2.830J / 6.780J / ESD.63J Control of Manufacturing Processes (SMA 6303)
Spring 2008

MIT 2.830/6.780 Problem Set 6 (2008) — Solutions

Problem 1

See the following pages for exemplary solutions (courtesy X. Su and K. Umeda)

For the t-test part of the question, two approaches were accepted: one in which the significance of each effect was tested in turn by looking for evidence of a mean shift (interactions are not probed in this approach); the other is to define a standard error as in example 12-7 of Montgomery and perform a t-test on all effects, including interactions. This second approach is exactly equivalent to doing ANOVA.

$SS_T = SS_A + SS_B + SS_{AB} + SS_E = 0.002386$, or $SS_T = \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{15}\left(y_{ijk} - \overline{y}_{\cdots}\right)^2$, where:

$$SS_A = (2)(15)\sum_{i=1}^{2}\left(\overline{y}_{i\cdot\cdot} - \overline{y}_{\cdots}\right)^2$$
$$= (2)(15)\left[(2.0377 - 2.03935)^2 + (2.041 - 2.03935)^2\right] = 0.000163$$

$$SS_B = (2)(15)\sum_{j=1}^{2}\left(\overline{y}_{\cdot j\cdot} - \overline{y}_{\cdots}\right)^2$$
$$= (2)(15)\left[(2.035567 - 2.03935)^2 + (2.043133 - 2.03935)^2\right] = 0.000859$$

$$SS_{AB} = (15)\sum_{i=1}^{2}\sum_{j=1}^{2}\left(\overline{y}_{ij\cdot} - \overline{y}_{i\cdot\cdot} - \overline{y}_{\cdot j\cdot} + \overline{y}_{\cdots}\right)^2 = 0.0000837$$

$$SS_E = \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{15}\left(y_{ijk} - \overline{y}_{ij\cdot}\right)^2 = 0.001279$$

SUMMARY	LH (j=1)	HH(j=2)	Total
LV(i=1)			
Count	15	15	30
Sum	30.491	30.64	61.131
Average	$\overline{y}_{11\cdot} = \frac{30.491}{15}$ =2.032733	$\overline{y}_{12\cdot} = \frac{30.64}{15}$ =2.042667	$\overline{y}_{1\cdot\cdot} = \frac{61.131}{30}$ = 2.0377
Variance	9.5E-06	2.24E-05	4.09E-05
HV(i=2)			
Count	15	15	30
Sum	30.576	30.654	61.23
Average	$\overline{y}_{21\cdot} = \frac{30.576}{15}$ =2.0384	$\overline{y}_{22\cdot} = \frac{30.654}{15}$ =2.0436	$\overline{y}_{2\cdot\cdot} = \frac{61.23}{30}$ =2.041
Variance	3.74E-05	2.21E-05	3.57E-05
Total			
Count	30	30	**120**
Sum	61.067	61.294	**122.361**
Average	$\overline{y}_{\cdot 1\cdot} = \frac{61.067}{30}$ =2.035567	$\overline{y}_{\cdot 2\cdot} = \frac{61.294}{30}$ =2.043133	$\overline{y}_{\cdots} = \frac{122.361}{120}$ =**2.03935**
Variance	3.09E-05	2.17E-05	

MANOVA table – Two-way with interactions:

Source of Variation	*SS*	*df*	*MS(=SS/df)*	*F*	*P-value*	*F crit*
Sample (Factor A)	SS_A = 0.000163	1	s_A^2 = 0.000163	7.149541	0.009808	4.012973
Columns (Factor B)	SS_B = 0.000859	1	s_B^2 = 0.000859	37.58889	9.3E-08	4.012973
Interaction	SS_{AB} = 8.4E-05	1	s_{AB}^2 = 8.4E-05	3.677261	0.060265	4.012973
Within	SS_E = 0.001279	56	s_E^2 = 2.28E-05			
Total	SS_T = 0.002386	59				

Comparing the computed F-ratios to a 5% upper critical value of the F distribution, where $F_{0.05,1,56} = 4.013$, we conclude that since computed F-ratio 7.15 > 4.013 and 37.59 > 4.013, both the injection speed and hold time affects the output diameter. However, as the computed F-ratio for interaction is smaller than the critical F value, there is no indication of interaction between the two factors.

b.

