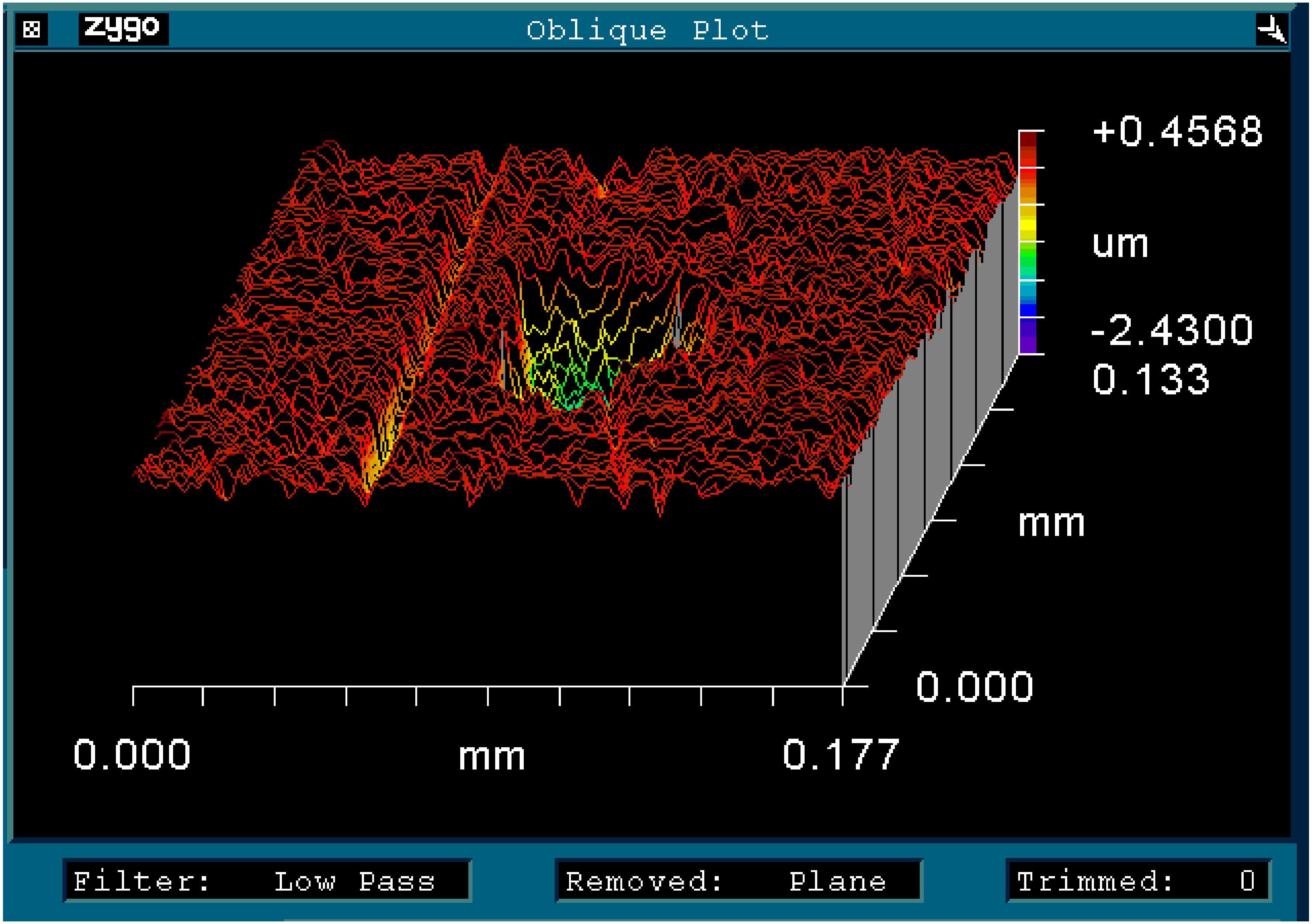
2.76 / 2.760 Lecture 12: Interfaces



Announcements

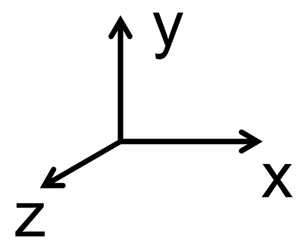
Schedule TA Meetings

Thursday design review (Oct. 21 not Nov. 21)

- FRs & DPs
- Concepts & approach
- Initial modeling (HTMs, systematic, random, actuation, flexures)
- Sensitivity and prioritization
- Scheduling & risk/mitigation

Homogeneous Transformation Matrices

Rotations:



$$^{A}H_{B}(\theta_x)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos(\theta_x) & -\sin(\theta_x) & 0\\ 0 & \sin(\theta_x) & \cos(\theta_x) & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$^{A}H_{B}(\theta_y)=\begin{bmatrix}\cos(\theta_y) & 0 & \sin(\theta_y) & 0\\ 0 & 1 & 0 & 0\\ -\sin(\theta_y) & 0 & \cos(\theta_y) & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

For small θ, cos(θ)~1 & sin(θ)~θ

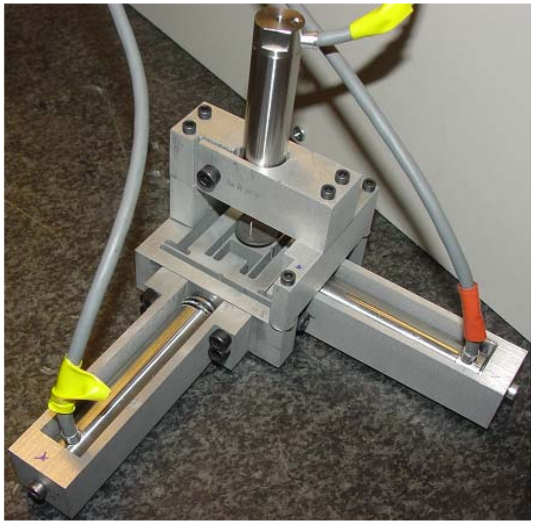
$$^{A}H_{B}(\theta_z)=\begin{bmatrix}\cos(\theta_z) & -\sin(\theta_z) & 0 & 0\\ \sin(\theta_z) & \cos(\theta_z) & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Order of multiplication is important for large θ

Purpose of today

Modeling of mechanical interfaces:

- "Rigid"
- Flexible
- Rigid-flexible



Examples:

- "Rigid": Kinematic couplings
- "Rigid"-Flexible: Flexure-couplings
- "Rigid"-Flexible: Quasi-kinematic coupling

Thursday: Manufacturing & assembly

- Cross-scale interfaces
- Bolted joints

Constraint concept review

Exact constraint

At some scale, everything is a mechanism

- Everything is compliant

What we've done: Kinematics

- Arranging constraints in optimum topology
- Ideal constraints & small motions
- Constraints = lines (ideal)

Now: Kinematics and mechanics

- Model & calculate stiffness/stress/motion

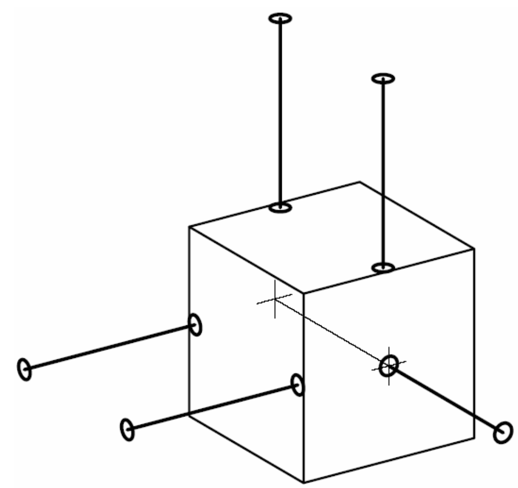


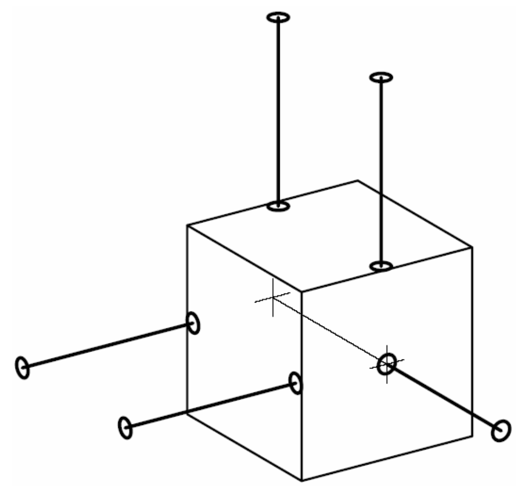
Figure: Layton Hales PhD Thesis, MIT.

Constraint fundamentals

Rigid bodies have 6 DOF

DOC = # of linearly independent constraints

DOF = 6 - DOC



A linear displacement can be visualized as a rotation about a point which is "far" away

Figure: Layton Hales PhD Thesis, MIT.

Statements

Points on a constraint line move perpendicular to the constraint line

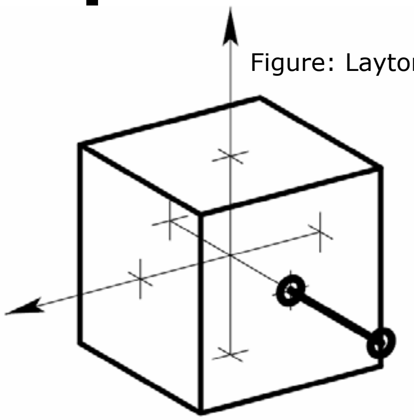
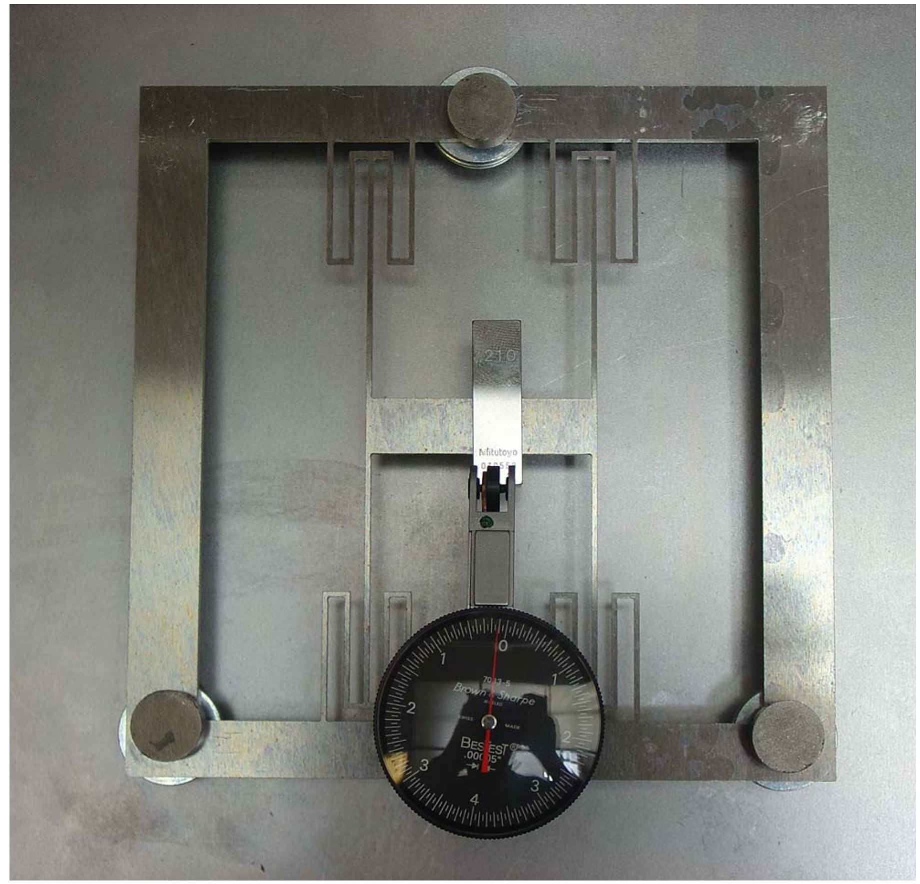


Figure: Layton Hales PhD Thesis, MIT.

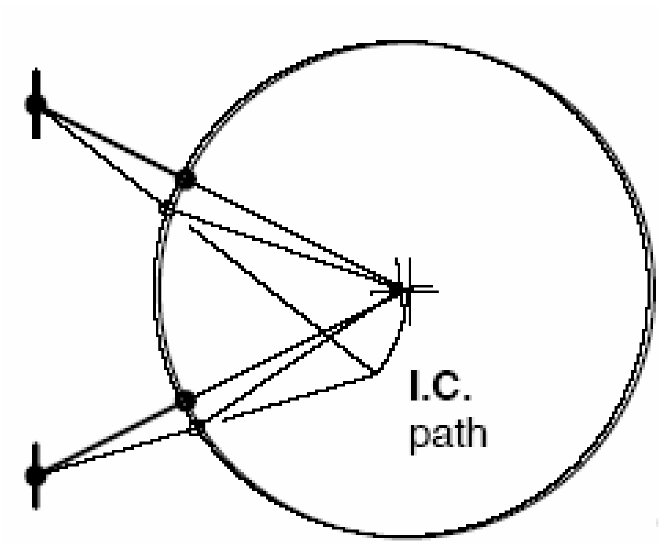
Constraints along this line are equivalent

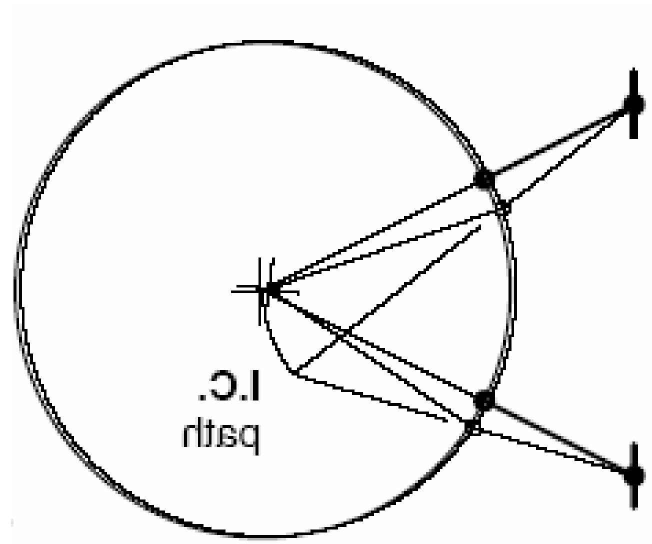
Diagram removed for copyright reasons.
Source: Blanding, D. L. *Exact Constraint: Machine Design using Kinematic Principles*. New York: ASME Press, 1999.



Statements

Intersecting, same-plane constraints are equivalent to other same-plane intersecting constraints

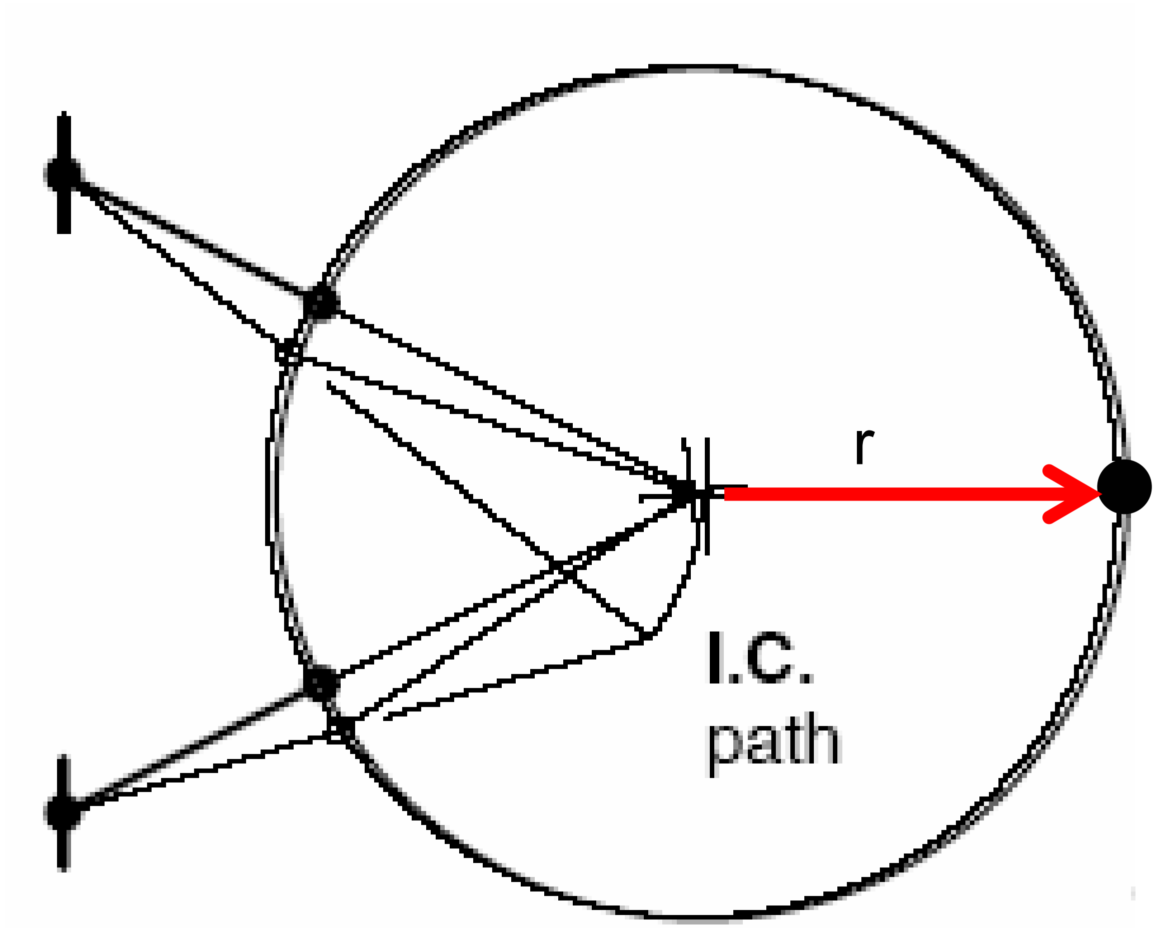




Instant centers are powerful tool for visualization, diagnosis, & synthesis

Abbe error

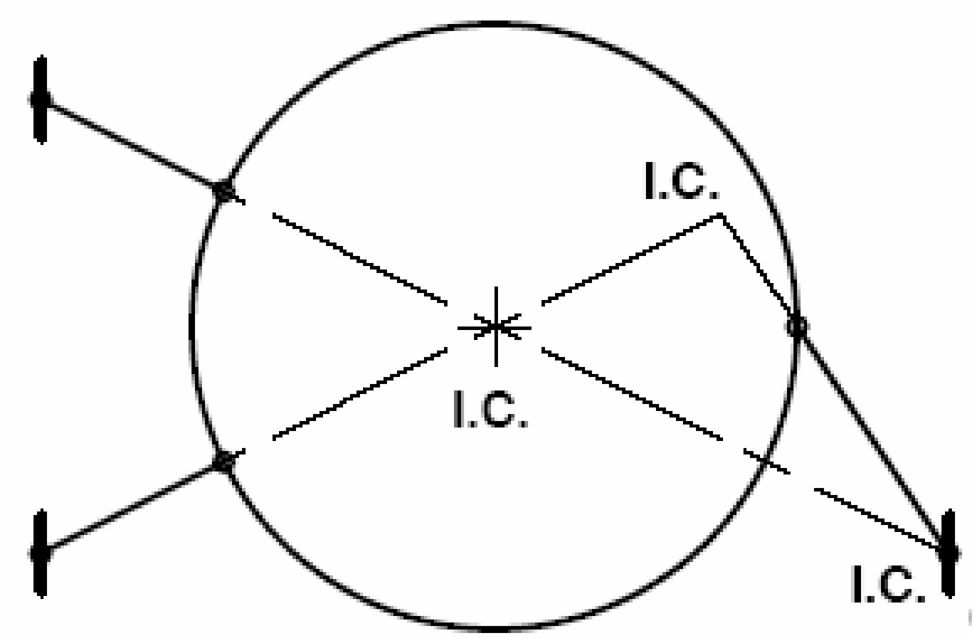
Error due to magnified moment arm



Statements

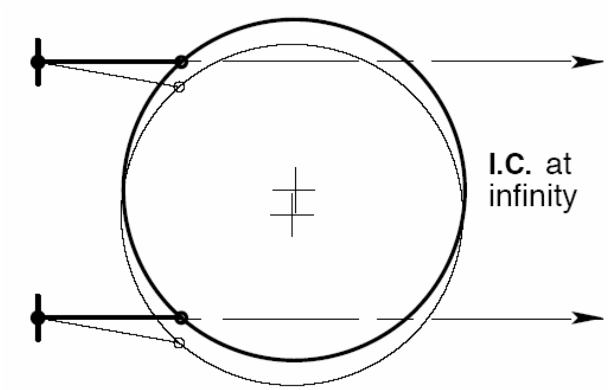
Constraints remove rotational degree of freedom

