2.853/4 Problem Set 2

1. **AGV repair service crew (40 pts)**

   In the semiconductor and liquid crystal display panel (LCD) industries, automated guided vehicles (AGVs) are widely used to transfer a lot (a cassette full of wafers or panes of glass) between processing machines. Figure 1 shows AGVs operating in a fab producing LCD panels *(source: Shinsung ENG)*.

   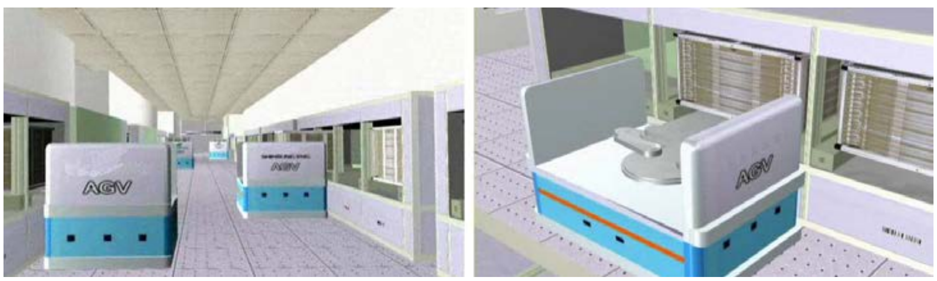

   Figure 1: AGVs in an LCD production line

   Due to their complicated mechanisms and high utilization, these AGVs are subject to failures. When an AGV fails, an AGV service crew in the factory takes the broken AGV to the repair shop and fixes it. The crew is stationed at a single location near the factory. AGV breakdowns occur randomly (according to a Poisson process) at a mean rate of one per hour for each AGV. The time required to fix an AGV has a negative exponential distribution (for any crew size). The expected repair time required by a one-worker crew is 2 hours. The cost per hour for each member of a repair crew is \$10.00. The cost that is attributable to not having an AGV in use (e.g., an AGV in the repair shop) is estimated to be \$40.00 per hour. This cost is due to a production loss caused by a lack of AGVs.

   (a) Assume that the mean service rate of the repair crew is proportional to its size. What should the crew size be in order to minimize the expected total cost of this operation per hour?

*A suggested timeline for P-set 2. However you are **not** required to submit your solutions as P-sets will **not** be graded.

(b) Repeat this question but with the mean service rate proportional to the square root of the crew size.

2. **Youngjae's Machine Shop (40 pts)**

In Youngjae's machine shop, there are several automated milling machines. Since he once learned a little about manufacturing systems from his neighbor Dr. Gershwin, he wanted to analyze the behavior of one of his milling machines in the shop. Let us focus on only one milling machine. It can work on one part at a time. There is a buffer with space for two parts waiting for the machine as shown in the figure below. The total buffer space is therefore three parts. In the figure, the milling area consists of the buffer station and the milling machine.

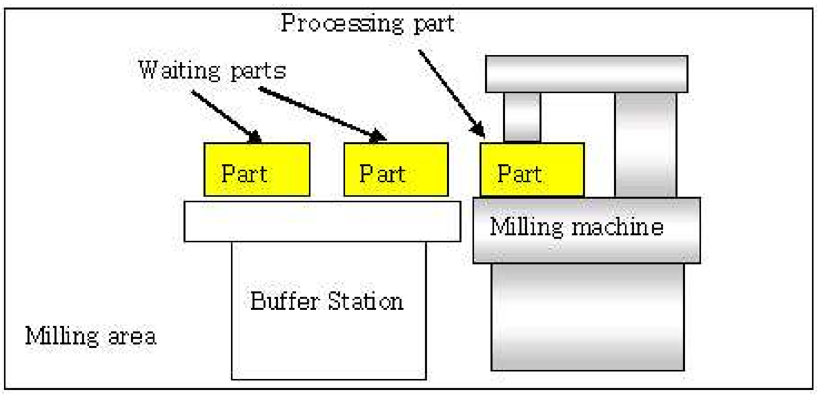

Figure 2: Milling Area

Parts arrive at the milling area according to a Poisson process at the rate of λ per hour. A part that finds the buffer station or the milling machine available will eventually receive service in a first-in-first-out (FIFO) manner. Parts that find the milling area full (the milling machine and the buffer are all occupied) go to other milling areas and never visit this milling area. It takes Youngjae $1/\mu$ hours, on average, to complete work on a part. All questions below refer to steady-state conditions.