Test on Factor A, injection speed input:
Let H_0 be the hypothesis that $\mu_{HV} = \mu_{LV}$
Let H_1 be the hypothesis that $\mu_{HV} \neq \mu_{LV}$

Test statistic: $t_0 = \dfrac{(\overline{y}_1 - \overline{y}_2) - (\overline{\mu}_1 - \overline{\mu}_2)}{s\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$, where $\mu_1 = \mu_{HV}$ and $\mu_2 = \mu_{LV}$.

Test if the mean diameters under low and high velocity are equal 95% of the time ($\alpha = 0.05$) If they do not, then the injection speed affects the output diameter.

Test on Factor B, hold time:
Let H_0 be the hypothesis that $\mu_{HH} = \mu_{LH}$
Let H_1 be the hypothesis that $\mu_{HH} \neq \mu_{LH}$

Test statistic: $t_0 = \dfrac{(\overline{y}_1 - \overline{y}_2) - (\overline{\mu}_1 - \overline{\mu}_2)}{s\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$, where $\mu_1 = \mu_{HH}$ and $\mu_2 = \mu_{LH}$.

Test if the mean diameters under low and high hold time are equal 95% of the time ($\alpha = 0.05$) If they do not, then the hold time affects the output diameter.

c.

Test on Factor A, injection speed input:
Two-tailed test:
To estimate with 95% degree of confidence, where $\alpha = 0.05$
$t_{\alpha,n_1+n_2-2} = t_{0.025,58} =$ (The rejection region is $|t_0| \geq 2.002$)

	High velocity, y_1	Low velocity, y_2
Sample mean, $\overline{y}_1$, $\overline{y}_2$	2.041	2.0377
Count, n_1, n_2	30	30
Sum squares, $\sum(y_1 - \overline{y}_1)^2, \sum(y_2 - \overline{y}_2)^2$	0.001036	0.0011863

Assuming that the variances for y_1 and y_2 are equal.
Pooled estimate of σ^2:

$$s^2 = \frac{0.001036 + 0.0011863}{30 + 30 - 2} = 0.0000383155$$

For $\overline{\mu}_1 - \overline{\mu}_2 = 0$, $$t_0 = \frac{(\overline{y}_1 - \overline{y}_2) - (\overline{\mu}_1 - \overline{\mu}_2)}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{0.0033}{0.00159824} = 2.0648 \text{ (with 58 d.o.f.)}$$

Since $|t_0| > 2.002$, we reject the null hypothesis: There is sufficient evidence of a difference in diameter caused by the variation of injection speed from low velocity to high velocity.

Test on Factor B, hold time input:
Two-tailed test:
To estimate with 95% degree of confidence, where α = 0.05
$t_{\alpha, n_1 + n_2 - 2} = t_{0.025,58} =$ (The rejection region is $|t_0| \geq 2.002$)

	High hold, y_1	Low hold, y_2
Sample mean, $\overline{y}_1$, $\overline{y}_2$	2.043133	2.035567
Count, n_1, n_2	30	30
Sum squares, $\sum(y_1 - \overline{y}_1)^2, \sum(y_2 - \overline{y}_2)^2$	0.000629467	0.000897367

Assuming that the variances for y_1 and y_2 are equal.
Pooled estimate of σ^2:

$$s^2 = \frac{0.000629467 + 0.000897367}{30 + 30 - 2} = 0.0000263247$$

For $\overline{\mu}_1 - \overline{\mu}_2 = 0$, $$t_0 = \frac{(\overline{y}_1 - \overline{y}_2) - (\overline{\mu}_1 - \overline{\mu}_2)}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{0.007567}{0.001324757} = 5.7117 \text{ (with 58 d.o.f.)}$$