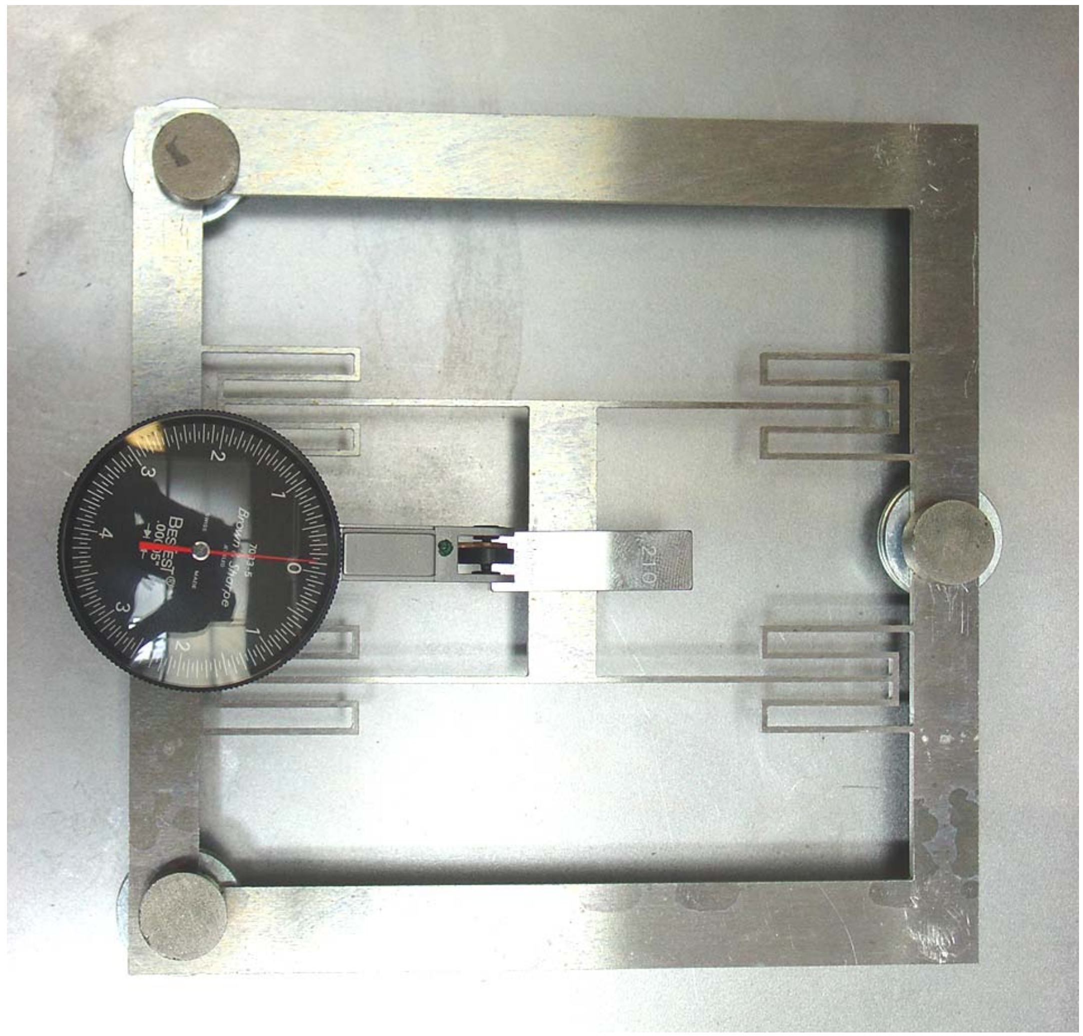
Length of moment arm determines the quality of the rotational constraint



Statements

Parallel constraints may be visualized/treated as intersecting at infinity





Past / next

We know how to lay out constraints to achieve a given behavior

Need to understand how to model and optimize them

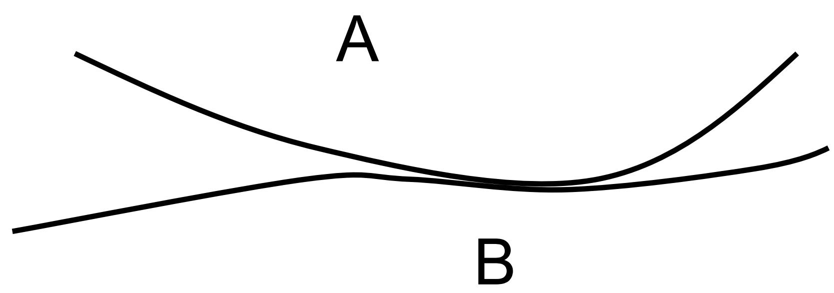
Reality checks and useful numbers for decision making

“Rigid” Constraints

Generic mechanical interface

Geometry & topology

- Relative form
- Surface finish



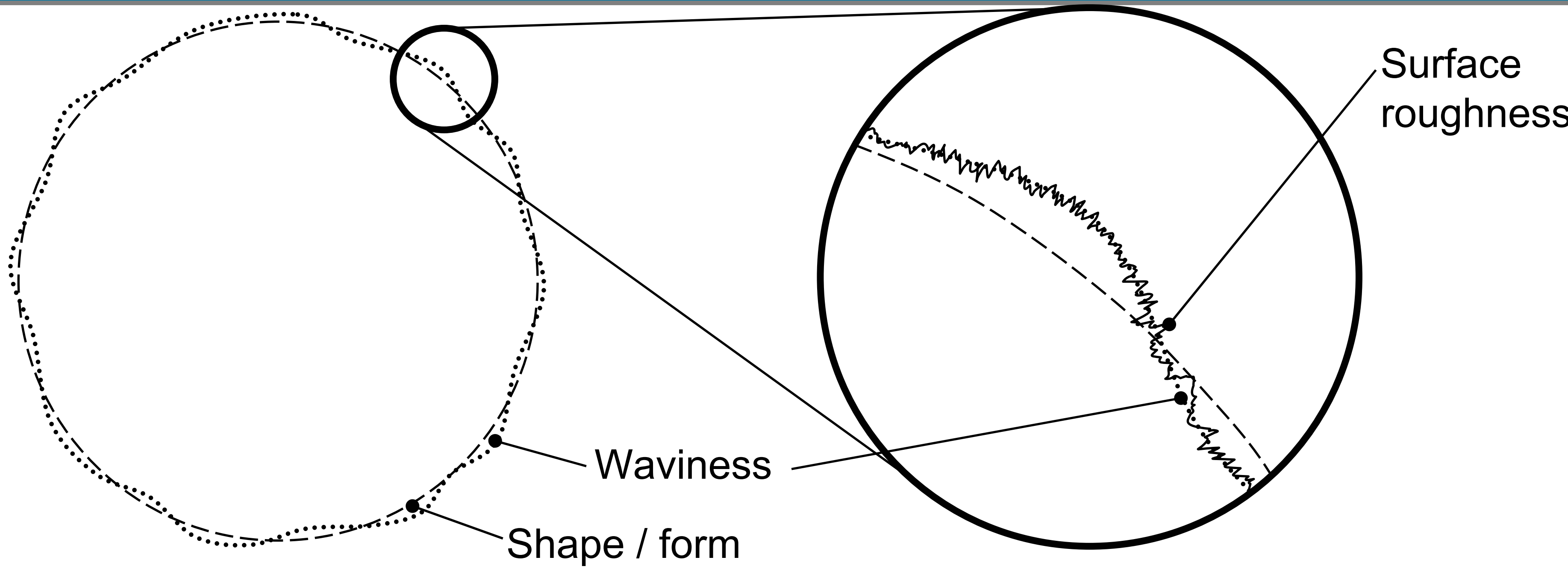
Material

- Modulus, Poisson's ratio, stress
- Oxide & contamination
- Surface energy

Flows

- Mass (wear)
- Momentum (interface force)
- Energy (elastic, thermal, etc…)

Local geometry



Waviness can be due to:

- ❑ Clamping forces
- ❑ Low frequency vibrations
- ❑ Equipment set up
- ❑ Bearing errors

Roughness can be due to:

- ❑ High frequency vibrations
- ❑ Tool geometry and rubbing
- ❑ Variations in material
- ❑ Chip contact

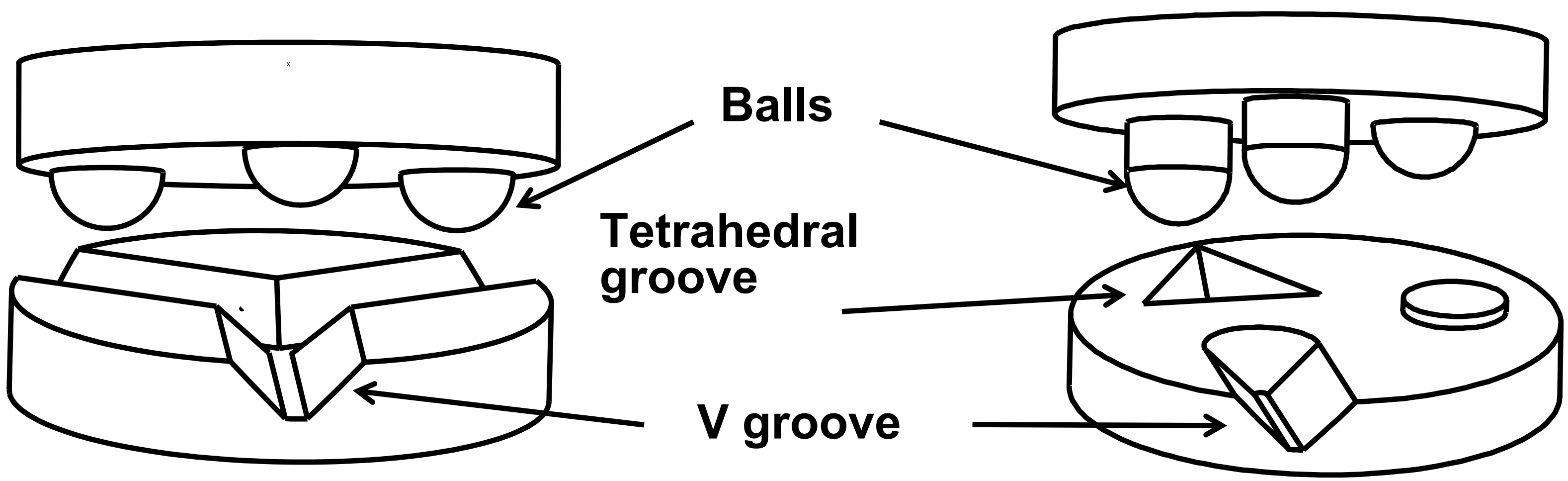
Exact constraint couplings

Exact constraint:

- Constraint points = DOF to be constrained
- Deterministic saves $
- Balls (inexpensive)
- Grooves (more difficult to make)

Kinematic design the issues are:

- KNOW what is happening in the system
- MANAGE forces, deflections, stresses and friction



Figures: Layton Hales PhD Thesis, MIT.

Passive kinematic couplings

Fabricate and forget: no active elements

Goal: Repeatable location

- (1/4 micron is the norm)

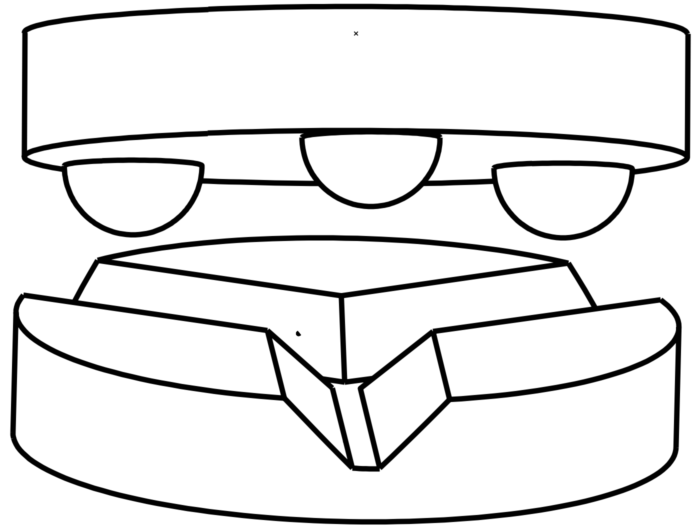


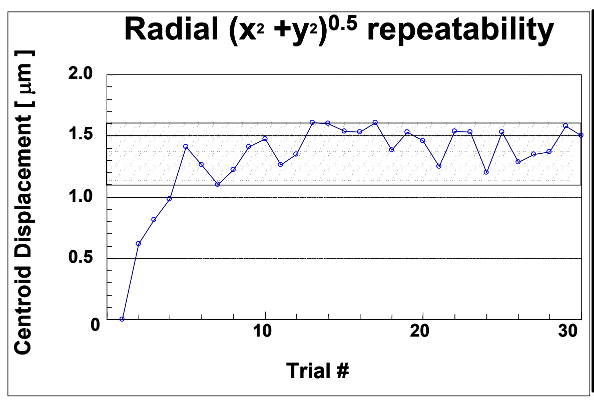
Figure: Layton Hales PhD Thesis, MIT.

What is important?

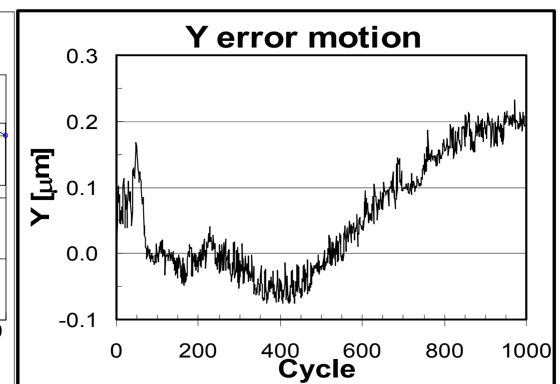
- Contact point & contact force direction
- Contact stress
- Stiffness
- Friction & hysteresis
- Order of engagement

