tau_i \leq T$$

- Define $u_i = U_i/T$ to be the *production rate* of type i parts. Then we must have

$$\sum_{i=1}^{k} \tau_i u_i \leq 1$$

- This is the *capacity constraint* of the machine.

Setups
Loss of Capacity

Assume

- there is one setup for every Q parts (Q=lot size),
- the setup time is S,
- the time to process a part is τ.

Then the time to process Q parts is $S + Q\tau$. The average time to process one part is $\tau + S/Q$.

Setups
Loss of Capacity

If the demand rate is d parts per time unit, then the demand is feasible only if

$$d < \frac{1}{\tau + S/Q} < \frac{1}{\tau}$$

This is not satisfied if S is too large or Q is too small.

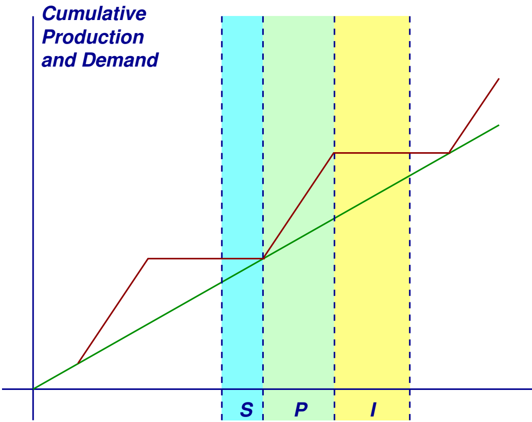
Setups
Deterministic Example

- Focus on a single part type *(simplification!)*
- Short time scale (hours or days).
- Constant demand.
- Deterministic setup and operation times.
- Setup/production/(idleness) cycles.
- *Policy:* Produce at maximum rate until the inventory is enough to last through the next setup time.

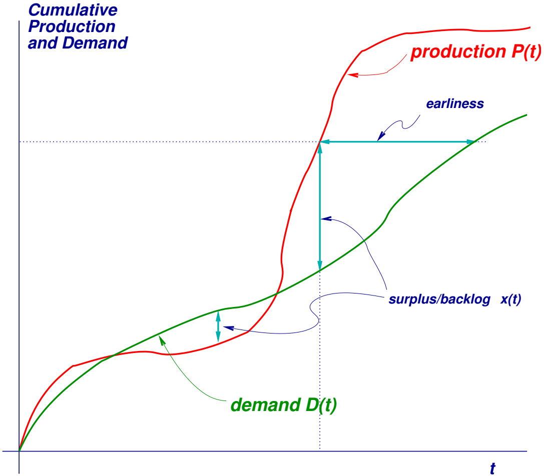
Setups
Deterministic Example

Cycle:

- S = setup period for the part type
- P = period the machine is operating on the part type
- I = period the machine is making or setting up for *other* parts, or idle



Production Objective



Objective is to keep th
cumulative production
line close to the
cumulative demand
line.

Setups

Deterministic Example

Cycle:

- *Setup period.* Duration: S. Production: 0. Demand: Sd. Net change of *surplus* , ie of $P - D$ is $\Delta_S = -Sd$.

- *Production period.* Duration: $t = Q\tau$. Production: Q. Demand: td. Net change of $P - D$ is $\Delta_P = Q - td = Q - Q\tau d = Q(1 - \tau d)$.

Setups

Deterministic Example

- *Idleness period (for the part we focus on).* Duration: I. Production: 0. Demand: Id. Net change of $P - D$ is $\Delta_I = -Id$.

- Total (desired) net change over a cycle: 0.

- Therefore, net change of $P - D$ over whole cycle is $\Delta_S + \Delta_P + \Delta_I = -Sd + Q(1 - \tau d) - Id = 0$.