Since $|t_0| > 2.002$, we reject the null hypothesis: There is sufficient evidence of a difference in diameter caused by the variation of hold time from low hold to high hold.

d. Results obtained show similar conclusions as previously obtained that both the injection speed and hold time affects the output diameter. However the difference is that there is no basis on which we can conclude if there was any interaction between the two factors. The similarity between these two methods could be due to absence of interactions, or it could be that interactions present are not of a significance level that is large enough and thus did not affect the significance of the main effects caused by both factors. This can also be seen from the rather near but lower values of computed F-ratio for interaction with the critical F value in the MANOVA table.

Second example solution to Problem 1 (K. Umeda)

(a)

ANOVA

source of variation	contract	effect	coefficient	sum of squares	d.o.f	mean square	Fo	P-value
A	0.227	0.028375	0.0141875	0.00085882	1	0.00085882	36.917661	1.2124E-07
B	0.099	0.012375	0.0061875	0.00016335	1	0.00016335	7.02187109	0.01049275
AB	-0.071	-0.008875	-0.0044375	8.4017E-05	1	8.4017E-05	3.61159598	0.06262443
Error				0.00127947	55	2.3263E-05		
Total				0.00238565	59			

The results of ANOVA shows the factor A has a very small p-value, which means that the distribution of factor A is different from the distribution of error with high confidence. I conclude the factor A has the largest impact on the output among these three factors.

(b)

I use t-test for the test statistic. Firstly, I compute the estimated error of the coefficient, using the error mean square from the ANOVA.

s.e. (b) = sqrt((the error of the mean square)/n2^k)

The t-value is calculated from the following equation, using s.e. (b) and the coefficients in the regression model.

t = b/ s.e. (b)

Then, the p-value is calculated, applying the degree of freedom of error and both side of t-distribution.

(c)

t-test

source of variation	contract	effect	coefficient	SE Coef	T	P
Constant			2.039	0.00062267	3275.17303	3.563E-147
A	0.227	0.00756667	0.00378333	0.00062267	6.07599054	1.2124E-07
B	0.099	0.0033	0.00165	0.00062267	2.64988134	0.01049275
AB	-0.071	-0.0023667	-0.0011833	0.00062267	-1.9004199	0.06262443

(d)

I got the same p-value for A, B, AB factors in both methods of ANOVA and t-test. Therefore, as the effect s of factors decrease, the corresponded p-values increase. A large p-value means that the factor has same distribution with the distribution of error, and does not affect the output. .

Problem 2

Montgomery 12–2.

Since the standard order (Run) is provided, one approach to solving this exercise is to create a 2^3 factorial design in MINITAB, then enter the data. Another approach would be to create a worksheet containing the data, then define a customer factorial design. Both approaches would achieve the same result. This solution uses the first approach.

Select **Stat > DOE > Factorial > Create Factorial Design**. Leave the design type as a 2-level factorial with default generators, and change the Number of factors to "**3**". Select "**Designs**", highlight **full factorial**, change number of replicates to "**2**", and click "**OK**". Select "**Factors**", enter the factor names, leave factor types as "**Numeric**" and factor levels as -1 and +1, and click "**OK**" twice. The worksheet is in run order, to change to standard order (and ease data entry) select **Stat > DOE > Display Design** and choose standard order. The design and data are in the MINITAB worksheet **Ex12-2.MTW**.

(a)
To analyze the experiment, select **Stat > DOE > Factorial > Analyze Factorial Design**. Select "**Terms**" and verify that all terms (A, B, C, AB, AC, BC, ABC) are included.