Wear in period

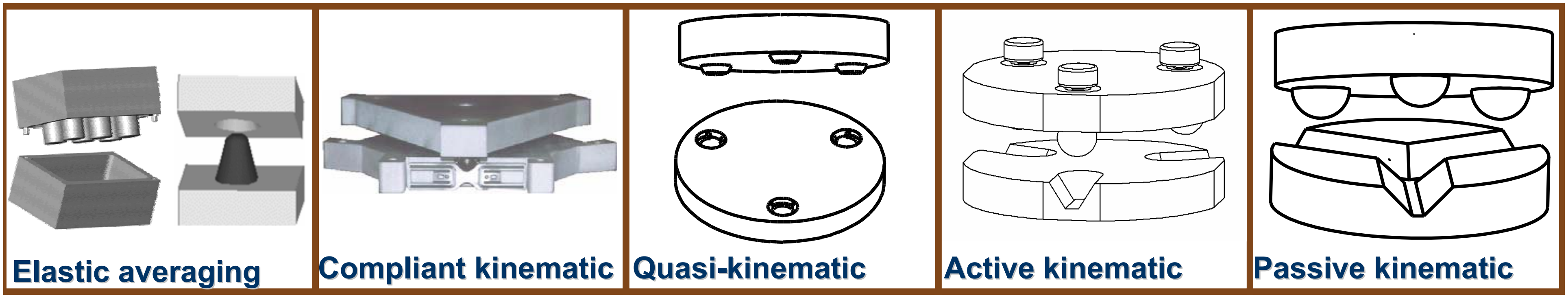
No flexures

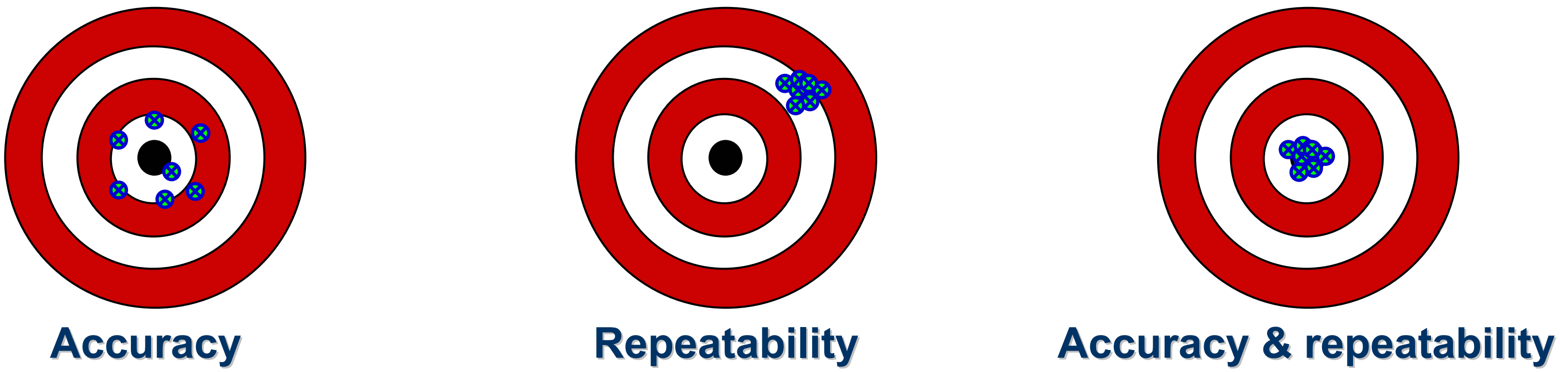


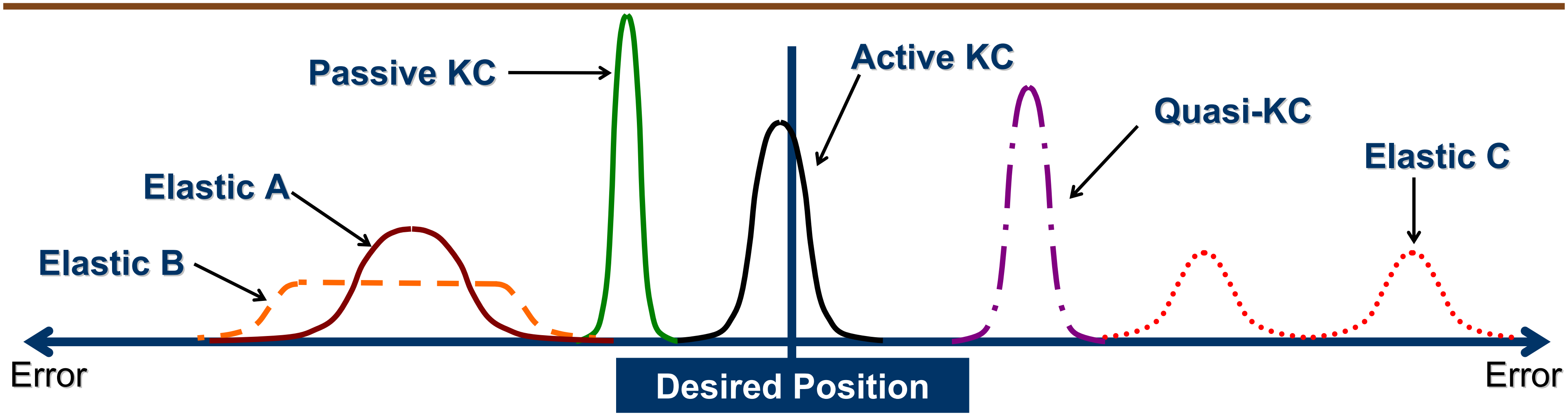
With flexures



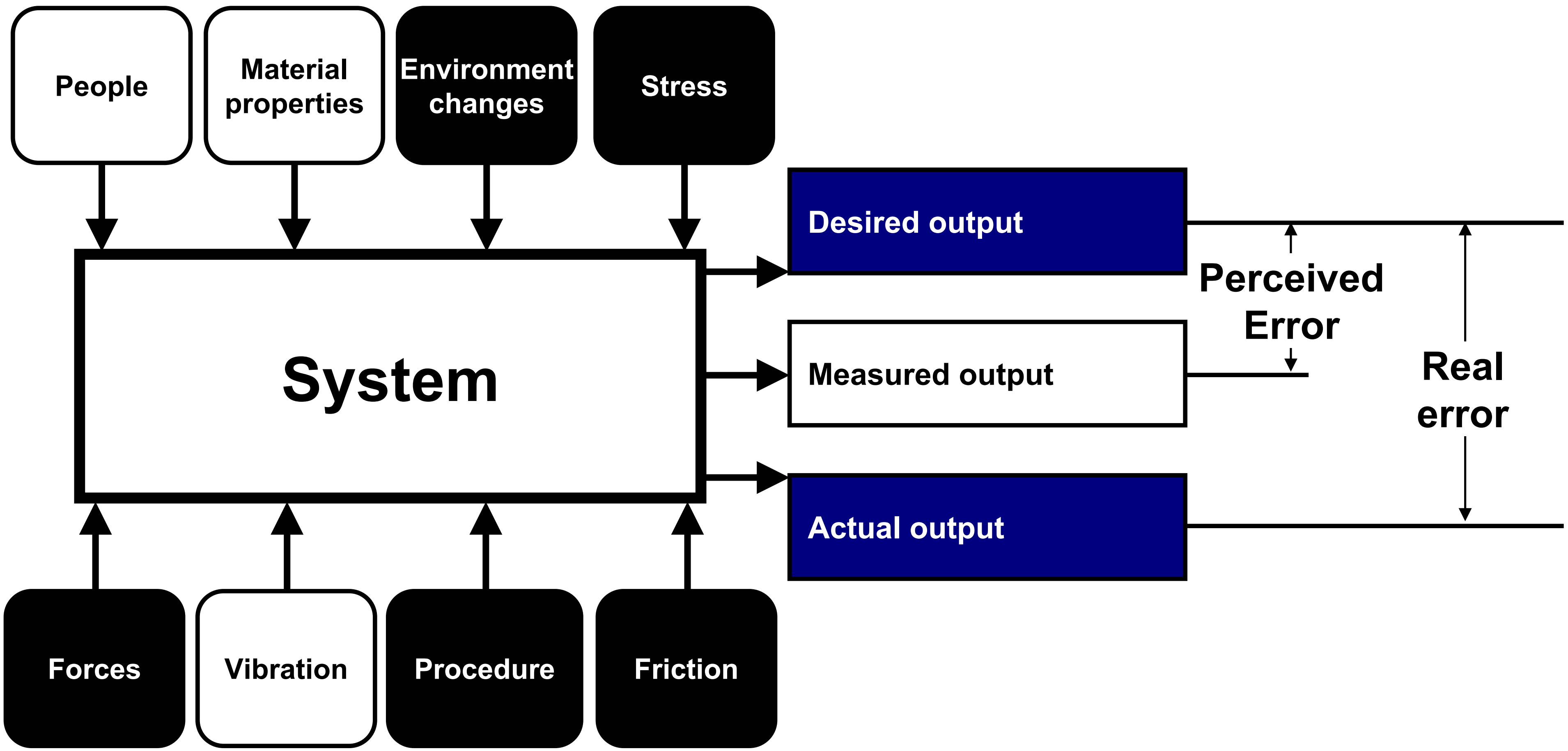
Common mechanical interfaces



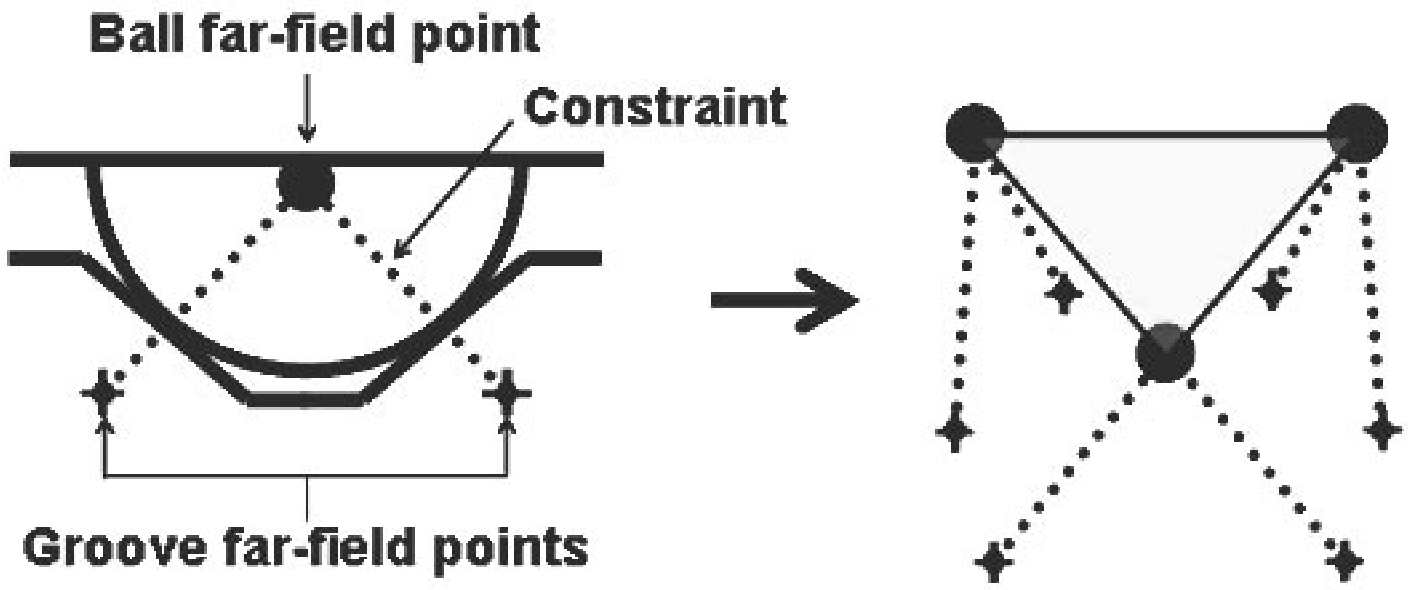




Minimize variation



Fixtures as mechanisms



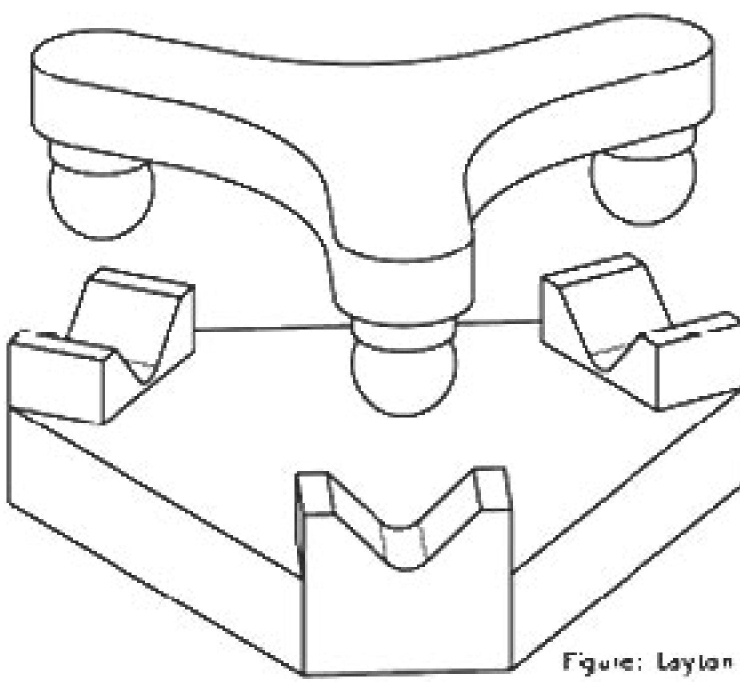


Figure: Layton Hales PhD Thesis, MIT.

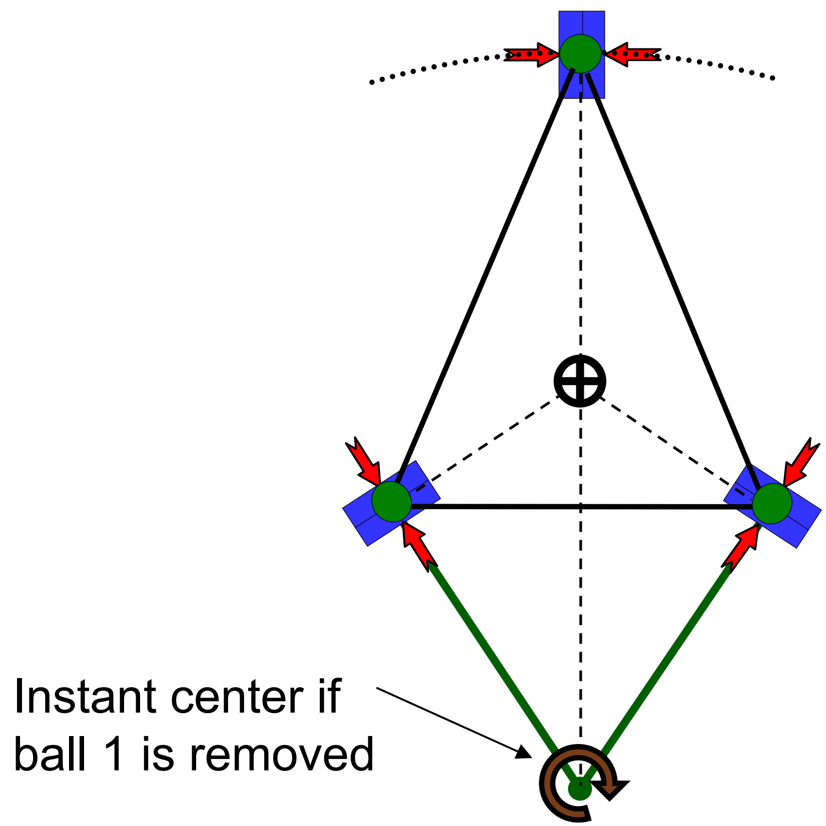
Stability & balanced stiffness

Instant center can help you identify how to best constrain or free up a mechanism

$$\frac{K_{\parallel}}{K_{\perp}} \cdot \frac{\delta_{\perp}}{\delta_{\parallel}} \rightarrow CM_k \cdot CM_{\delta} << 1$$

Diagram removed for copyright reasons.
Source: Alex Slocum, *Precision Machine Design*.

Poor **Good**



*Pictures from Precision Machine Design

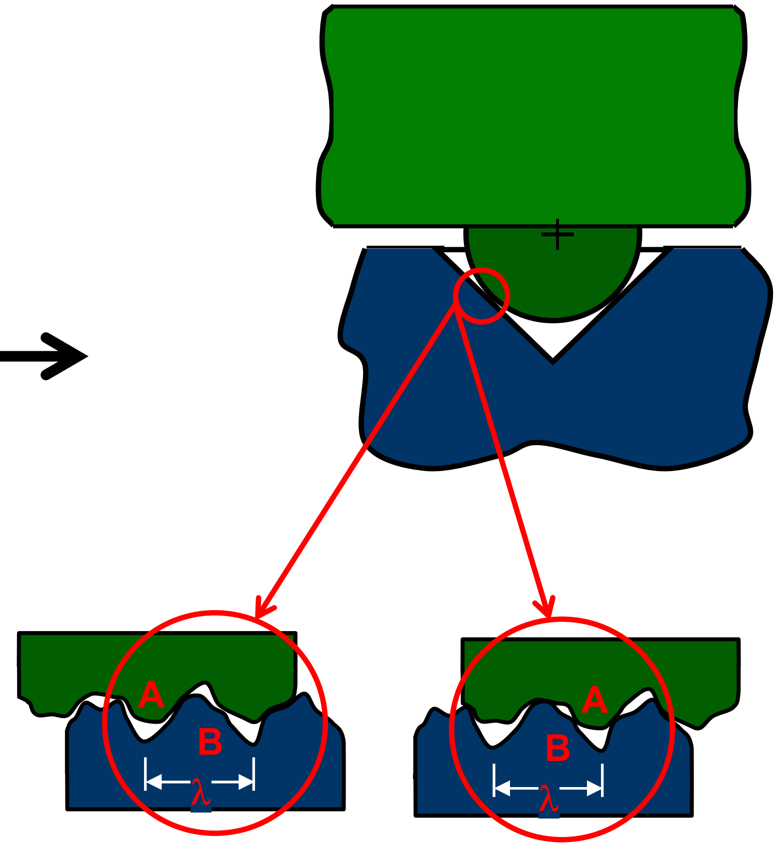
Source of error motion

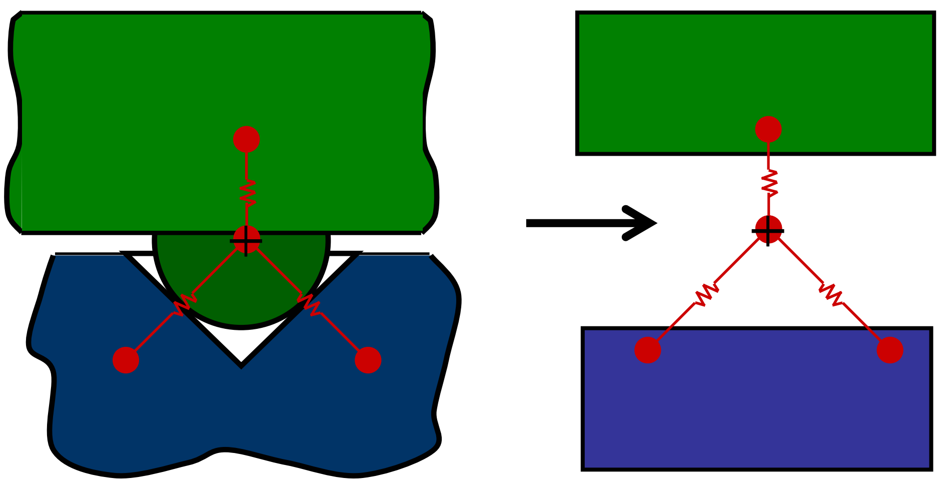
Perspective on couplings

- ❑Ideal: Couplings are static
- ❑Reality: Deflections = motion

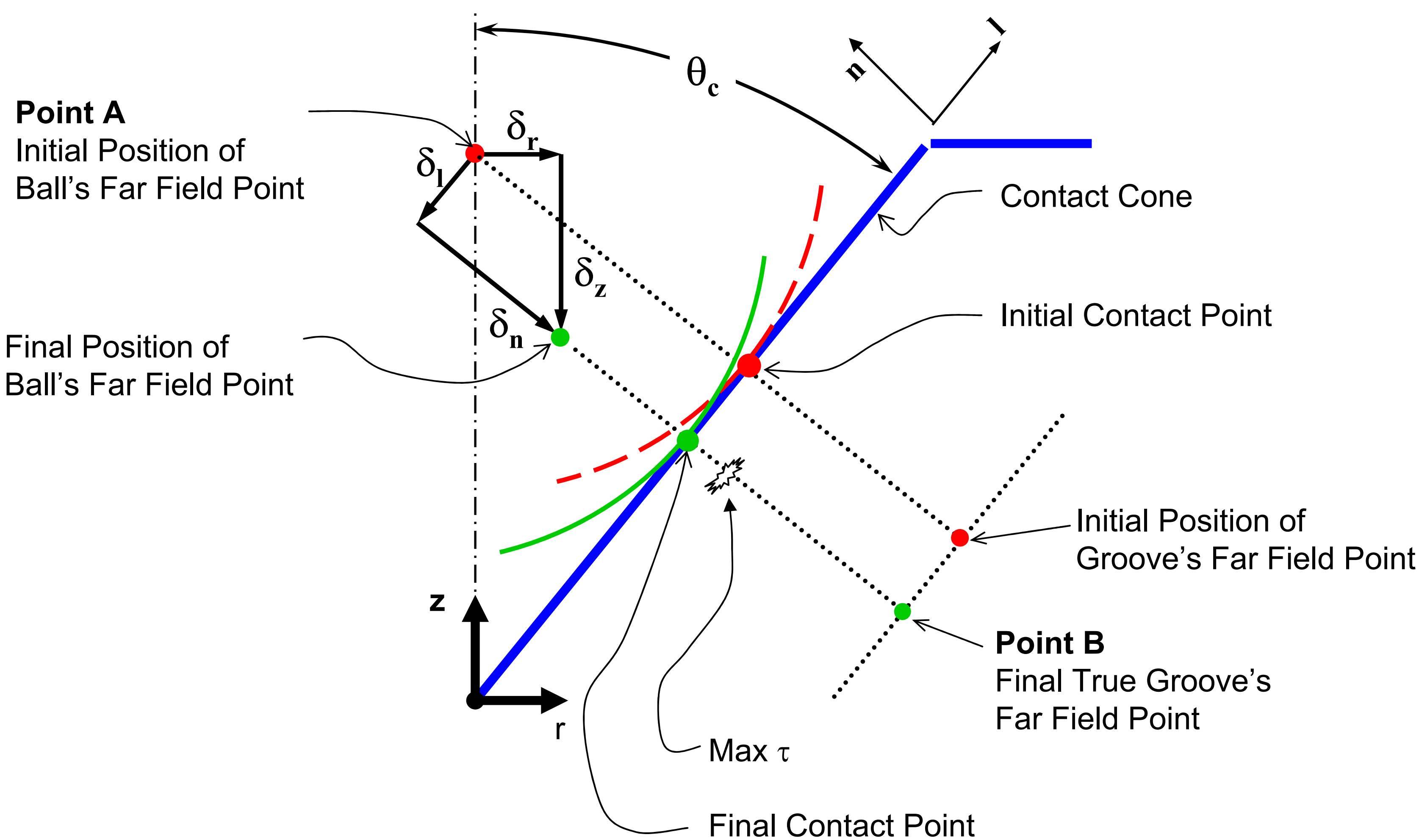
How/why they move

- ❑Friction and mating hysteresis
- ❑Contact deflections





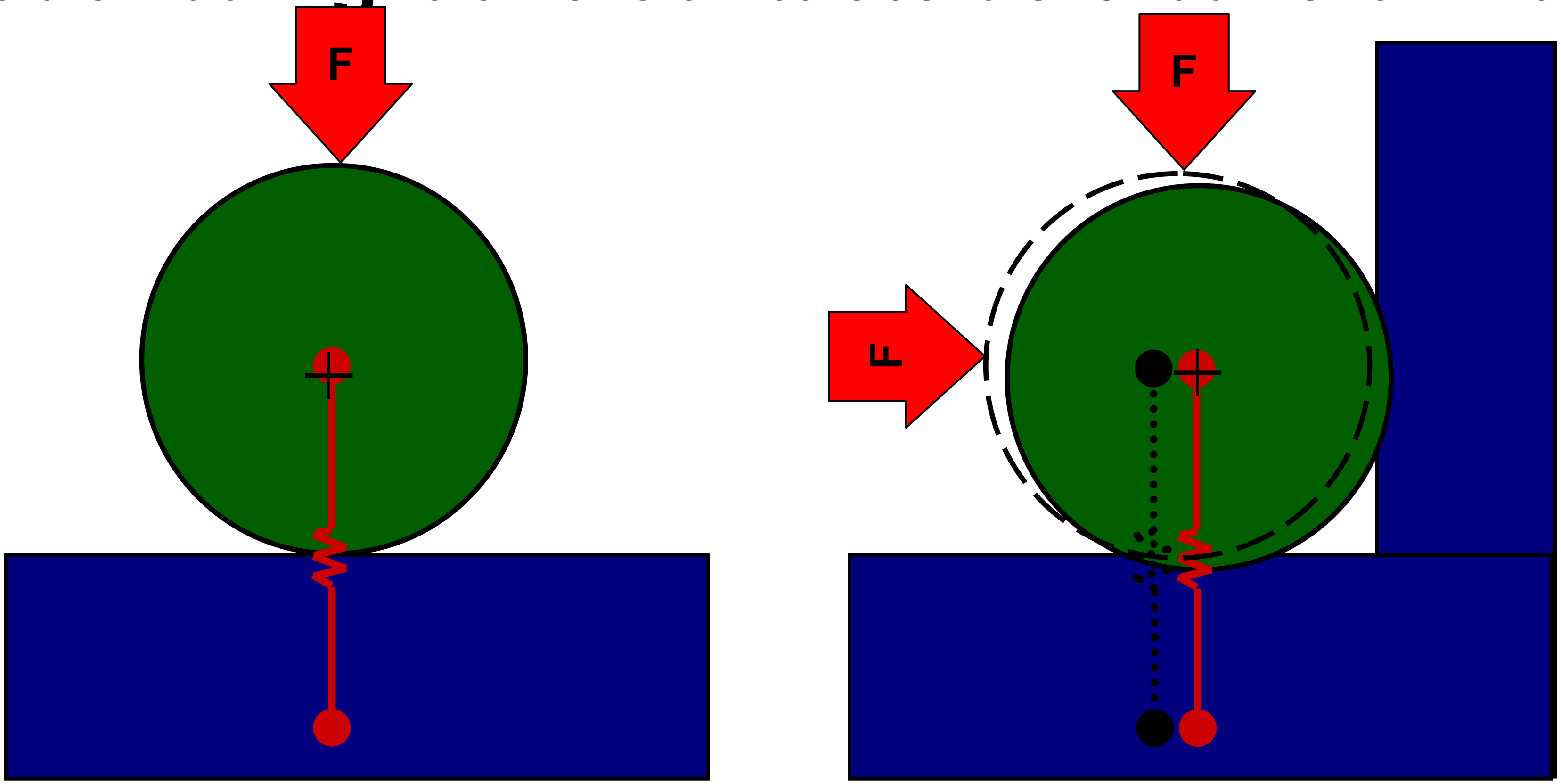
Distance of approach (δ_n)