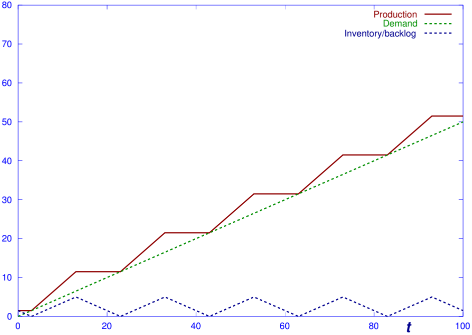
Setups
Deterministic Example

- Since $I \geq 0$, $Q(1 - \tau d) - Sd \geq 0$.
- If $I = 0$, $Q(1 - \tau d) = Sd$.
- If $\tau d > 1$, net change in $P - D$ will be negative.

Setups

Production & inventory history

$S = 3,\ Q = 10,\ \tau = 1,\ d = .5$

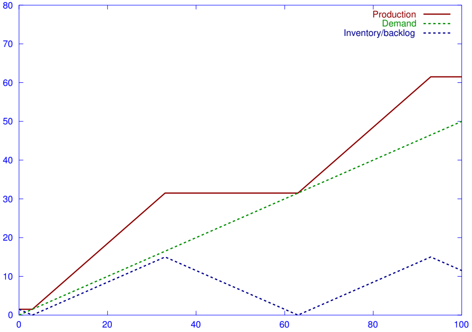


- Production period duration = $Q\tau = 10$.
- Idle period duration = 7.
- Total cycle duration = 20.
- Maximum inventory is $Q(1 - \tau d) = 5$.

Setups

Not frequent enough

$S = 3,\ Q = 30,\ \tau = 1,\ d = .5$

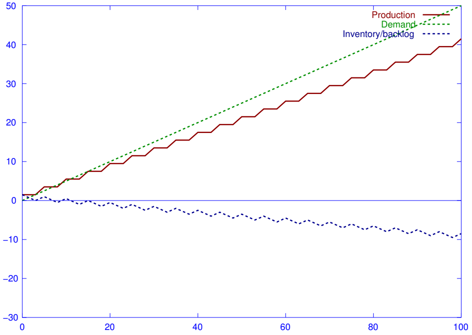


- Production period duration = $Q\tau = 30$.
- Idle period duration = 27.
- Total cycle duration = 60.
- Maximum inventory is $Q(1 - \tau d) = 15$.

Setups

Too frequent

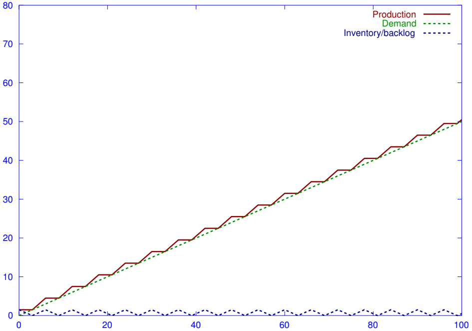
$S = 3,\ Q = 2,\ \tau = 1,\ d = .5$



- *Batches too small – demand not met.*
- $Q(1 - \tau d) - Sd = -0.5$
- Backlog grows.
- Too much capacity spent on setups.

Setups
Just right!

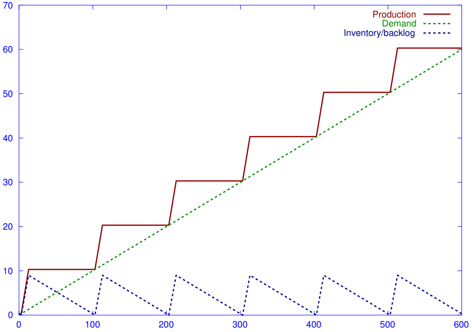
$S = 3,\ Q = 3,\ \tau = 1,\ d = .5$



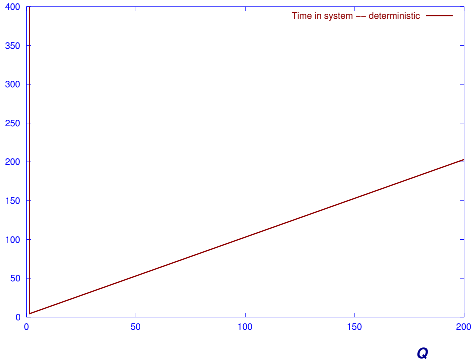
- *Small batches – small inventories.*
- Maximum inventory is $Q(1 - \tau d) = 1.5$.