Factorial Fit: Life versus Cutting Speed, Metal Hardness, Cutting Angle

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Estimated Effects and Coefficients for Life (coded units)
Term                              Effect    Coef  SE Coef      T      P
Constant                                  413.13    12.41  33.30  0.000
Cutting Speed                      18.25    9.13    12.41   0.74  0.483
Metal Hardness                     84.25   42.12    12.41   3.40  0.009 **
Cutting Angle                      71.75   35.88    12.41   2.89  0.020 **
Cutting Speed*Metal Hardness      -11.25   -5.62    12.41  -0.45  0.662
Cutting Speed*Cutting Angle      -119.25  -59.62    12.41  -4.81  0.001 **
Metal Hardness*Cutting Angle      -24.25  -12.12    12.41  -0.98  0.357
Cutting Speed*Metal Hardness*     -34.75  -17.37    12.41  -1.40  0.199
  Cutting Angle

S = 49.6236   R-Sq = 85.36%   R-Sq(adj) = 72.56%

Analysis of Variance for Life (coded units)
Source              DF   Seq SS   Adj SS   Adj MS     F      P
Main Effects         3    50317    50317    16772  6.81  0.014
2-Way Interactions   3    59741    59741    19914  8.09  0.008
3-Way Interactions   1     4830     4830     4830  1.96  0.199
Residual Error       8    19700    19700     2462
  Pure Error         8    19700    19700     2463
Total               15   134588
…
```

Based on ANOVA results, a full factorial model is not necessary. Based on *P*-values less than 0.10, a reduced model in Metal Hardness, Cutting Angle, and Cutting Speed*Cutting Angle is more appropriate. Cutting Speed will also be retained to maintain a hierarchical model.