Max shear stress occurs below surface, in the member with largest R

Solving for coupling errors

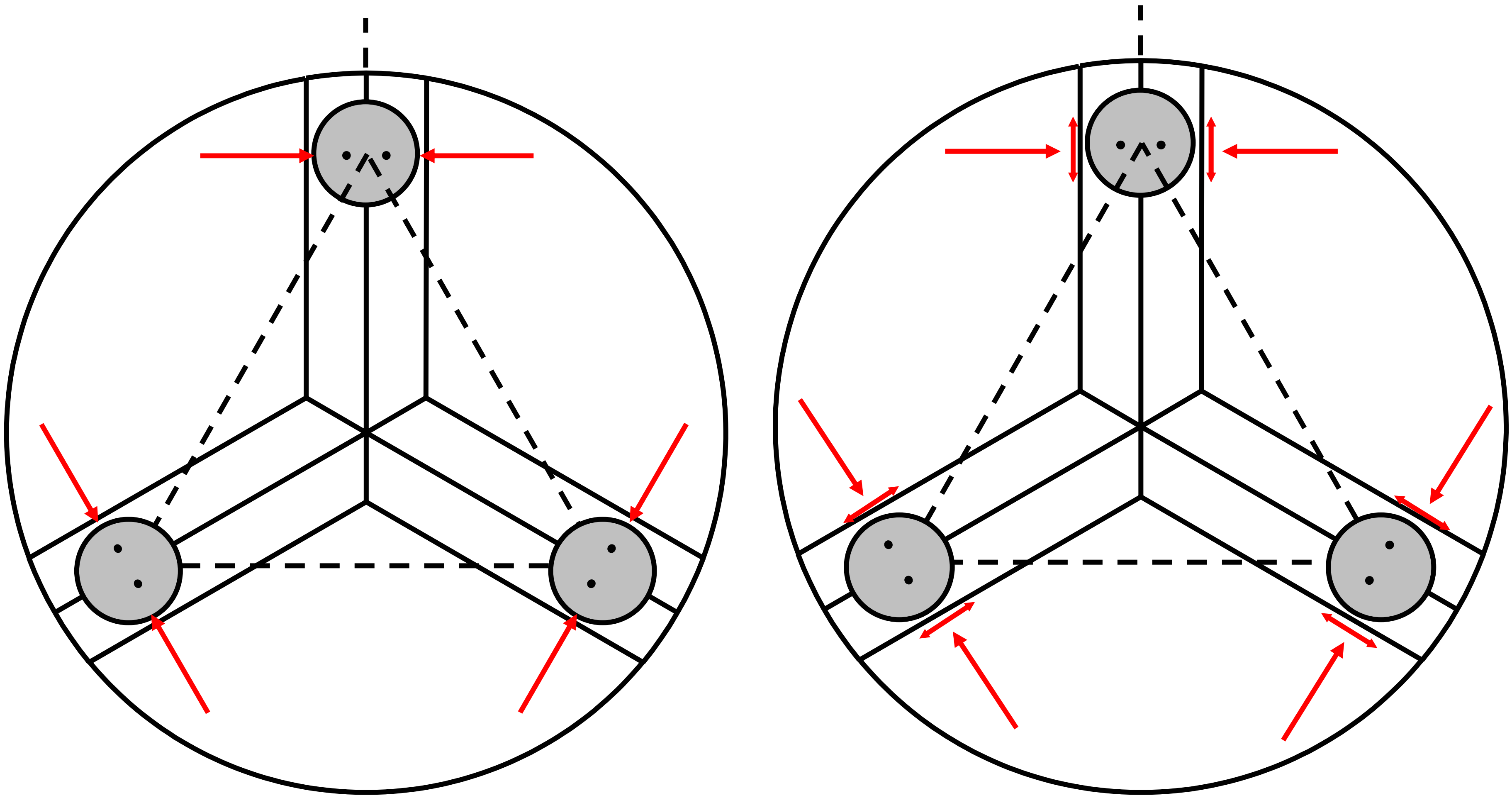
Model ball-groove contacts as 6 balls on flats



Important relationships for ball-flat contact

Contact pressure	Distance of approach	Stiffness [normal to groove surface]
$a=\left(\frac{3}{4}\cdot\frac{F_n\cdot R_e}{E_e}\right)^{1/3}$ $q=\frac{3}{2\pi}\cdot\frac{F}{a^2}<1.5\cdot\sigma_{tensile}$ for metals	$\delta_n=\left(\frac{9}{16}\cdot\frac{F^2}{R\cdot E_e^{\ 2}}\right)^{1/3}$	$k_n=2\cdot\delta^{1/2}\cdot R^{1/2}\cdot E_e$

System of six contact points



Ideal in-plane constraints **Real in-plane constraints**

Modeling round interfaces

Equivalent radius

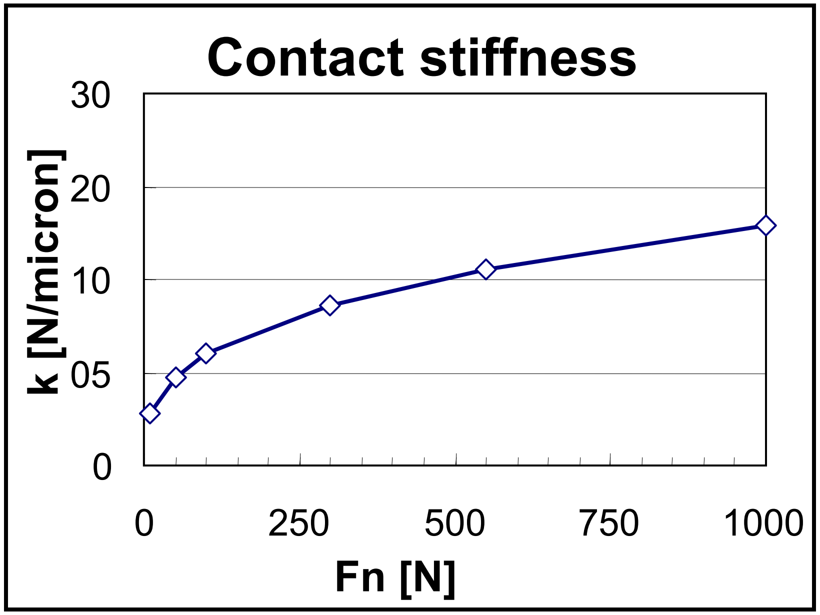
$$R_e = \frac{1}{\frac{1}{R_{1major}} + \frac{1}{R_{1minor}} + \frac{1}{R_{2major}} + \frac{1}{R_{2minor}}}$$

Equivalent modulus

$$E_e = \frac{1}{\frac{1-\eta_1^2}{E_1} + \frac{1-\eta_2^2}{E_2}}$$

Poisson's ratio

Young's modulus



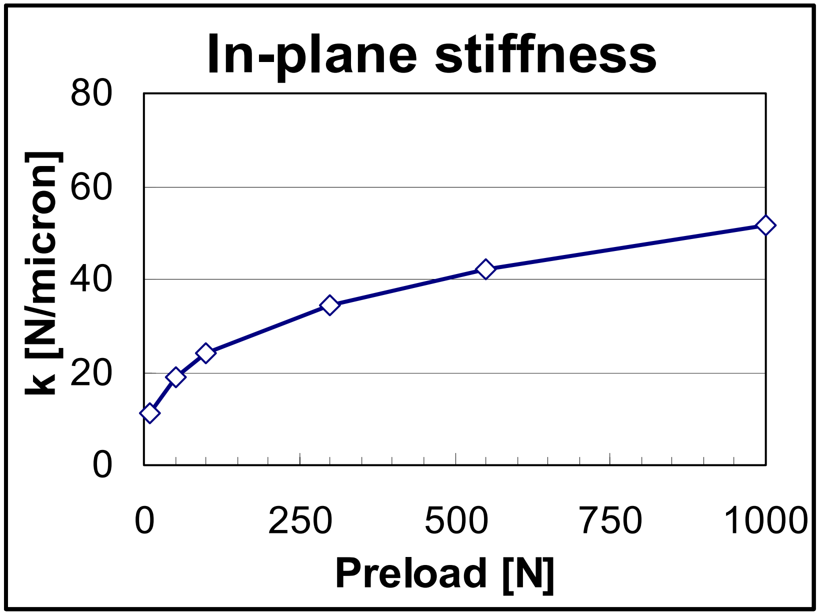
$$\delta_n = \left(\frac{9}{16} \cdot \frac{F_n^2}{R_e \cdot E_e^2} \right)^{1/3}$$

Important scaling law

$$k_n(\delta_n) = \left(2 \cdot R_e^{0.5} \cdot E_e\right) \cdot \delta_n^{0.5} \longrightarrow k_n(F_n) = \text{Constant} \cdot \left(R_e^{1/3} \cdot E_e^{2/3}\right) \cdot F_n^{1/3}$$

Importance of preload

Preload keeps the fixture parts together



Increase preload:

- increase ball-groove in contact
- increase contact stiffness

$$\delta_n = \left(\frac{9}{16} \cdot \frac{F_n^{\,2}}{R_e \cdot E_e^{\,2}} \right)^{1/3} \qquad a = \left(\frac{3}{4} \cdot \frac{F_n \cdot R_e}{E_e} \right)^{1/3}$$

Stiffness scales as:

$$k_n(\delta_n) = \frac{dF}{d\delta_n}$$

Important scaling law

$$k_n(\delta_n) = \left(2 \cdot R_e^{\,0.5} \cdot E_e\right) \cdot \delta_n^{\,0.5} \qquad k_n(F_n) = \text{Constant} \cdot \left(R_e^{1/6} \cdot E_e^{\,2}\right) \cdot F_n^{1/3}$$

Preload should have high repeatability

Load balance: Ford and moment

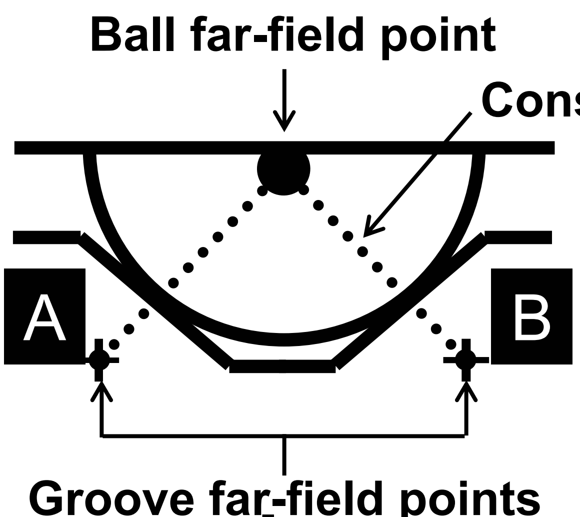
Force balance (3 equations)

$$\Sigma\vec{F}_{relative} = 0 = \left(\vec{F}_{preload} + \vec{F}_{Error}\right) + \left(\vec{F}_{Ball\_1} + \vec{F}_{Ball\_2} + \vec{F}_{Ball\_3} + \vec{F}_{Ball\_4} + \vec{F}_{Ball\_5} + \vec{F}_{Ball\_6}\right)$$

Moment balance (3 equations)

$$\vec{M}_{relative} = \Sigma^{6}{}_{i=1}\left(\vec{M}_{Ball\_i}\right) + \left(\vec{M}_{preload}\right) + \left(\vec{M}_{error}\right) = \left(\vec{r}_{preload} \times \vec{F}_{preload} + \vec{r}_{error} \times \vec{F}_{Error}\right) + \Sigma\vec{r}_{Ball\_i} \times \vec{F}_{Ball\_i}$$

$$\Sigma\vec{M}_{relative} = \left(\vec{r}_{preload} \times \vec{F}_{preload} + \vec{r}_{error} \times \vec{F}_{Error}\right) + \left(\vec{r}_{Ball\_1} \times \vec{F}_{Ball\_1} + \vec{r}_{Ball\_2} \times \vec{F}_{Ball\_2} + \vec{r}_{Ball\_3} \times \vec{F}_{Ball\_3} + \vec{r}_{Ball\_4} \times \vec{F}_{Ball\_4} + \vec{r}_{Ball\_5} \times \vec{F}_{Ball\_5} + \vec{r}_{Ball\_6} \times \vec{F}_{Ball\_6}\right)$$



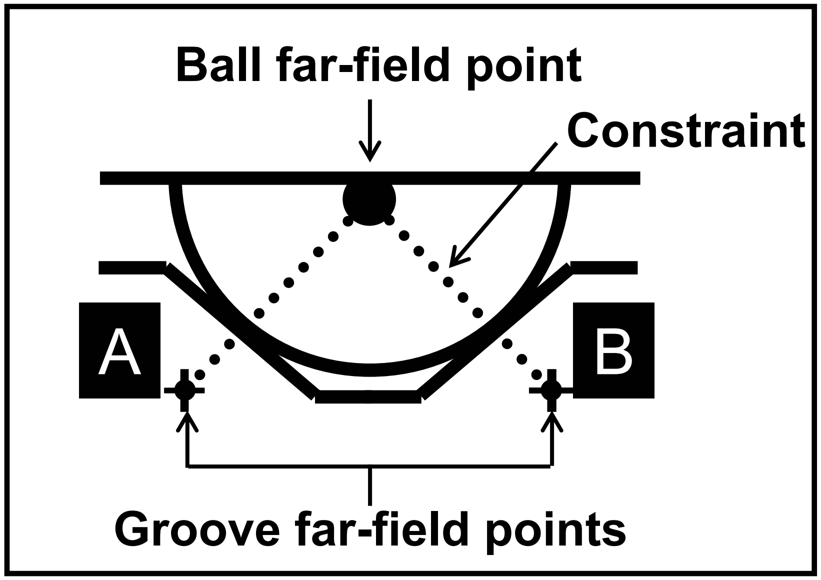
Given geometry, materials, preload force, error force, solve for local distance of approach

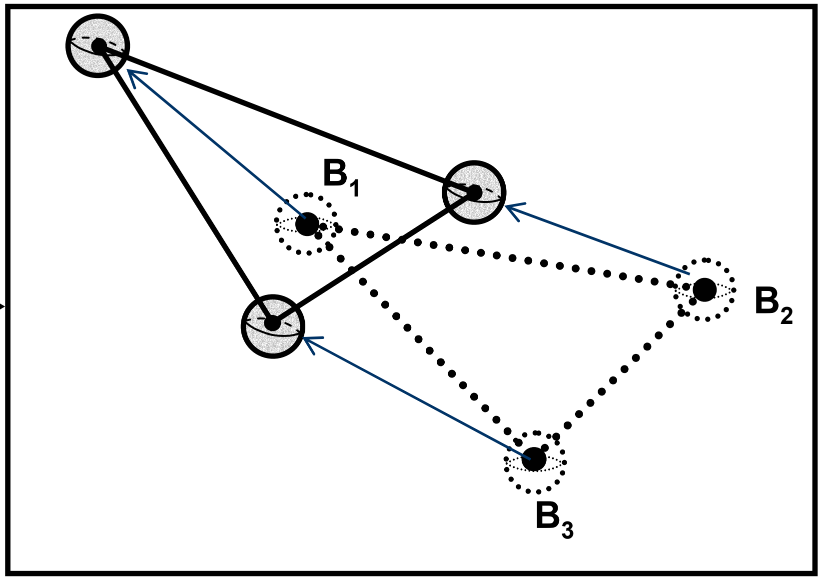
Ball motions: Displacements

$$\Sigma\vec{\delta}_{n\_Ball\_iA} = -\left(\frac{9}{16}\cdot\frac{\left|F_n{}^2\right|}{R_e\cdot E_e{}^2}\right)^{1/3}\Bigg|_{Ball\_iA}\cdot\hat{n}_{Ball\_iA}$$

$$\Sigma\vec{\delta}_{n\_Ball\_iB} = -\left(\frac{9}{16}\cdot\frac{\left|F_n{}^2\right|}{R_e\cdot E_e{}^2}\right)^{1/3}\Bigg|_{Ball\_iB}\cdot\hat{n}_{Ball\_iB}$$

$$\Sigma\vec{\delta}_{Ball\_i} = \vec{\delta}_{Ball\_iA} + \vec{\delta}_{Ball\_iB}$$





Coupling motions

Given 3 points (e.g. ball centers)

- ❑(1) Define centroid of the points & attach CS
- ❑(2) Define plane equation & normal vector at CS

Compliance errors cause plane/normal to shift

Differences in coefficients give:

- ❑Centroid (δx, δy, θz)
- ❑Normal vector (θx, θy, δz)

- ❑For small θ_i

  $\varepsilon_z \sim \theta_i$

  $\sin(\theta_i) \sim \theta_i$

  $\cos(\theta_i) \sim 1$

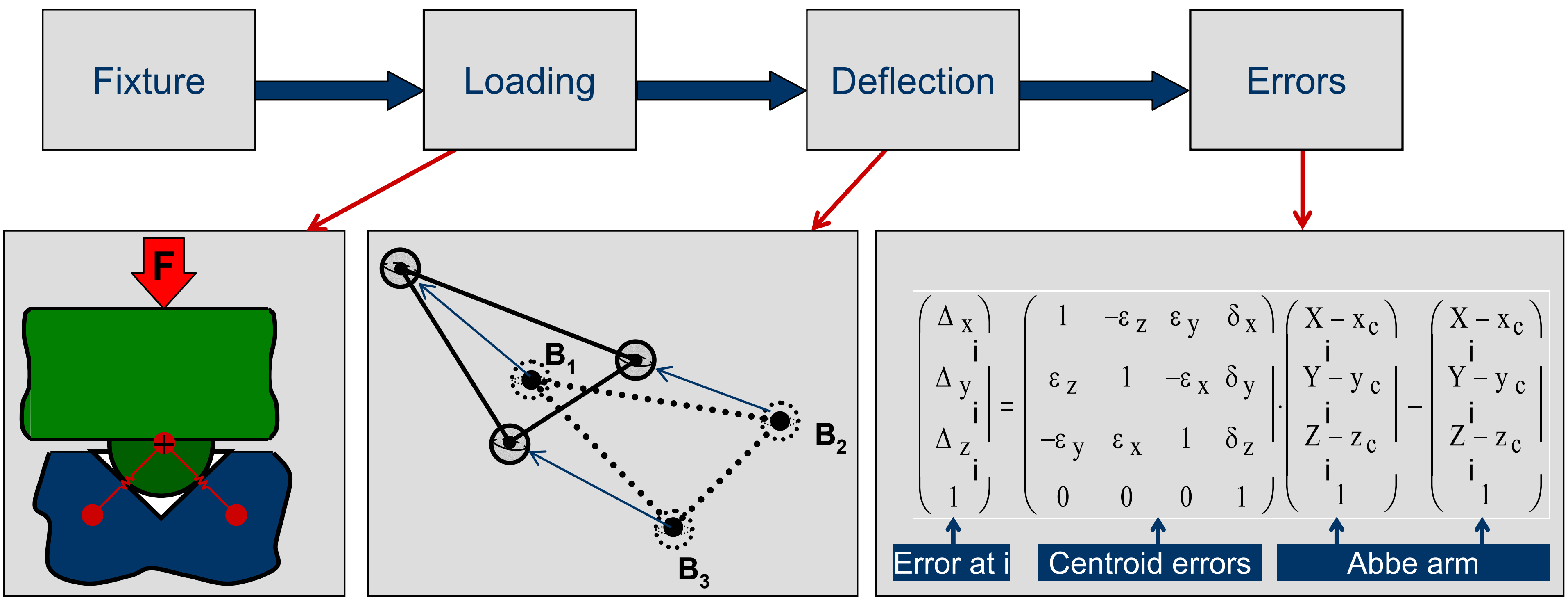
Solving for coupling errors

Step 1: Analyze ball center displacements

Step 2: Obtain centroid displacements

- Translations: $\delta_x, \delta_y, \delta_z$ Rotations: $\varepsilon_x, \varepsilon_y, \varepsilon_z$