Setups
Another set of parameters

$S = 3,\ Q = 10,\ \tau = 1,\ d = .1$



Setups
Time in the system



- Each batch spends $Q\tau + S$ time units in the system *if* $Q(1 - \tau d) - Sd \geq 0$.
- Optimal batch size: $Q = Sd/(1 - \tau d)$

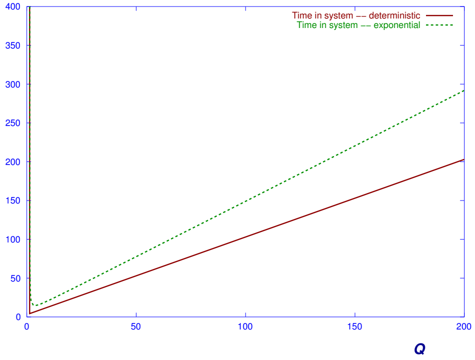
Setups

Time in the system — Stochastic Example

- Batch sizes equal (Q); processing times random.
  - ⋆ Average time to process a batch is $Q\tau + S = 1/\mu$.
- Random arrival times (exponential inter-arrival times)
  - ⋆ Average time between arrivals of batches is $Q/d = 1/\lambda$.
- Infinite buffer for waiting batches

Setups

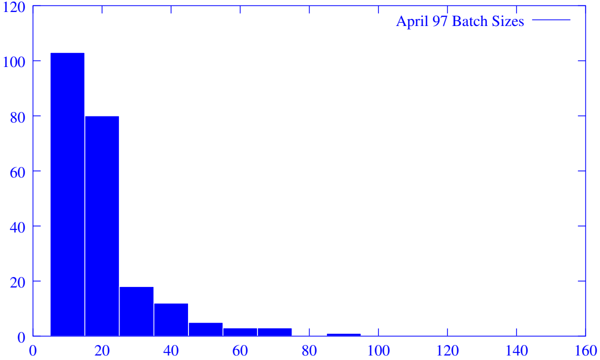
Time in the system — Stochastic Example



- Treat system as an $M/M/1$ queue in batches.
- Average delay for a batch is $1/(\mu - \lambda)$.
- *Variability increases delay* .