Factorial Fit: Life versus Cutting Speed, Metal Hardness, Cutting Angle

Estimated Effects and Coefficients for Life (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		413.13	12.47	33.12	0.000
Cutting Speed	18.25	9.13	12.47	0.73	0.480
Metal Hardness	84.25	42.12	12.47	3.38	0.006
Cutting Angle	71.75	35.88	12.47	2.88	0.015
Cutting Speed*Cutting Angle	-119.25	-59.62	12.47	-4.78	0.001

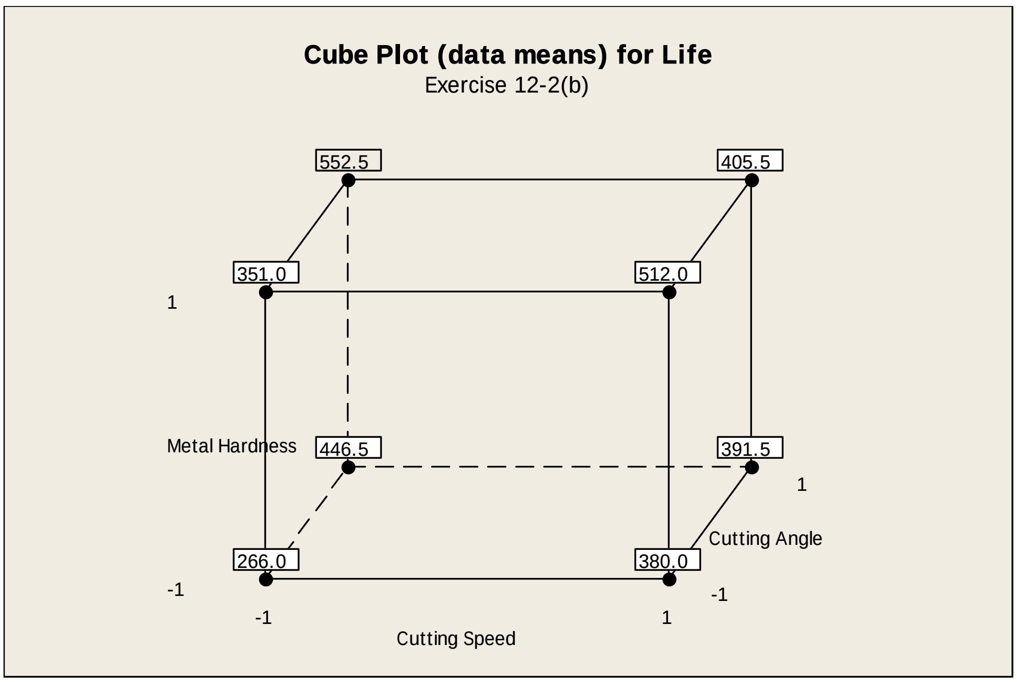
S = 49.8988   R-Sq = 79.65%   R-Sq(adj) = 72.25%

Analysis of Variance for Life (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	50317	50317	16772	6.74	0.008
2-Way Interactions	1	56882	56882	56882	22.85	0.001
Residual Error	11	27389	27389	2490		
Lack of Fit	3	7689	7689	2563	1.04	0.425
Pure Error	8	19700	19700	2463		
Total	15	134588				

(b)

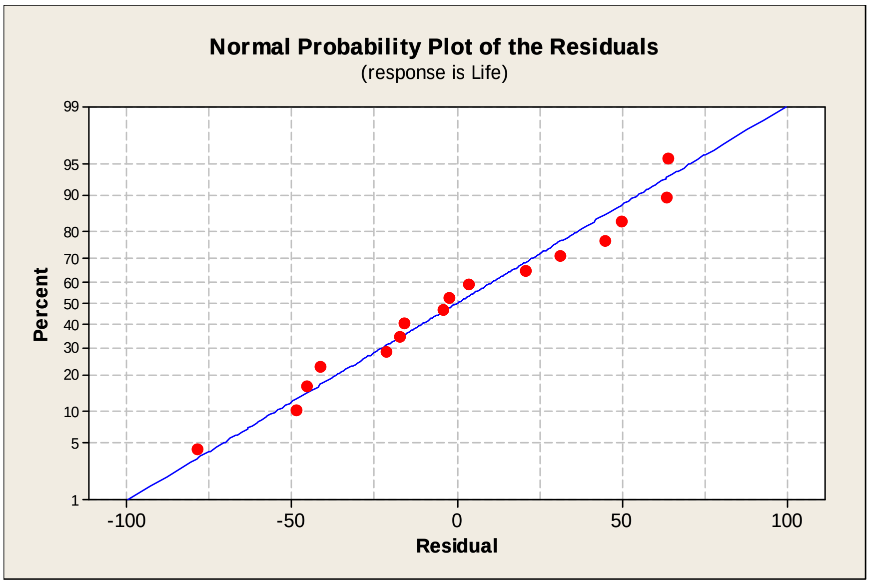
The combination that maximizes tool life is easily seen from a cube plot. Select **Stat > DOE > Factorial > Factorial Plots**. Choose and set-up a "**Cube Plot**".

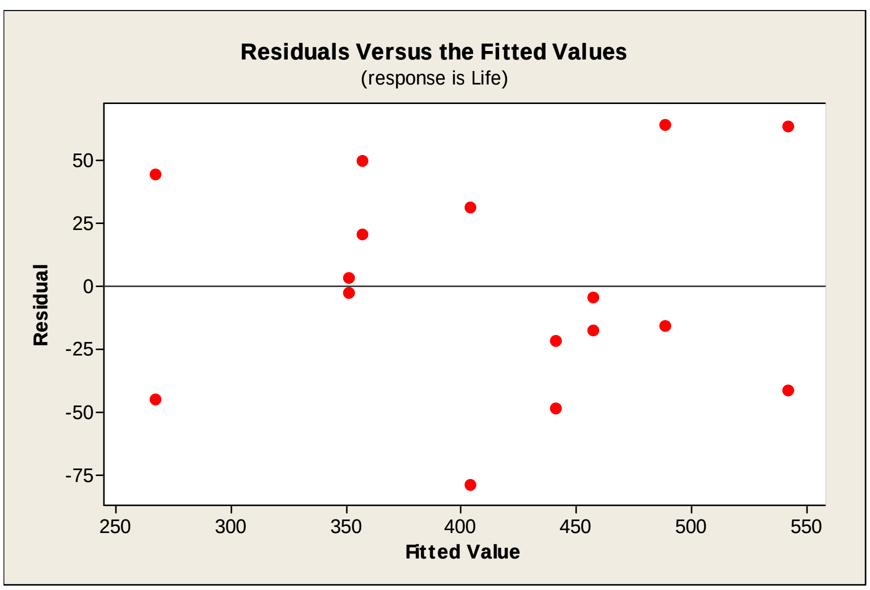


Longest tool life is at A-, B+ and C+, for an average predicted life of 552.5.

(c)
From examination of the cube plot, we see that the low level of cutting speed and the high level of cutting angle gives good results regardless of metal hardness.

Montgomery 12–3
To find the residuals, select **Stat > DOE > Factorial > Analyze Factorial Design**. Select "**Terms**" and verify that all terms for the reduced model (A, B, C, AC) are included. Select "**Graphs**", and for residuals plots choose "**Normal plot**" and "**Residuals versus fits**". To save residuals to the worksheet, select "**Storage**" and choose "**Residuals**".