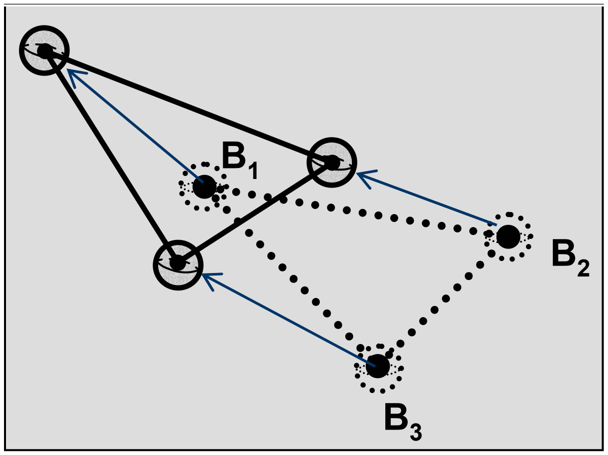
Step 3: Calculating error at any point



Solving for coupling errors

Error | **HTM** | **Amplification Arm** | **Distance from centroid**

$$\begin{pmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & -\varepsilon_z & \varepsilon_y & \delta_x \\ \varepsilon_z & 1 & -\varepsilon_x & \delta_y \\ -\varepsilon_y & \varepsilon_x & 1 & \delta_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} X_i - x_c \\ Y_i - y_c \\ Z_i - z_c \\ 1 \end{pmatrix} - \begin{pmatrix} X_i - x_c \\ Y_i - y_c \\ Z_i - z_c \\ 1 \end{pmatrix}$$



Environmental: Temperature

Thermal expansion

$$\Delta x = \alpha \cdot \Delta T \cdot x$$

	α μm/°C/m	μinch/°F/inch
Cast iron	13	7
Carbon steel [AISI 1005]	13	7
Stainless steel [AISI 303]	18	10
Aluminum 6061 T6	24	13
Yellow brass	20	11
Delrin (polymer)	85	47

Which is better with respect to in-plane errors if:

- ❑ $\Delta T_{top} > \Delta T_{bottom}$
- ❑ $\Delta T_{top} = \Delta T_{bottom}$
- ❑ $\Delta T_{top} < \Delta T_{bottom}$

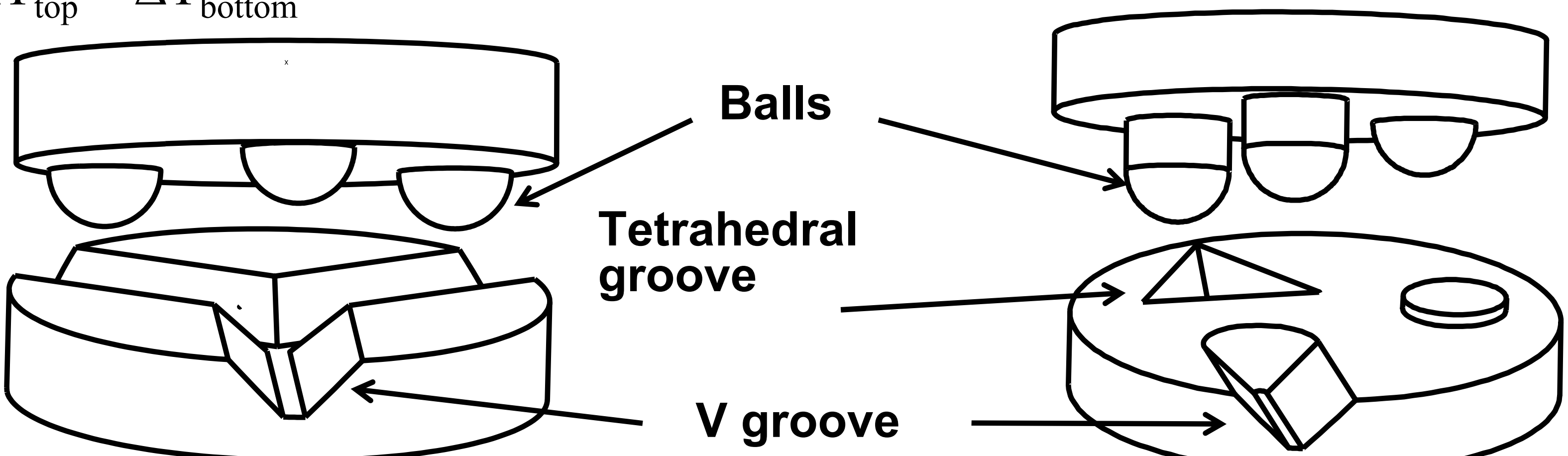
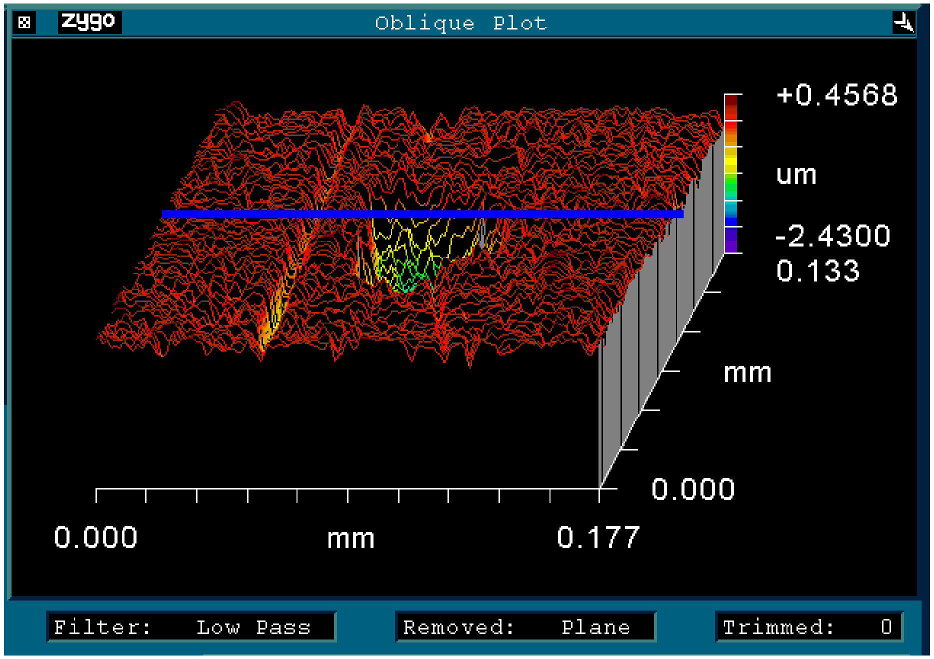
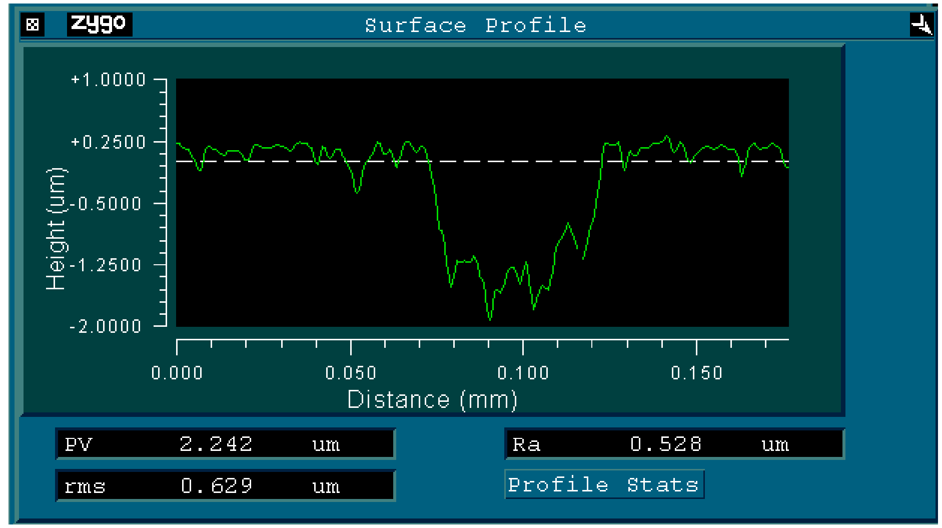


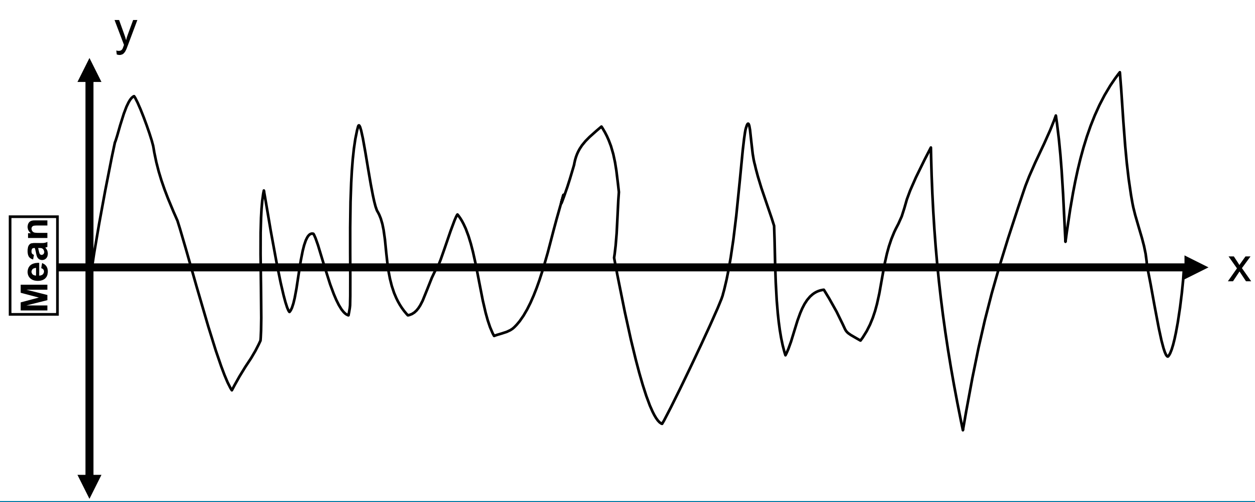
Figure: Layton Hales PhD Thesis, MIT.

Example: Surface finish trace & metric





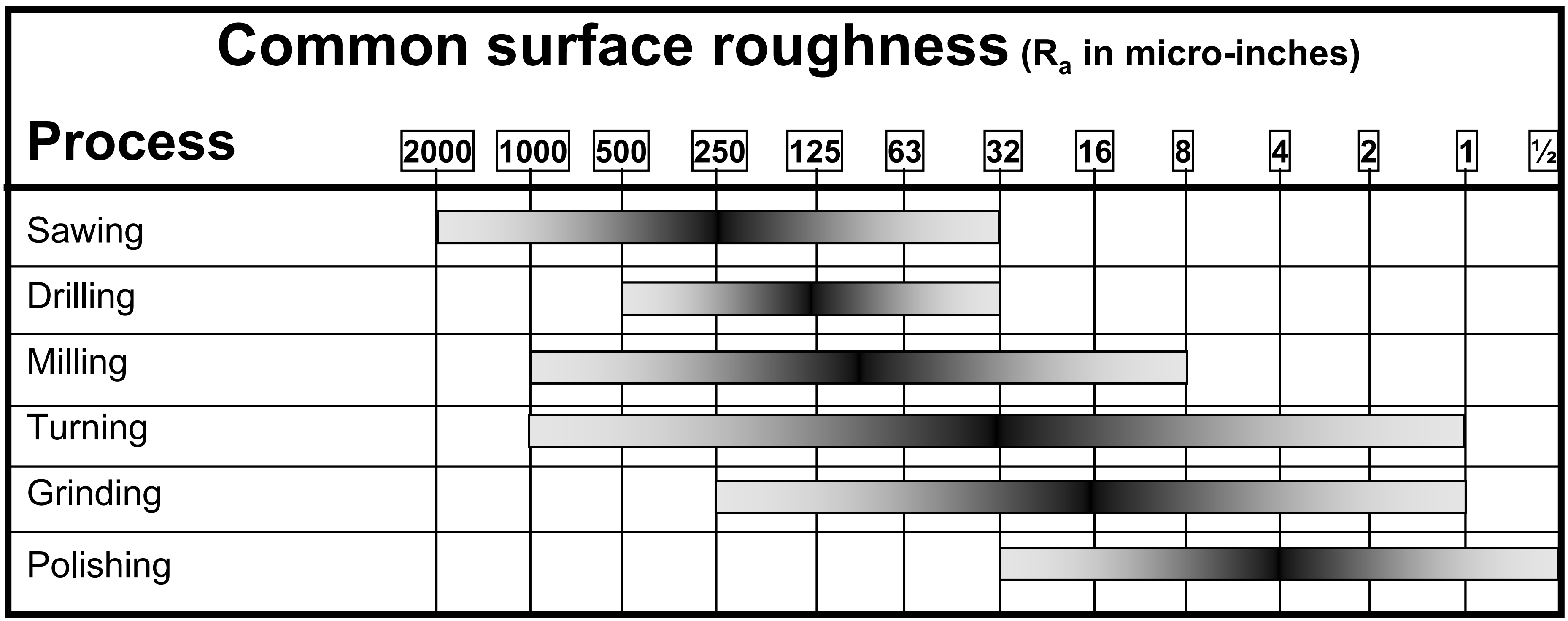
$$R_a = \frac{1}{L} \cdot \int_0^L |y| \, dx$$



Surface finish

Producing fine surface is expensive/time consuming

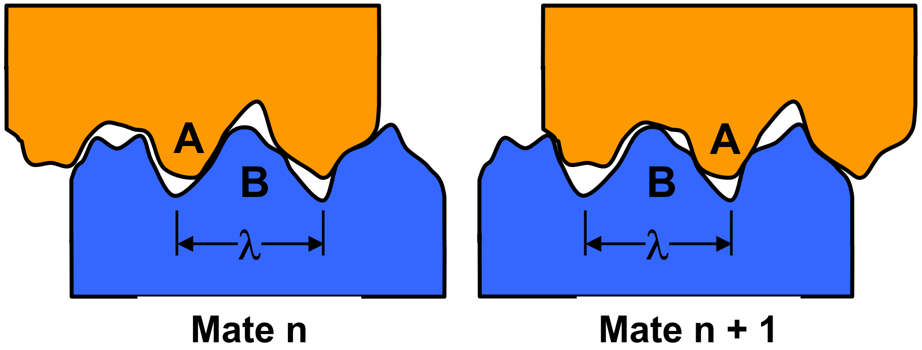
Reference chart:



Contact problems

Surface topology (finish):

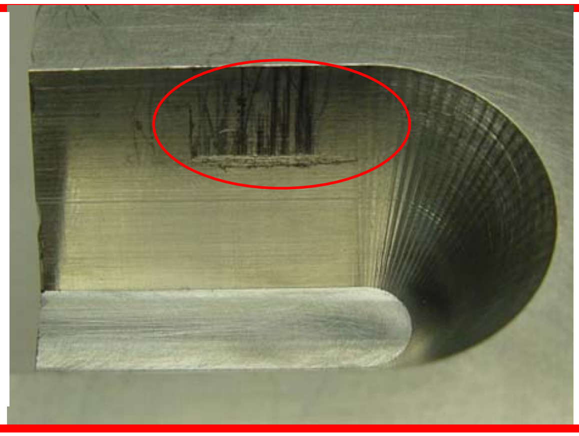
- 50 cycle repeatability ~ 1/3 μm Ra
- Friction depends on surface finish!
- Finish should be a design spec
- Surface may be brinelled if possible



Wear and Fretting:

- High stress + sliding = wear
- Metallic surfaces = fretting
- Use ceramics if possible (low μ and high strengt
- Dissimilar metals avoids "snowballing"

Wear on Groove



Friction:

- Friction = Stored energy, over constraint
- Flexures can help (see right)
- Lubrication (high pressure grease) helps
- Beware of settling time

Image removed for copyright reasons.
See Figure 4, "Ball in a V Groove…" in
Schouten, et. al., "Design of a kinematic
coupling for precision applications."
Precision Engineering, vol. 20, 1997.

Repeatability with & w/o lubrication

The trend of the data is important

Magnitude depends on coupling design and test conditions

Slocum, A. H., Precision Engineering, 1988: Kinematic couplings for precision fixturing–Experimental determination of repeatability and stiffness

Radial Repeatability (Unlubricated)

Displacement, µm

2

Graph removed for copyright reasons. See Slocum, A. H. "Kinematic couplings for precision fixturing: Experimental determination of repeatability and stiffness." Precision Engineering, 1988.

0

Number of Trials

600

Radial Repeatability (Lubricated)

Displacement, µm

2

Graph removed for copyright reasons. See Slocum, A. H. "Kinematic couplings for precision fixturing: Experimental determination of repeatability and stiffness." Precision Engineering, 1988.