Setups

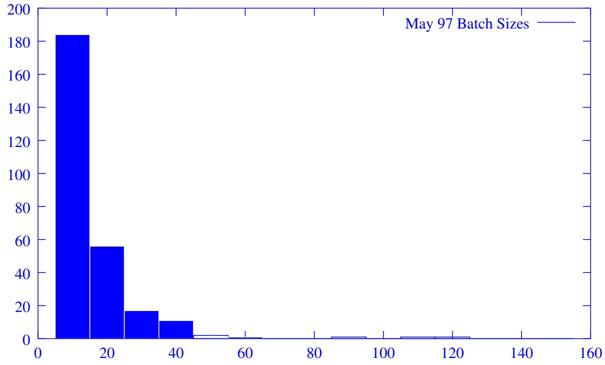
Batch size data from a factory



Random batch sizes

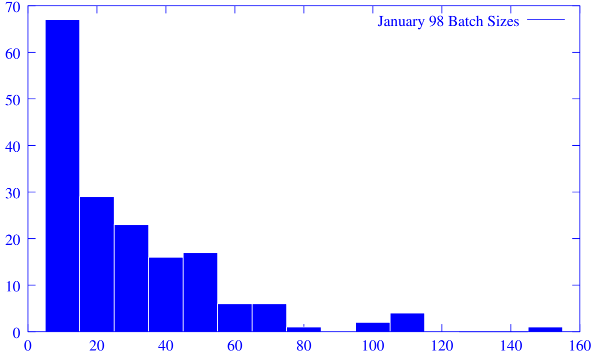
Setups

Batch size data from a factory



Random batch sizes

Setups
Batch size data from a factory



Avg Batch Size=25
Std Dev=27

Setups

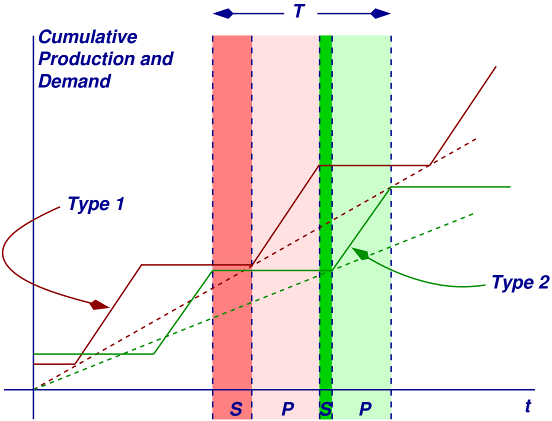
Two Part Types

- Assumptions:
  - Cycle is *produce Type 1, setup for Type 2, produce Type 2, setup for Type 1* .
  - Unit production times: τ_1, τ_2.
  - Setup times: S_1, S_2.
  - Batch sizes: Q_1, Q_2.
  - Demand rates: d_1, d_2.
  - No idleness.

Setups

Two Part Types

Cycle:



Setups

Two Part Types

Let T be the length of a cycle. Then

$$S_1 + \tau_1 Q_1 + S_2 + \tau_2 Q_2 = T$$

To satisfy demand,

$$Q_1 = d_1 T; \qquad Q_2 = d_2 T$$

This implies

$$T = \frac{S_1 + S_2}{1 - (\tau_1 d_1 + \tau_2 d_2)}$$

Setups

Two Part Types

- $\tau_i d_i$ is the fraction of time that is devoted to producing part i.
- $1 - (\tau_1 d_1 + \tau_2 d_2)$ is the fraction of time that is *not* devoted to production.
- We must therefore have $\tau_1 d_1 + \tau_2 d_2 < 1$. This is a *feasibility condition* .

Setups

More Than Two Part Types

- New issue: *Setup sequence* .
  - ⋆ In what order should we produce batches of different part types?
- S_{ij} is the setup time (or setup cost) for changing from Type i production to Type j production.
- *Problem:*
  - ⋆ Select the setup sequence $\{i_1, i_2, ..., i_n\}$ to minimize $S_{i_1 i_2} + S_{i_2 i_3} + ... + S_{i_{n-1} i_n} + S_{i_n i_1}$.

Setups

More Than Two Part Types

Cases

- Sequence-independent setups: $S_{ij} = S_j$. Sequence does not matter.
- Sequence-dependent setups: traveling salesman problem.

Setups

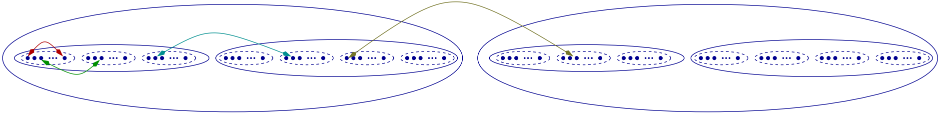
More Than Two Part Types

Cases

- Paint shop: i indicates paint color number.
- S_{ij} is the time or cost of changing from Color i to Color j.
- If $i > j$, i is darker than j and $S_{ij} > S_{ji}$.

Setups
More Than Two Part Types

Cases



- Hierarchical setups.
- Operations have several attributes.
- Setup changes between some attributes can be done quickly and easily.
- Setup changes between others are lengthy and expensive.

Setups
Dynamic Lot Sizing

- *Wagner-Whitin (1958)* problem
- Assumptions:
  - ⋆ Discrete time periods (weeks, months, etc.); $t = 1, 2, ..., T$.
  - ⋆ Known, but non-constant demand D_1, D_2, ..., D_T.
  - ⋆ Production, setup, and holding cost.
  - ⋆ Infinite capacity.