Normal probability plot of residuals indicates that the normality assumption is reasonable. Residuals versus fitted values plot shows that the equal variance assumption across the prediction range is reasonable.

Problem 3

On of many excellent solutions submitted (courtesy K. Lee).

Since $\sum(\text{contrast coefficient})^2 = 2^k$

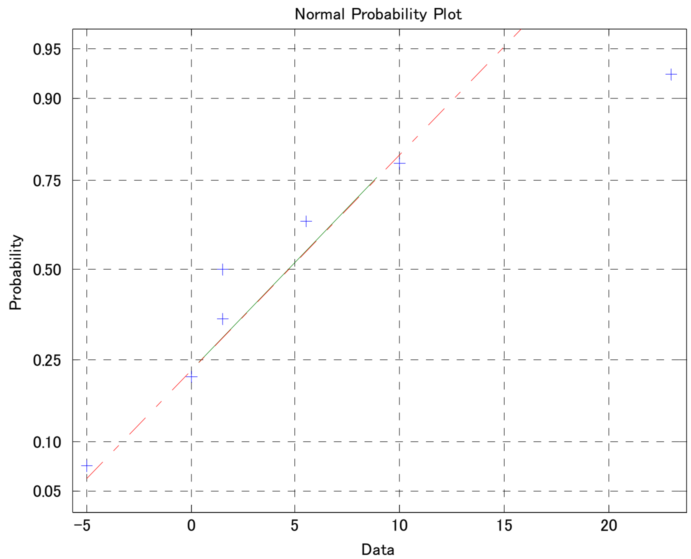
$$SS_A = nm\beta_A^2 = n2^k\,\beta_A^2 = n2^k\left(\frac{Effect_A}{2}\right)^2 = n2^k\left(\frac{Contrast_A}{n2^k}\right)^2 = \frac{Contrast_A^2}{n2^k} = \frac{Contrast_A^2}{n\sum(contrast\ coefficients)^2}$$

Problem 4

May and Spanos solutions removed due to copyright restrictions.

An example of one of many solutions submitted for Problem 4 (courtesy K. Umeda):

From the normal probability plot of effects, the effect of A stays away from the straight line, so I estimate A has an impact.



In general, higher-order interactions are negligible, so here, **ABC can be treated as an estimate of error**. Then, the table below shows the result of ANOVA. From the table below, the p- value of A is the smallest, so could conclude A is a significant factor [at the 15% level].

ANOVA								
source of variation	contrast	effect	coefficient	sum of squares	d.o.f	mean square	Fo	P−value
A	92	23	11.5	1058	1	1058	17.4876	0.149429
B	−20	−5	−2.5	50	1	50	0.826446	0.530292
C	6	1.5	0.75	4.5	1	4.5	0.07438	0.830499
AB	6	1.5	0.75	4.5	1	4.5	0.07438	0.830499
AC	40	10	5	200	1	200	3.305785	0.32012
BC	0	0	0	0	1	0	0	1
Error				60.5	1	60.5		
Total				1377.5	7			