0

Number of Trials

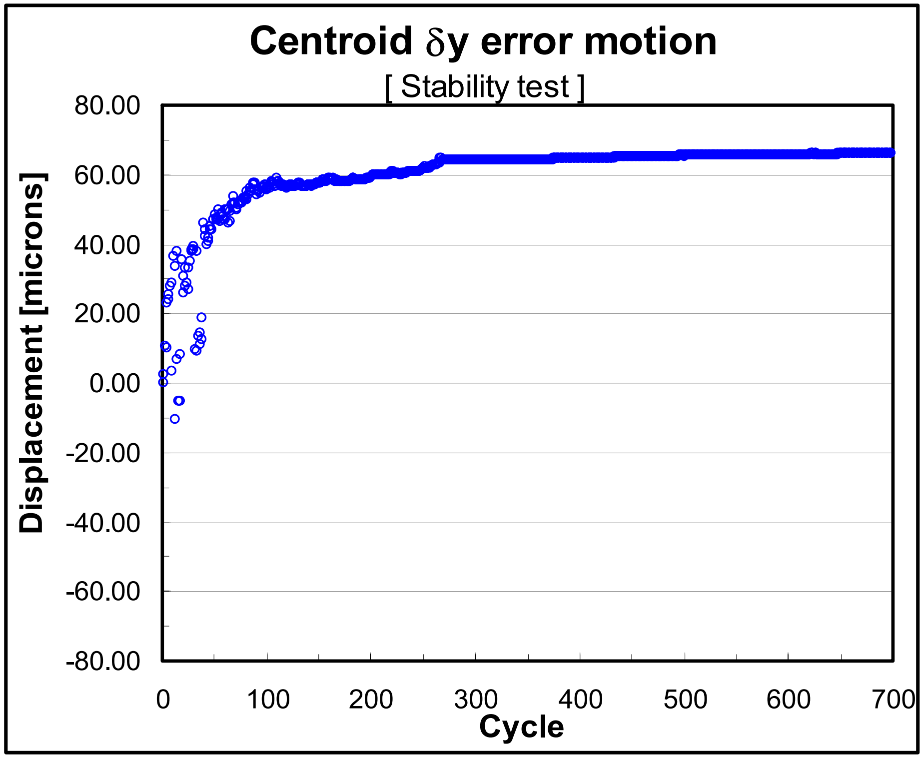
600

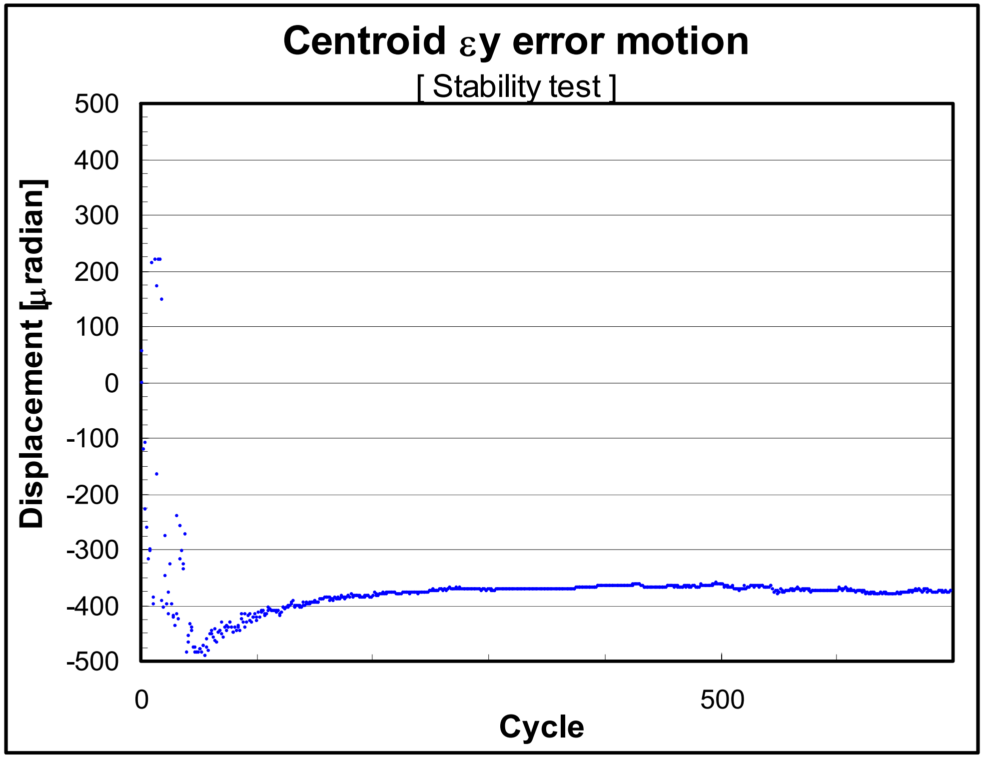
Effect of Hard Coatings

What should be the affect of adding TiN coating?

- ❑ TiN = low friction
- ❑ TiN = high surface energy

Results of non-lubricated, TiN coated tests





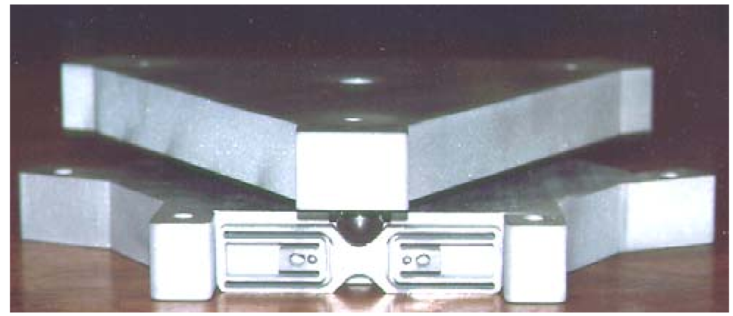
Flexible constraints

Adding and taking away constraints

Why? Want less than 6 DOF constraint

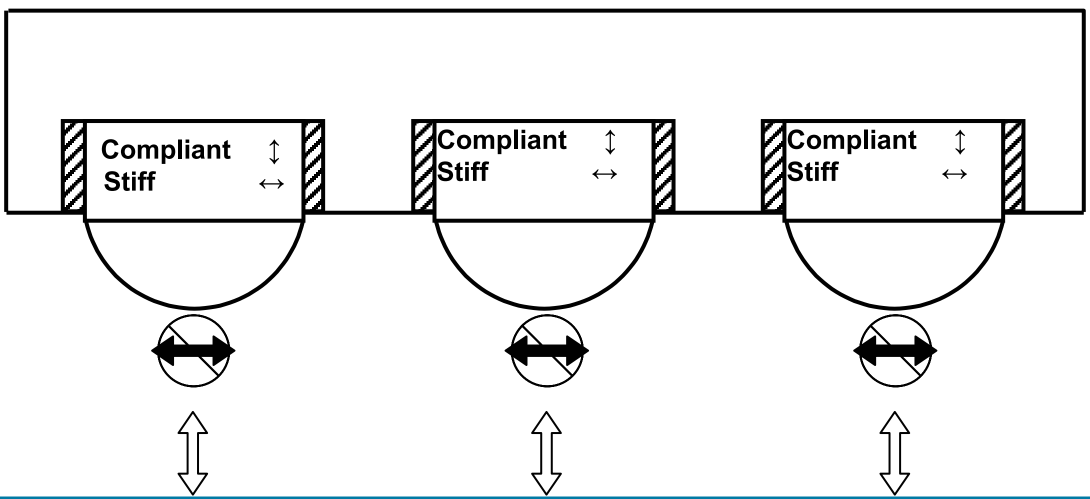
E.g.: Planar alignment Δz for sealing/stiffness

- ❑ Add compliance to permit z motion
- ❑ This is equivalent to adding a DOF

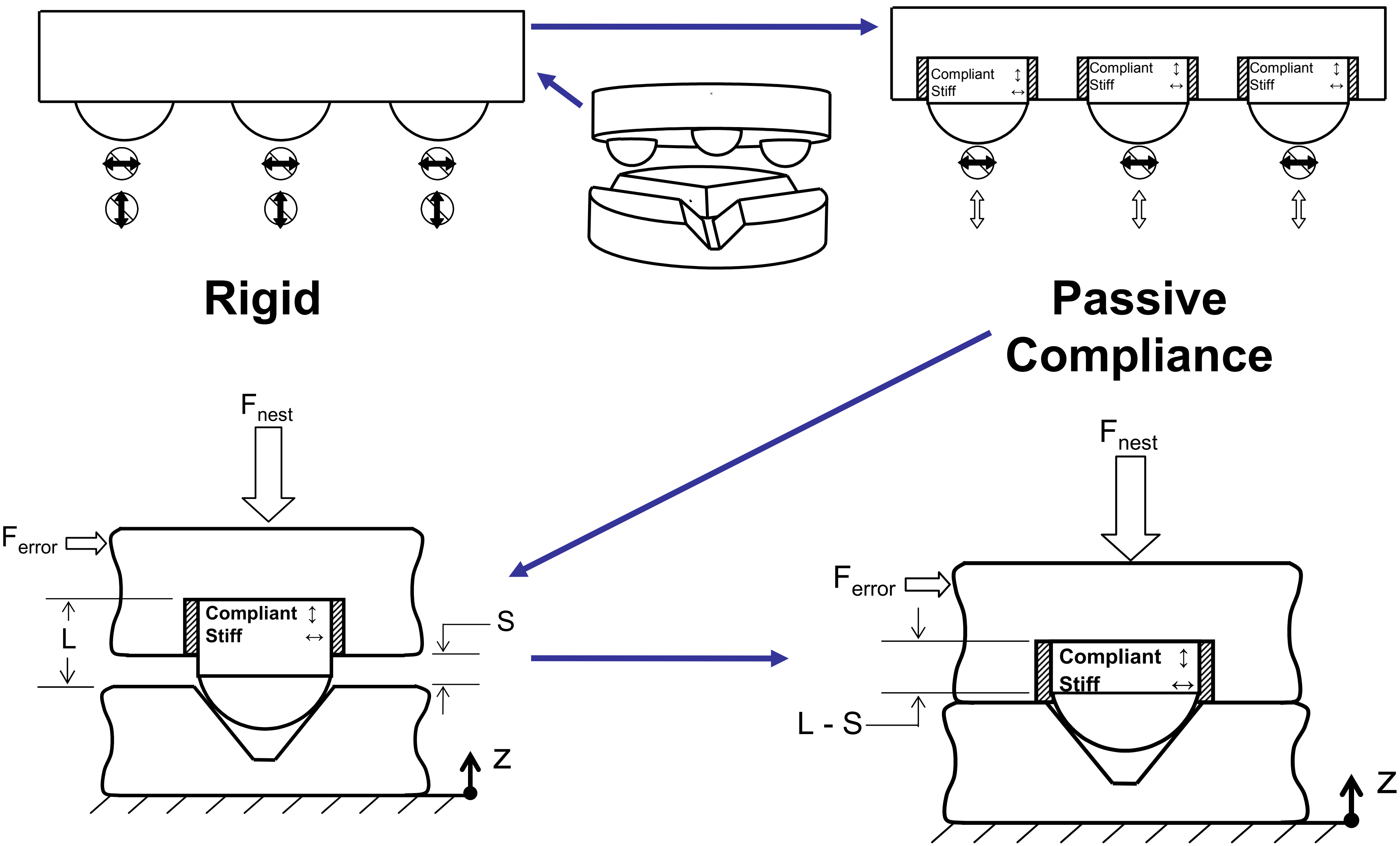


Care must be taken to make sure

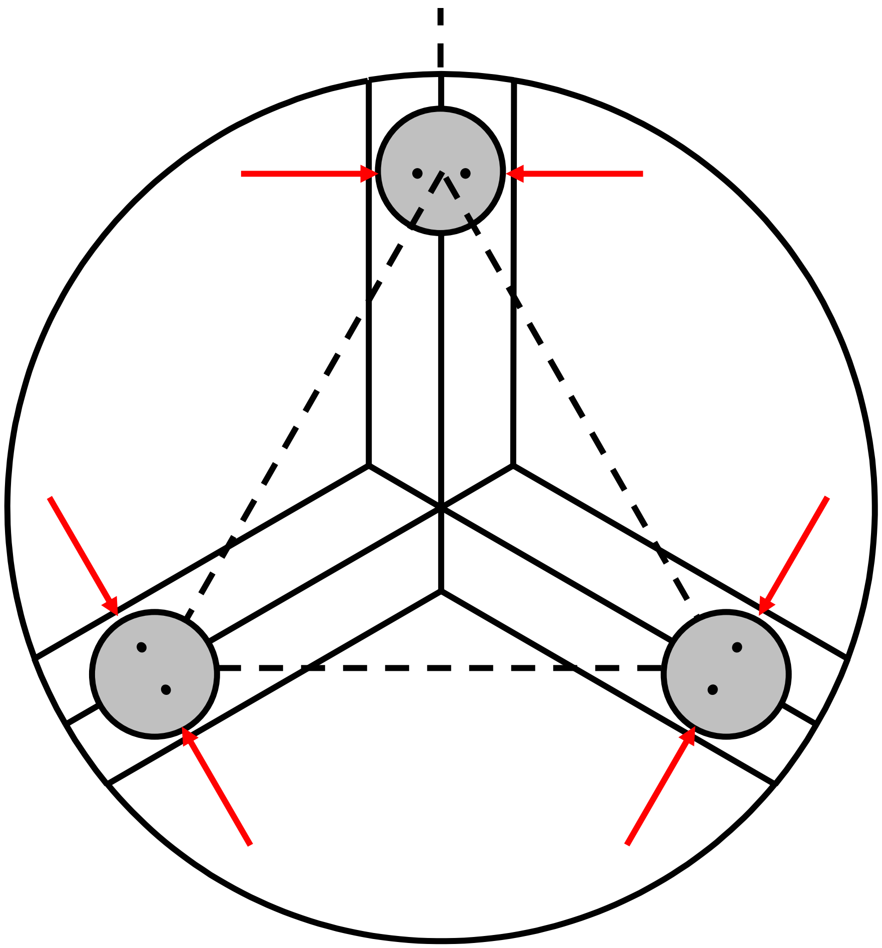
- ❑ Compliant direction is not in a sensitive direction
- ❑ Parasitic errors in sensitive directions are acceptable



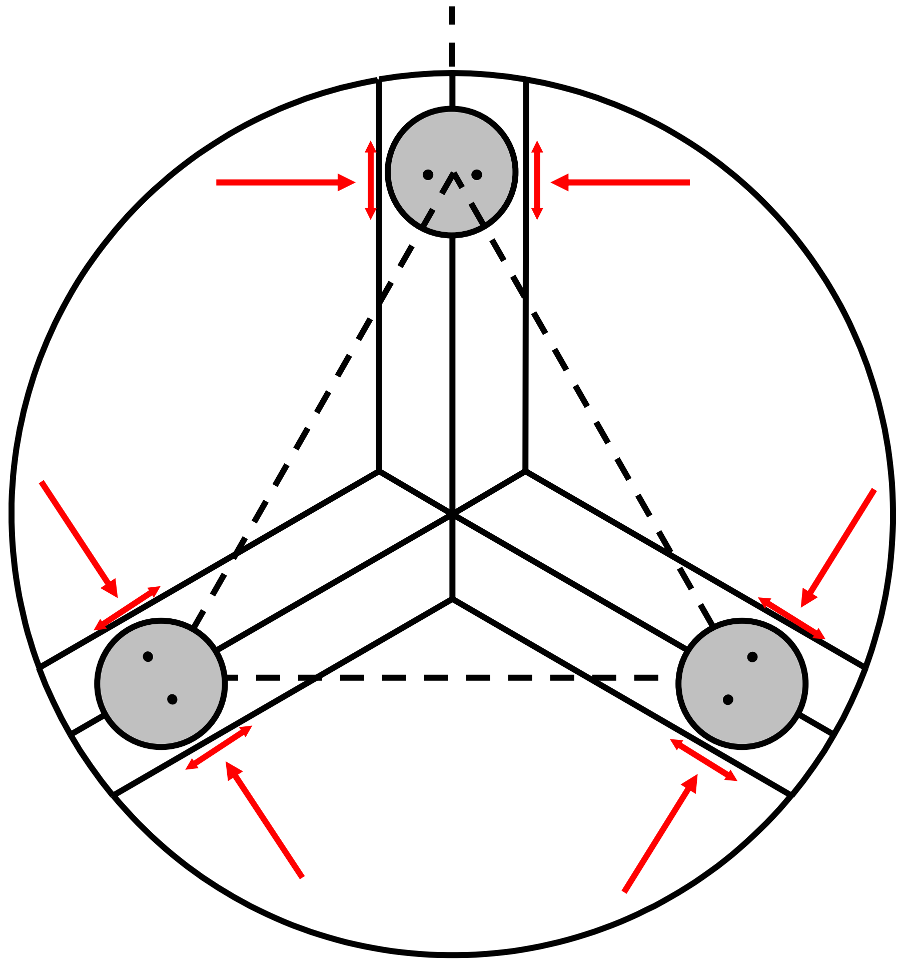
Out-of-plane use of flexures



System of six contact points

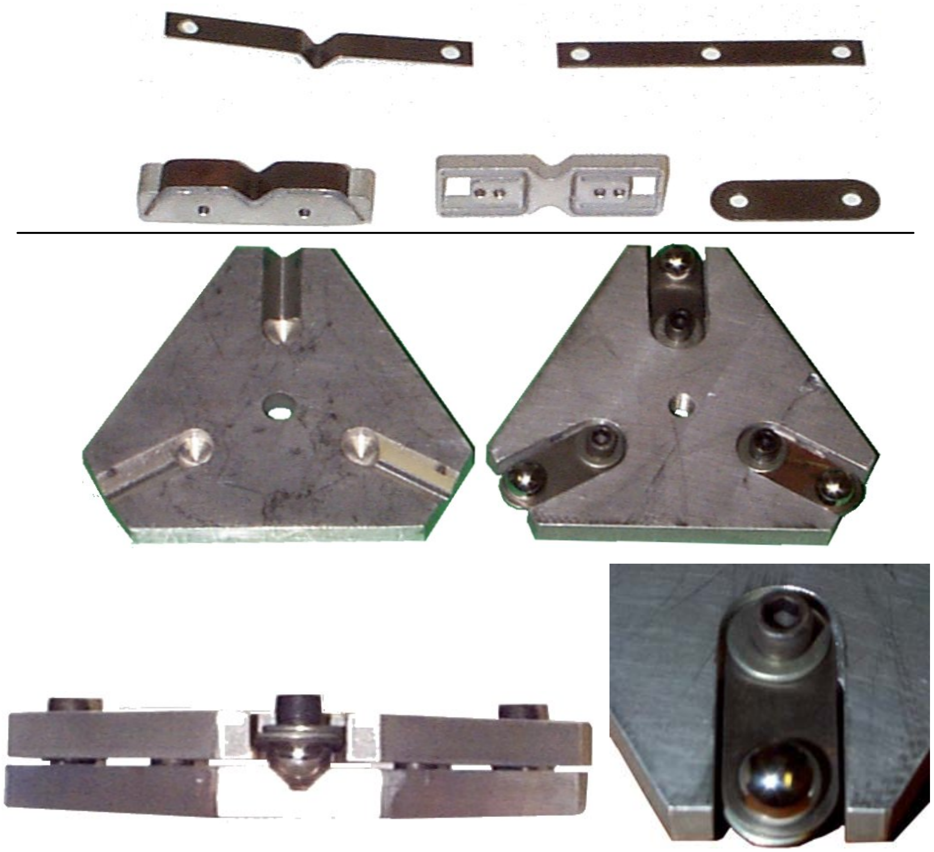


Ideal in-plane constraints

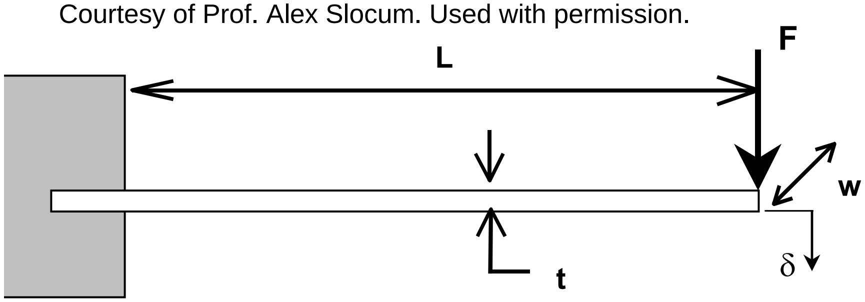


Real in-plane constraints

Example: Cantilevered balls



Courtesy of Prof. Alex Slocum. Used with permission.