Setups
Dynamic Lot Sizing

- c_t = production cost (dollars per unit) in period t
- A_t = setup or order cost (dollars) in period t
- h_t = holding cost; cost to hold one item in inventory from period t to period $t+1$
- I_t = inventory at the end of period t — the state variable
- Q_t = lot size in period t — the decision variable

Setups
Dynamic Lot Sizing

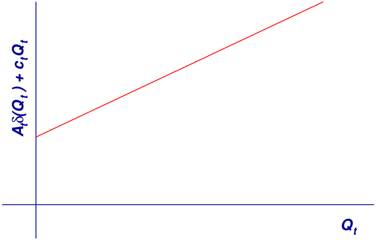
Problem formulation:

minimize $\sum_{t=1}^{T} (A_t \delta(Q_t) + c_t Q_t + h_t I_t)$

(where $\delta(Q) = 1$ if $Q > 0$; $\delta(Q) = 0$ if $Q = 0$)

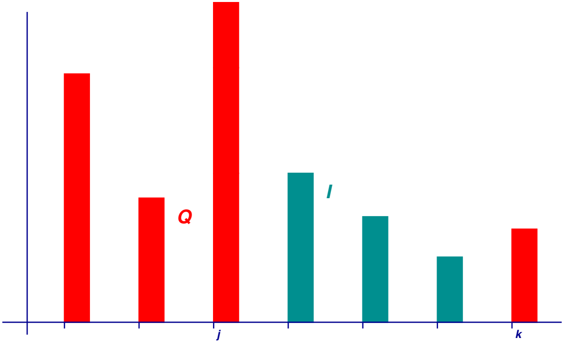
subject to

- $I_{t+1} = I_t + Q_t - D_t$
- $I_t \geq 0$



Setups
Dynamic Lot Sizing

Characteristic of Solution:



Setups

Dynamic Lot Sizing

Characteristic of Solution:

- *Either* $I_t = 0$ *or* $Q_{t+1} = 0$. *That is, produce only when inventory is zero.* Or,
    - ⋆ If we assume $I_j = 0$ and $I_k = 0$ $(k > j)$ and $I_t > 0, t = j+1, ..., k$,
    - ⋆ then $Q_j > 0$, $Q_k > 0$, and $Q_t = 0, t = j+1, ..., k$.

Setups
Dynamic Lot Sizing

Then

- $I_{j+1} = Q_j - D_j$,
- $I_{j+2} = Q_j - D_j - D_{j+1}$, ...
- $I_k = 0 = Q_j - D_j - D_{j+1} - ... - D_k$

Or, $Q_j = D_j + D_{j+1} + ... + D_k$

which means *produce enough to exactly satisfy demands for some number of periods, starting now.*

This is not enough to determine the solution, but it means that the search for the optimal is limited.

Setups
Real-Time Scheduling

- *Problem:* How to decide on batch sizes (ie, setup change times) in response to events.

- *Issue:* Same as before.
  - ⋆ Changing too often causes capacity loss; changing too infrequently leads to excess inventory and lead time.

Setups

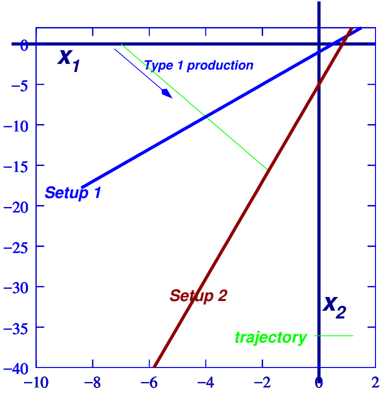
Real-Time Scheduling — One Machine, Two Part Types

Model:

- d_i = demand rate of Type i
- $\mu_i = 1/\tau_i$ = maximum production rate of Type i
- S = setup time
- $u_i(t)$ = production rate of Type i at time t
- $x_i(t)$ = surplus (inventory or backlog) of Type i
- $\dfrac{dx_i}{dt} = u_i(t) - d_i, i = 1, 2$