(a) Assume that the pdf for service times is negative exponential. For the cases (i) $\lambda = \mu$ and (ii) $\lambda = 2\mu$, compute L, the expected number of customers in the milling area. To do this first define the state space. Then draw the Markov transition graph. Then write the transition or balance equations. Compute the steady state probabilities and then calculate L.

(b) Youngjae wants to add a sufficient number of buffer space for waiting parts to make sure that at least 92% of parts that try to enter to the milling area get served there. For $\lambda = \mu$, what is the minimum number of buffer space he will need in the milling area? (Assume that all parts that try to enter to the milling area find at least one empty buffer space will become actual customers, while the parts that do not will be gone for another milling area.)

(c) Return to the situation described in part (a). Suppose that Youngjae decided not to work on a part in FIFO order but in random order. That is, if Youngjae finishes a process and finds two parts waiting, he will chose between the two parts at random, i.e., each waiting part will have probability 0.5 of being the next one to be processed. In which of the two cases, if any, will W time be greater? Please justify your answer briefly. (W is the expected total amount of time that a part has to spend in the milling area.)

(d) Assume now that Youngjae has added a sufficiently large number of buffer space to the milling area so that parts are always processed there. For the case $\lambda = 0.9\mu$, compute L, the expected number of parts in the milling area at any randomly chosen time with the system in steady state. A numerical answer is expected for this part.

3. **More on the Newsvendor (40 pts)**

Recall the triangular density, with parameters a, b and m.

$$F(t) = P(T \leq t) = \begin{cases} 0 & t \leq a \\ \dfrac{(t-a)^2}{(m-a)(b-a)} & a \leq t \leq m \\ 1 - \dfrac{(b-t)^2}{(b-m)(b-a)} & m \leq t \leq b \\ 1 & t \geq b \end{cases}$$

$$f(t) = \frac{dF}{dt} = \begin{cases} 0 & t \leq a \\ \dfrac{2(t-a)}{(m-a)(b-a)} & a \leq t \leq m \\ \dfrac{2(b-t)}{(b-m)(b-a)} & m \leq t \leq b \\ 0 & t \geq b \end{cases}$$

See Figure 3.

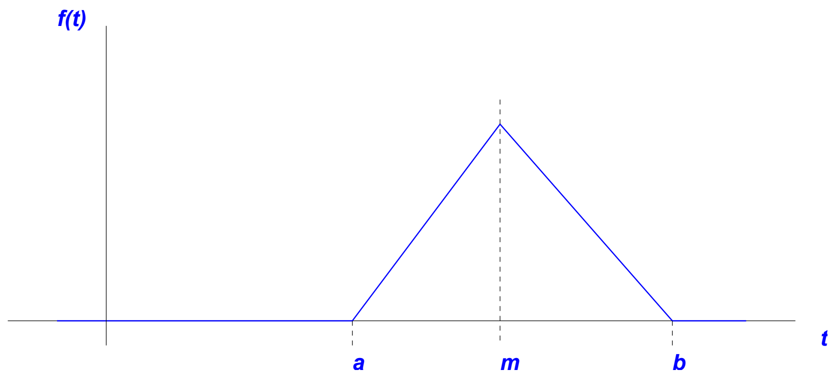

Figure 3: Triangular density function

(a) Show that the mean and variance are given by

$$\mu = \frac{a+m+b}{3}$$

$$\sigma^2 = \frac{a^2+b^2+m^2-ab-am-bm}{18}$$

Consider the symmeteric distribution, in which $m = (a+b)/2$. Show that

$$\mu = \frac{a+b}{2}$$

$$\sigma^2 = \frac{(a-b)^2}{24}$$

Write equations for a and b as functions of μ and σ.

(b) Consider a newsvendor problem with purchase price c, sales price (revenue per unit) r, salvage value s and demand given by a triangular distribution with parameters a, b, and $m = (a+b)/2$.

i. How many newspapers should the vendor buy? Express this in terms of a and b; also express it in terms of μ and σ.

ii. In the next part of this question, you are asked to draw graphs. What are the appropriate bounds on μ (when σ is fixed) and σ (when μ is fixed)?

iii.
- Let $r = 1, c = .25, s = 0, \sigma = 10$. Draw the graph of x^* vs μ.
- Let $r = 1, c = .75, s = 0, \sigma = 10$. Draw the graph of x^* vs μ.

- Let $r = 1, c = .25, s = 0, \mu = 100$. Draw the graph of x^* vs σ.
- Let $r = 1, c = .75, s = 0, \mu = 100$. Draw the graph of x^* vs σ.