Characteristics

Stroke	≤ 0.25 inches
Repeatability	5 -10 microns

Ball movement in non-sens. direction

Applications/Processes

1. Assembly
2. Casting

Design Issues (flexure)

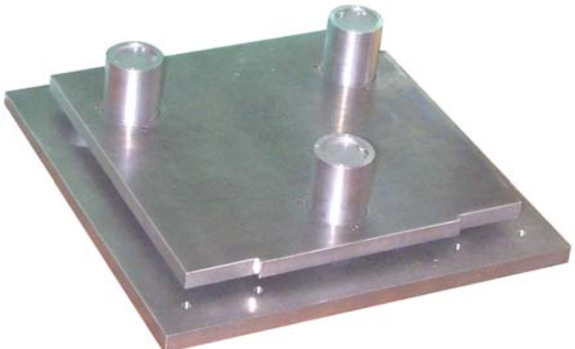
1. $K_r \sim \frac{w^2}{t^2}$
2. Tolerances affect K_r

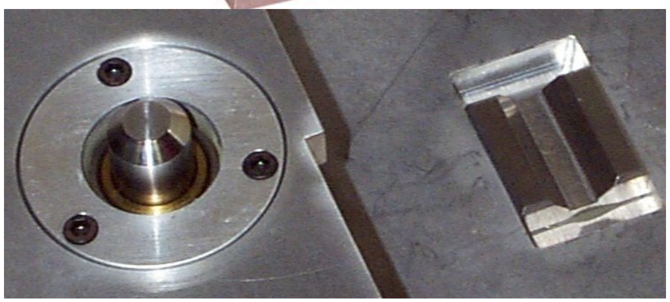
Cost

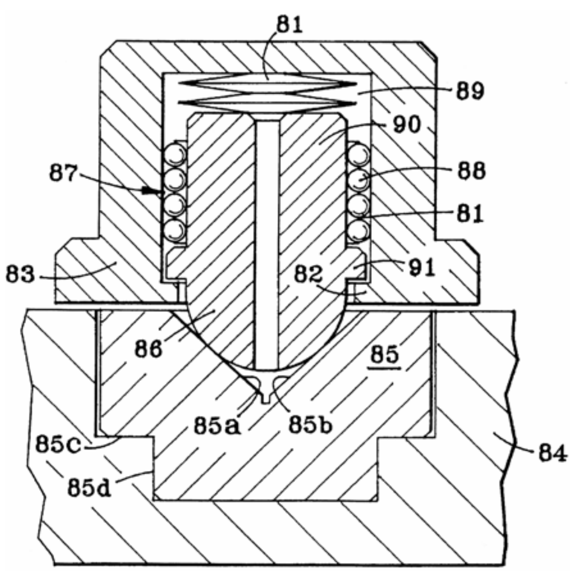
$ 10 - 200

U.S. Patent 5, 678, 944,
Slocum, Muller, Braunstein

Axial bushings with spring preload







U.S. Patent 5, 678, 944,
Slocum, Muller, Braunstein

Characteristics

1. Repeatability (2.5 micron)
2. Stroke ~ 0.5 inches

Applications/Processes

1. Assembly
2. Casting
3. Fixtures

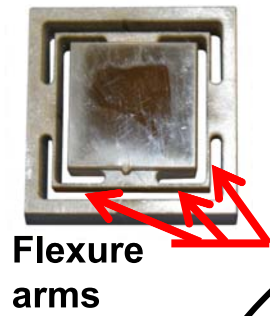
Design Issues (flexures)

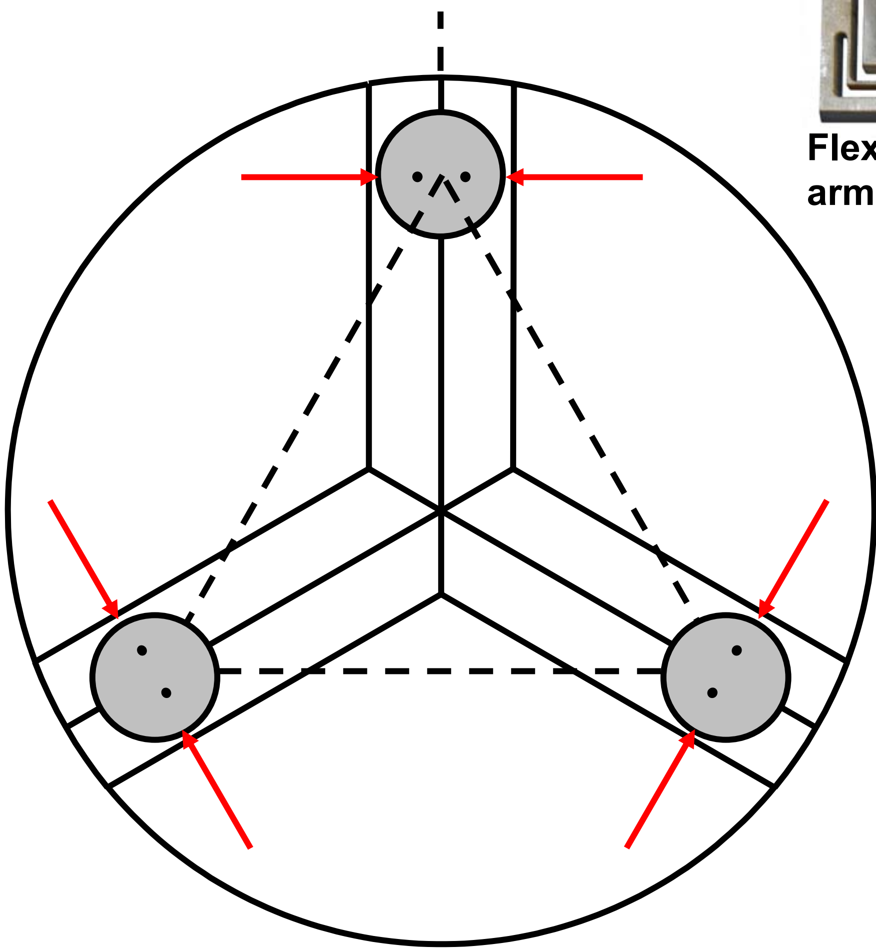
1. $K_r = \dfrac{K_{guide}}{K_{spring}}$
2. Press fit tolerances

Cost

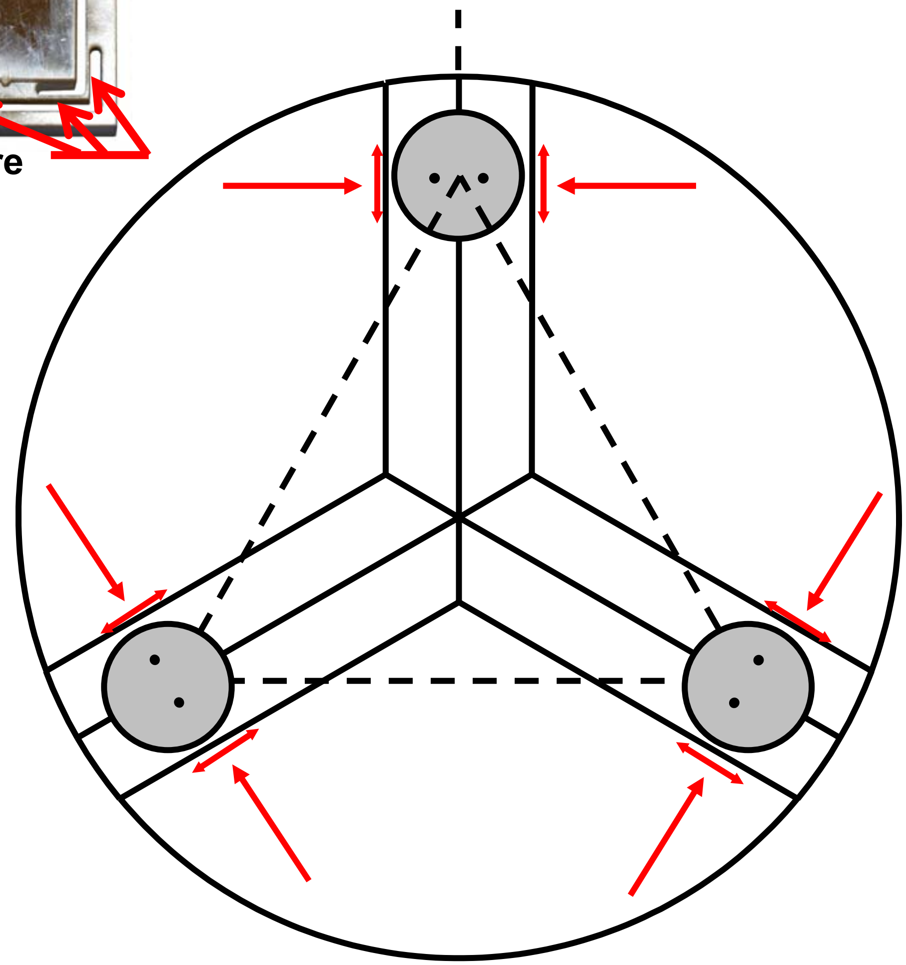
$ 2000

System of six contact points



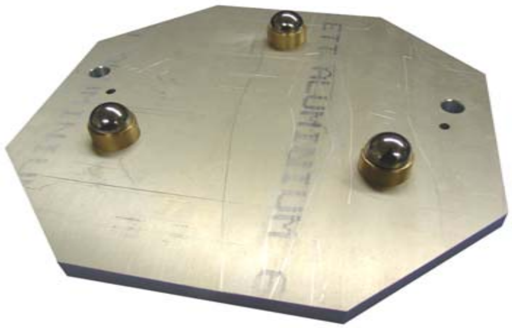


Ideal in-plane constraints



Real in-plane constraints

Flexure grooves reduce friction effect



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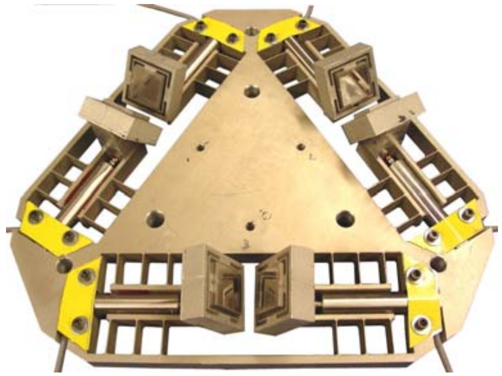

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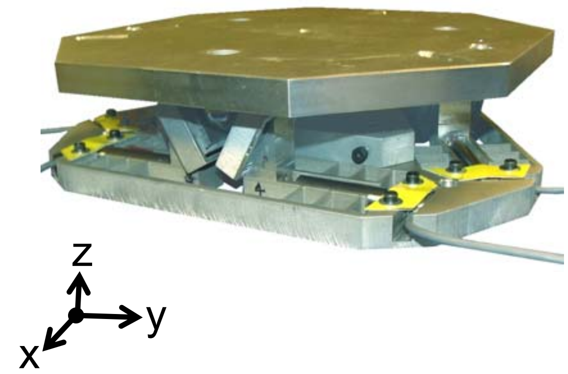
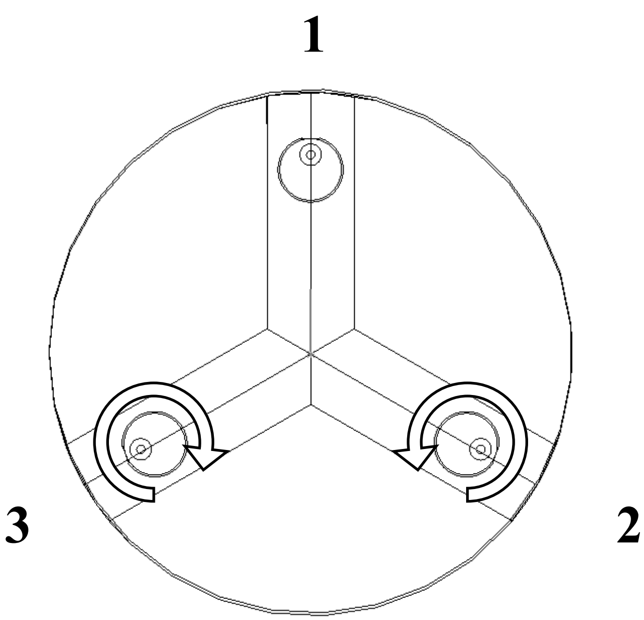
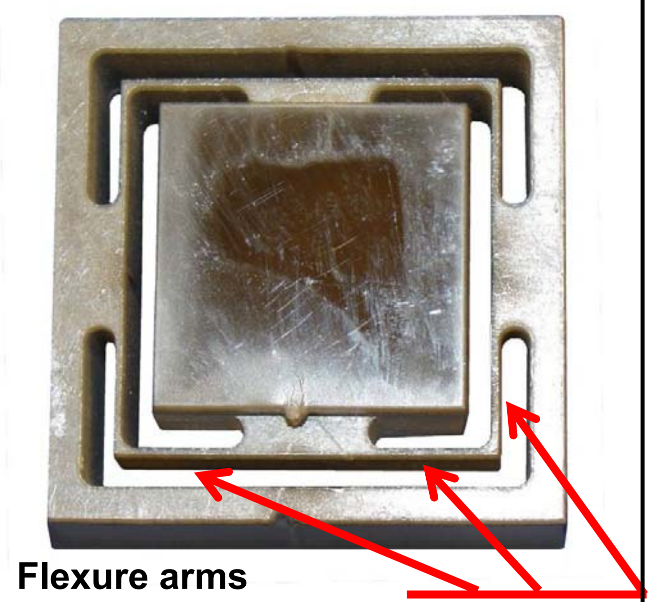
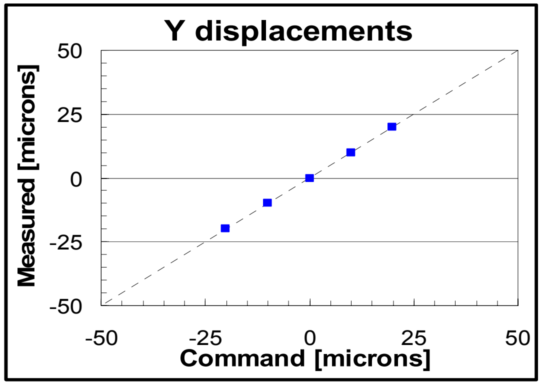
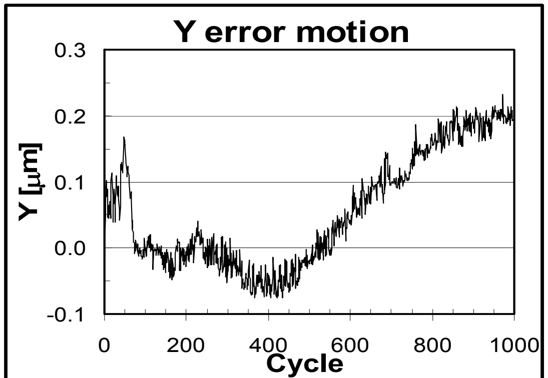




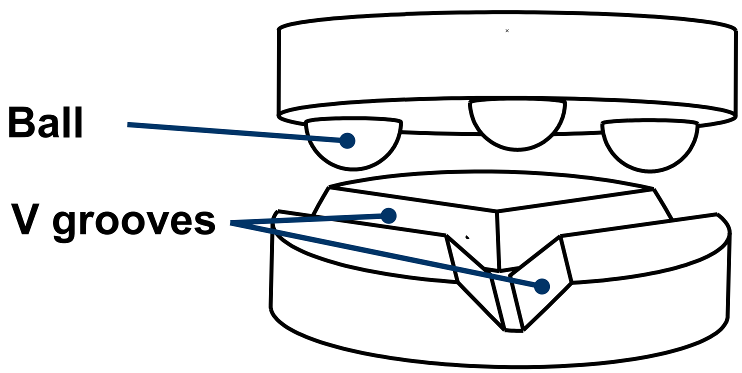


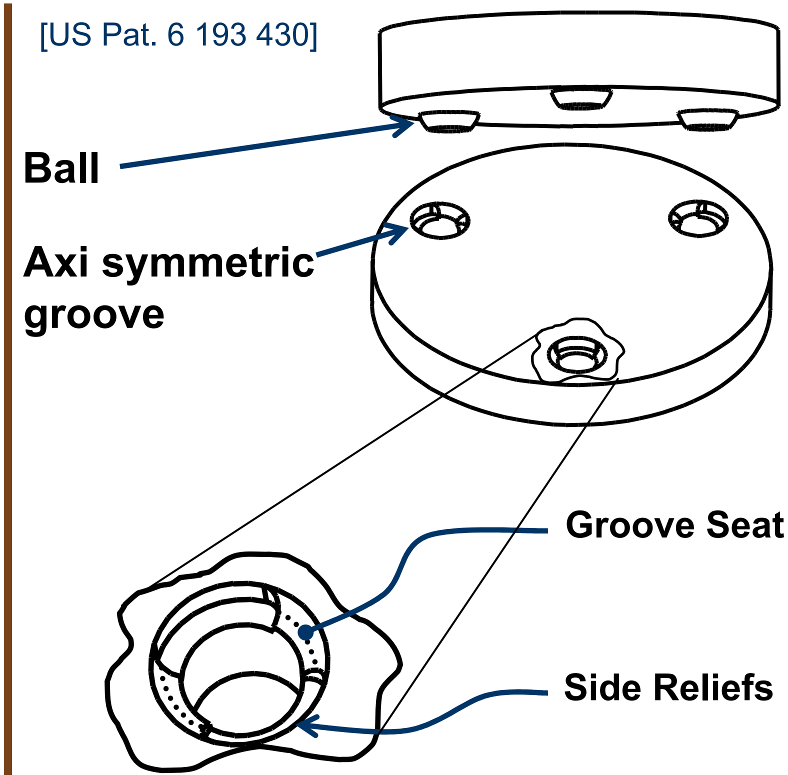




“Rigid”-flexible constraints

Exact & near kinematic constraint

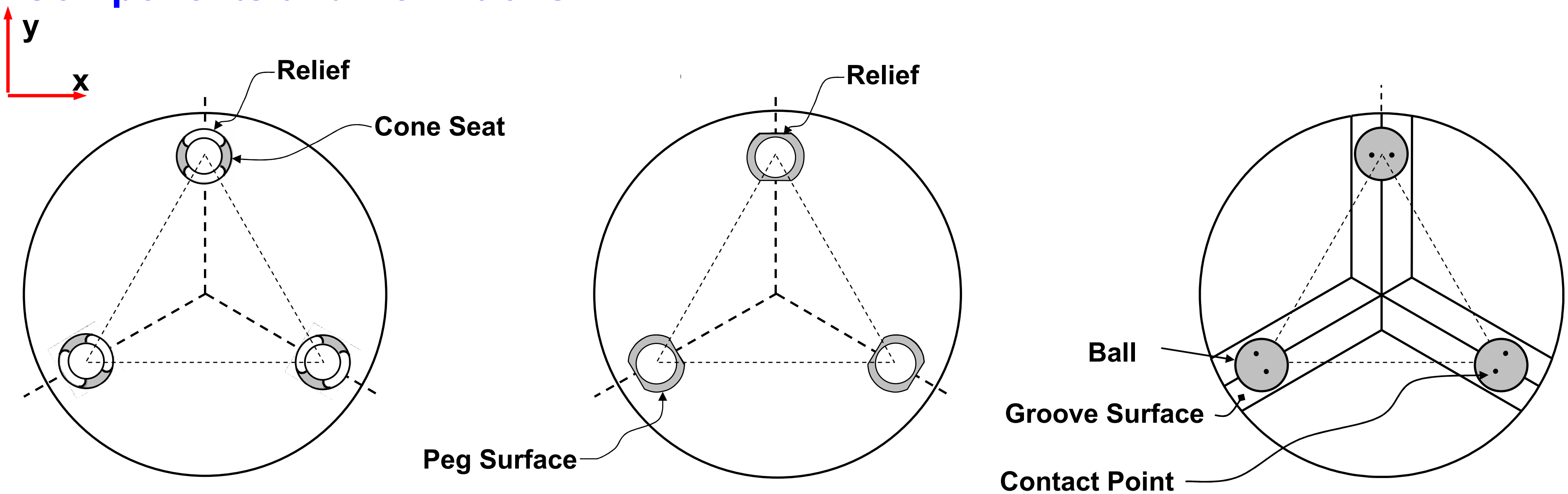




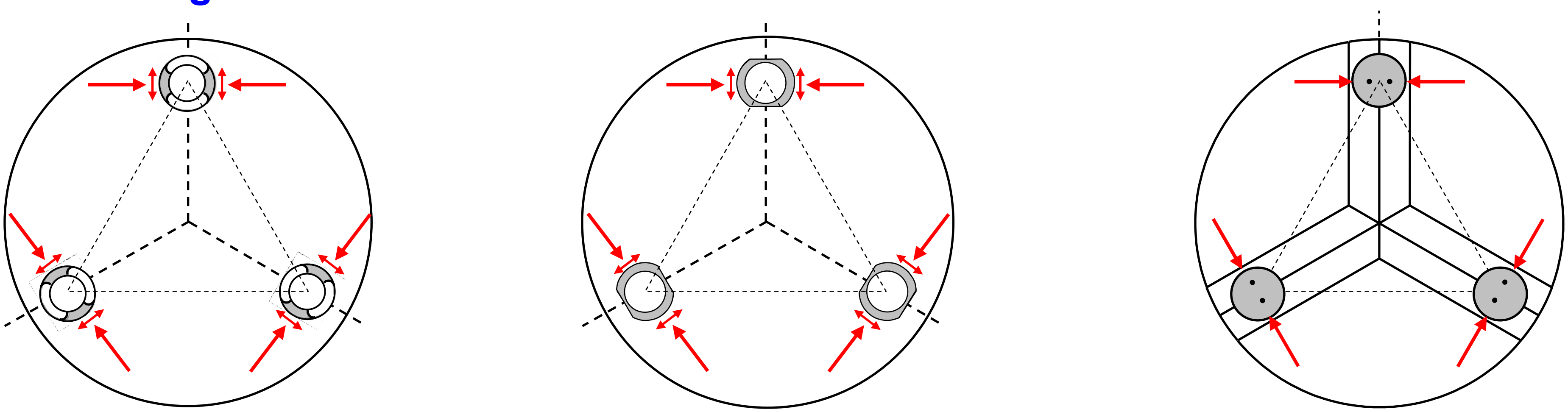
	Kinematic	Quasi-kinematic
Contact:	Point / small area	Arc
Repeatability:	80 nm	250 nm
Cost:	$10s to $1000s	$ 0.75
Sealing:	With flexures	Integrated
Time:	> 60 minutes	< 20 s