Setups

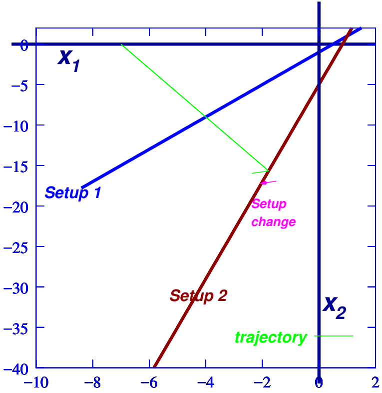
Real-Time Scheduling — Corridor Policy Heuristic



- Draw two lines, labeled *Setup 1* and *Setup 2.*
- Keep the system in setup i until $x(t)$ hits the *Setup* j line.
- Change to setup j.
- Etc.

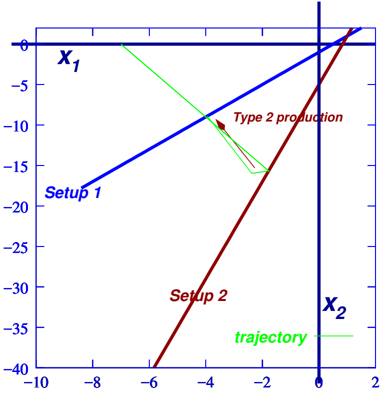
Setups

Real-Time Scheduling — Corridor Policy Heuristic



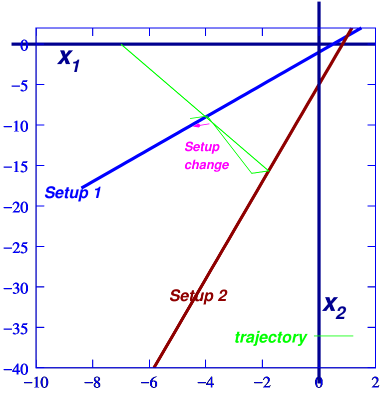
Setups

Real-Time Scheduling — Corridor Policy Heuristic



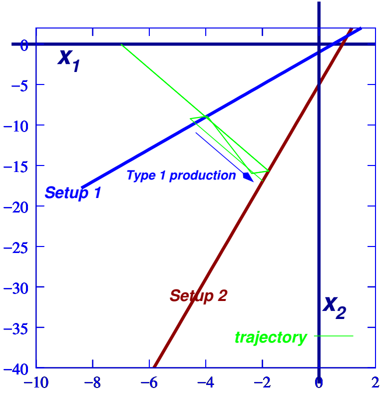
Setups

Real-Time Scheduling — Corridor Policy Heuristic

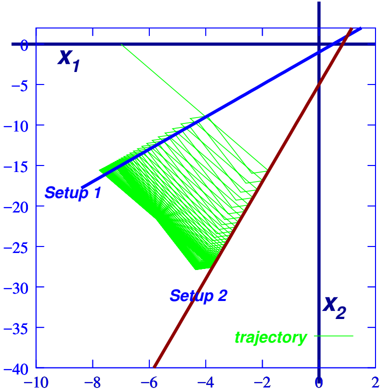


Setups

Real-Time Scheduling — Corridor Policy Heuristic



Setups
Real-Time Scheduling — Corridor Policy Heuristic



Setups
Real-Time Scheduling — Corridor Policy Heuristic

- In this version, batch size is a function of time.
- Also possible to pick parallel boundaries, with an upper limit. Then batch size is constant until upper limit reached.

Setups

Real-Time Scheduling — Corridor Policy Heuristic

Two possibilities (for two part types):

- Converges to limit cycle — only if demand is within capacity, ie if $\sum_i \tau_i d_i < 1$.
- Diverges — if
  - ⋆ demand is not within capacity, or
  - ⋆ corridor boundaries are poorly chosen.

Setups

Real-Time Scheduling — Corridor Policy Heuristic

Three possibilities for more than two part types:

- Limit cycle — only if demand is within capacity,
- Divergence — if
  - ⋆ demand is not within capacity, or
  - ⋆ corridor boundaries are poorly chosen.
- *Chaos* if demand is within capacity, and corridor boundaries chosen ... not well?

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