QKC methods vs kinematic method

Components and Definitions

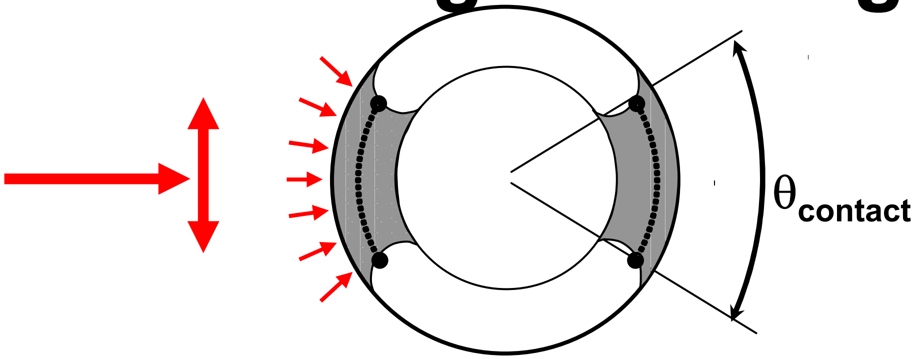


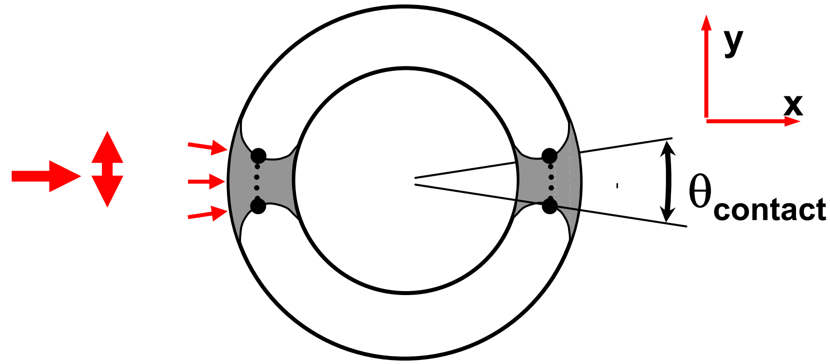
Force Diagrams



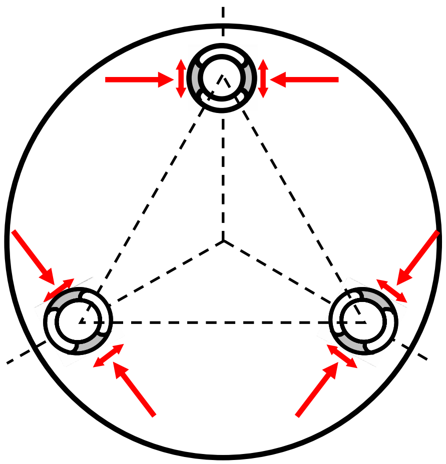
Constraint in QKCs

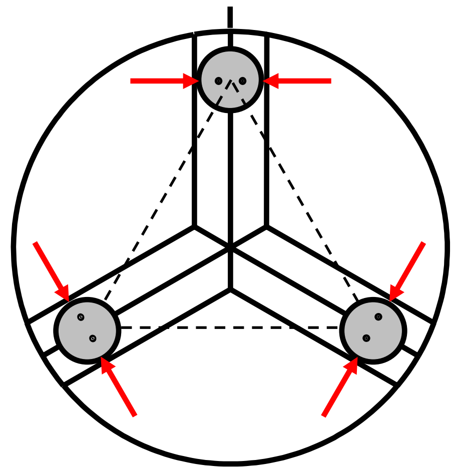
Contact angle and degree of over constraint





QKC constraint compared to "ideal" coupling





$$CM_k = \frac{\text{Stiffness parallel to bisecting planes}}{\text{Stiffness perpendicular to bisecting planes}} = \frac{k_{\|\text{Bisector}}}{k_{\perp\text{Bisector}}}$$

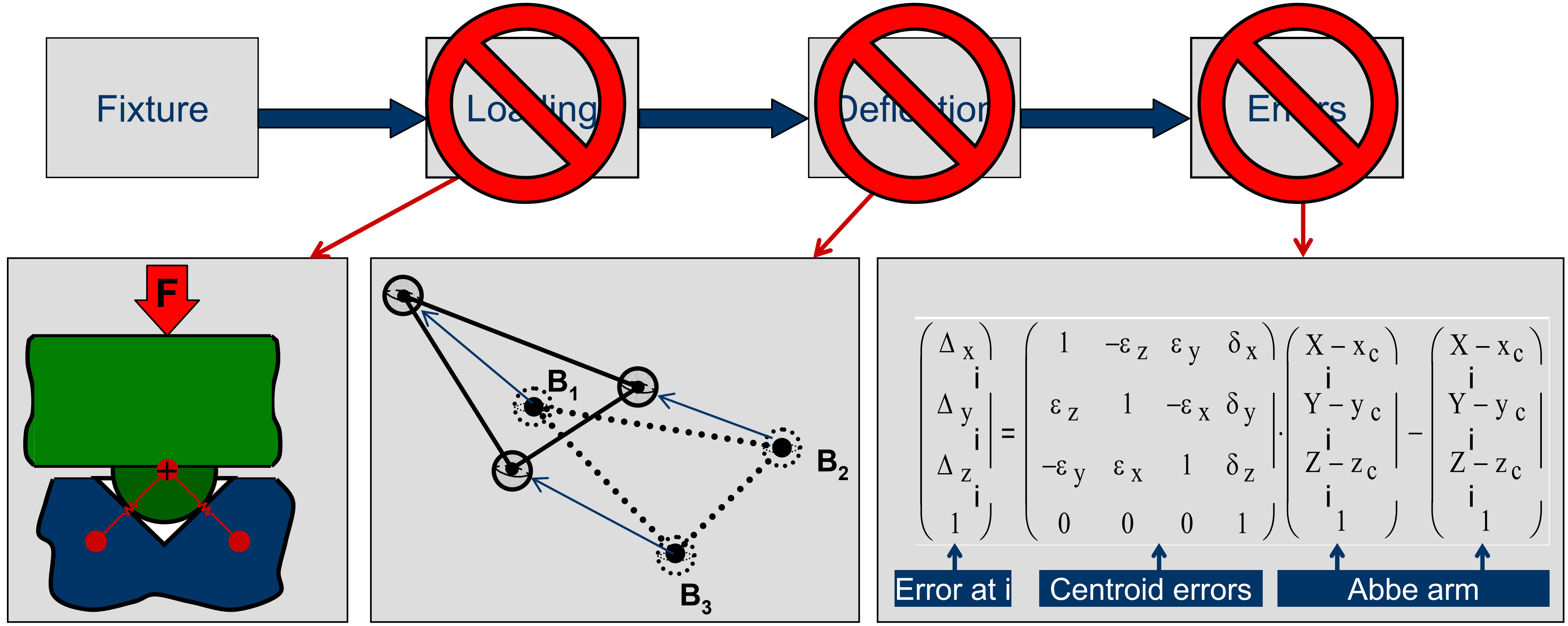
Solving for coupling errors

Step 1: Analyze ball center displacements

Step 2: Obtain centroid displacements

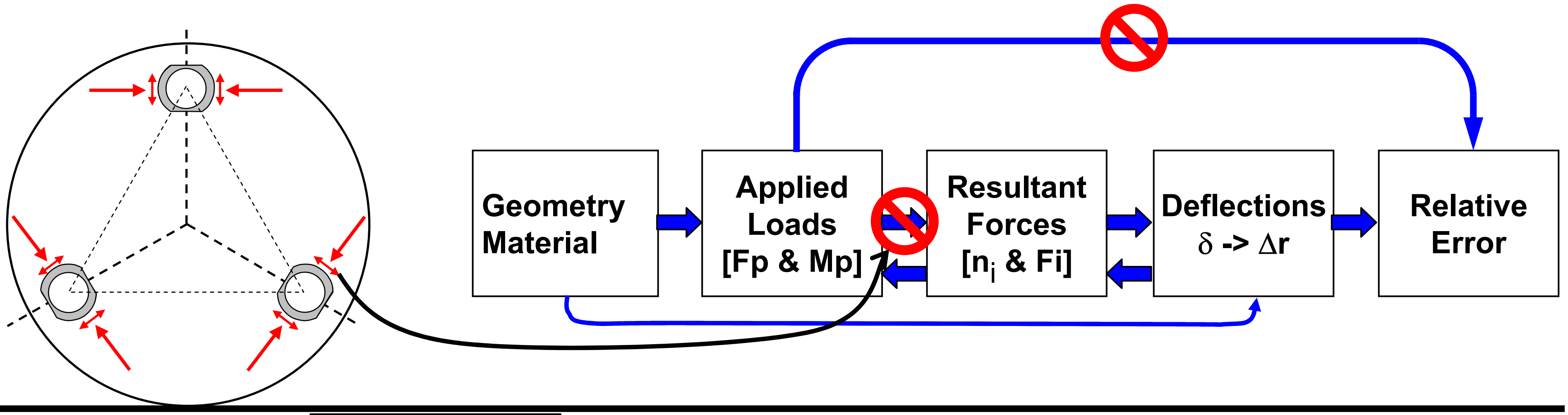
- Translations: $\delta_x, \delta_y, \delta_z$ Rotations: $\varepsilon_x, \varepsilon_y, \varepsilon_z$

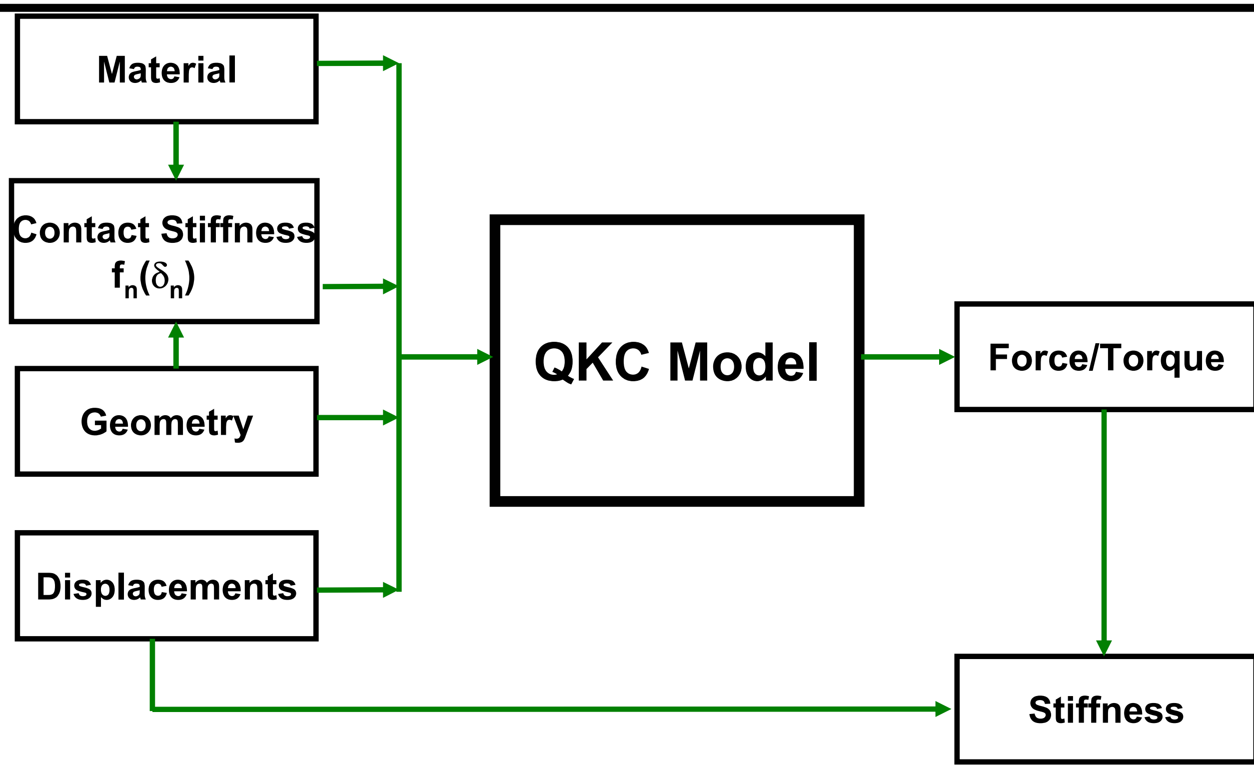
Step 3: Calculating error at any point



$$\begin{pmatrix} \Delta_{x_i} \\ \Delta_{y_i} \\ \Delta_{z_i} \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & -\varepsilon_z & \varepsilon_y & \delta_x \\ \varepsilon_z & 1 & -\varepsilon_x & \delta_y \\ -\varepsilon_y & \varepsilon_x & 1 & \delta_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} X_i - x_c \\ Y_i - y_c \\ Z_i - z_c \\ 1 \end{pmatrix} - \begin{pmatrix} X_i - x_c \\ Y_i - y_c \\ Z_i - z_c \\ 1 \end{pmatrix}$$

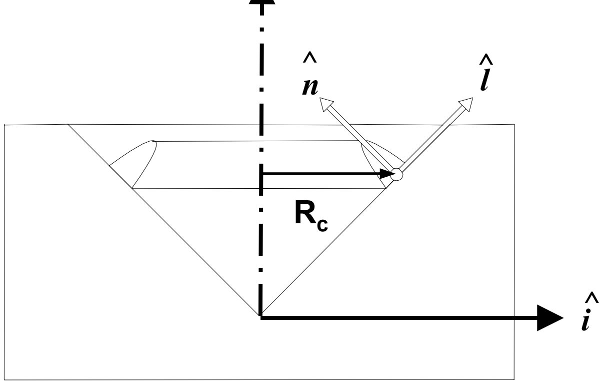
New model for QKC stiffness

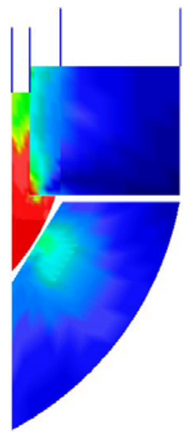


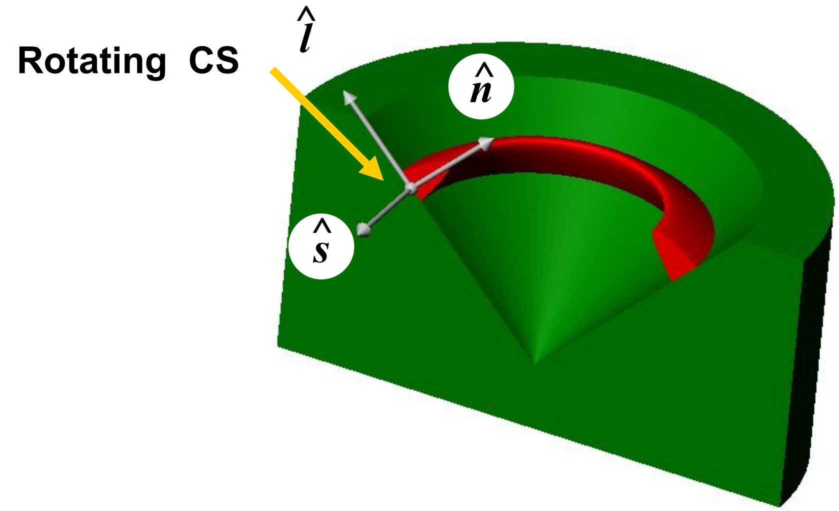


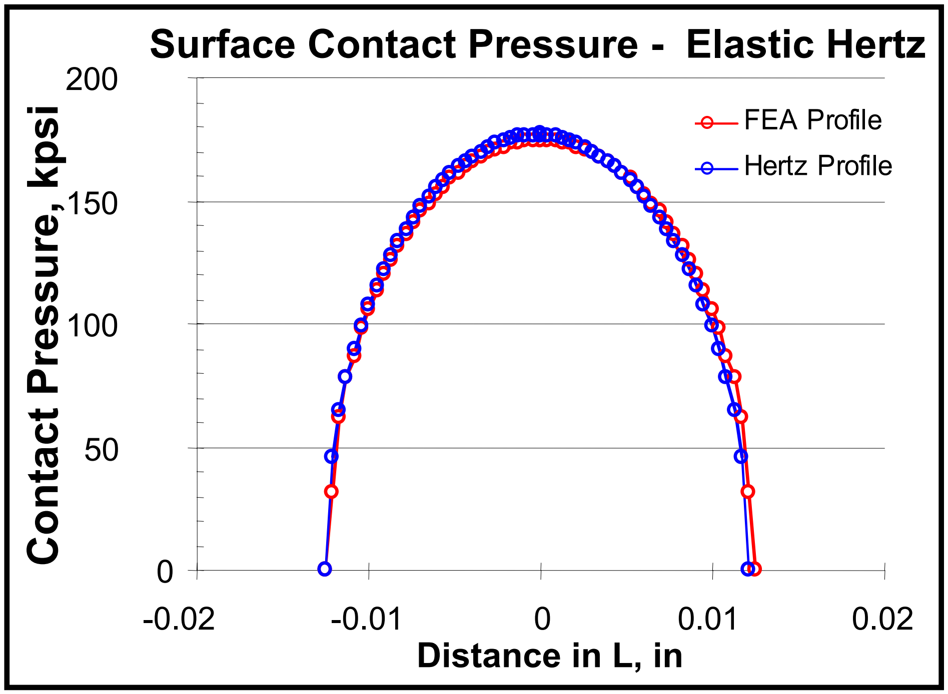
Contact mechanics

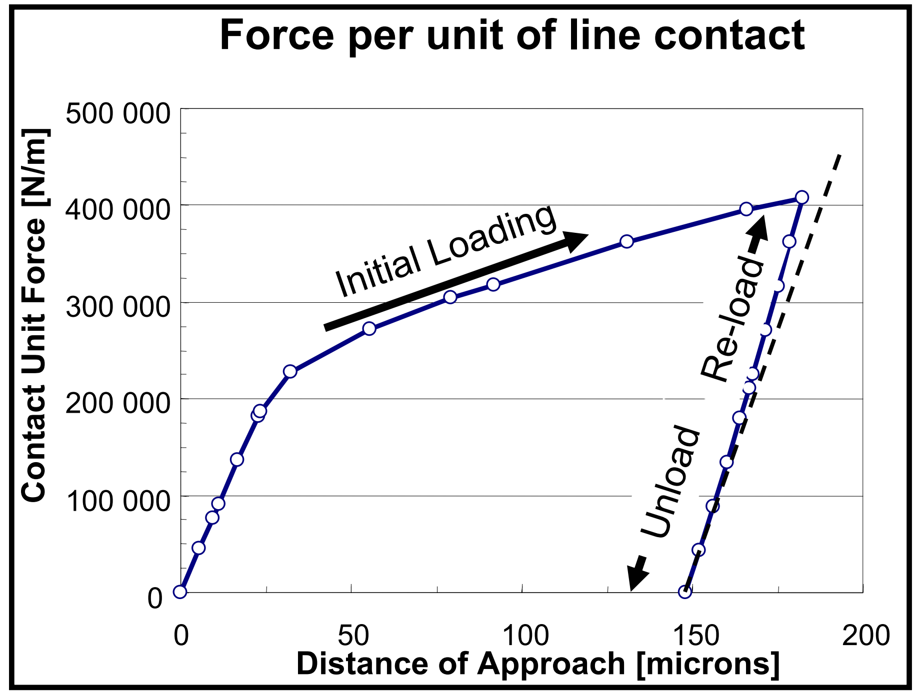
$$f_n(\theta_r) = K[\delta_n(\theta_r)]^b$$



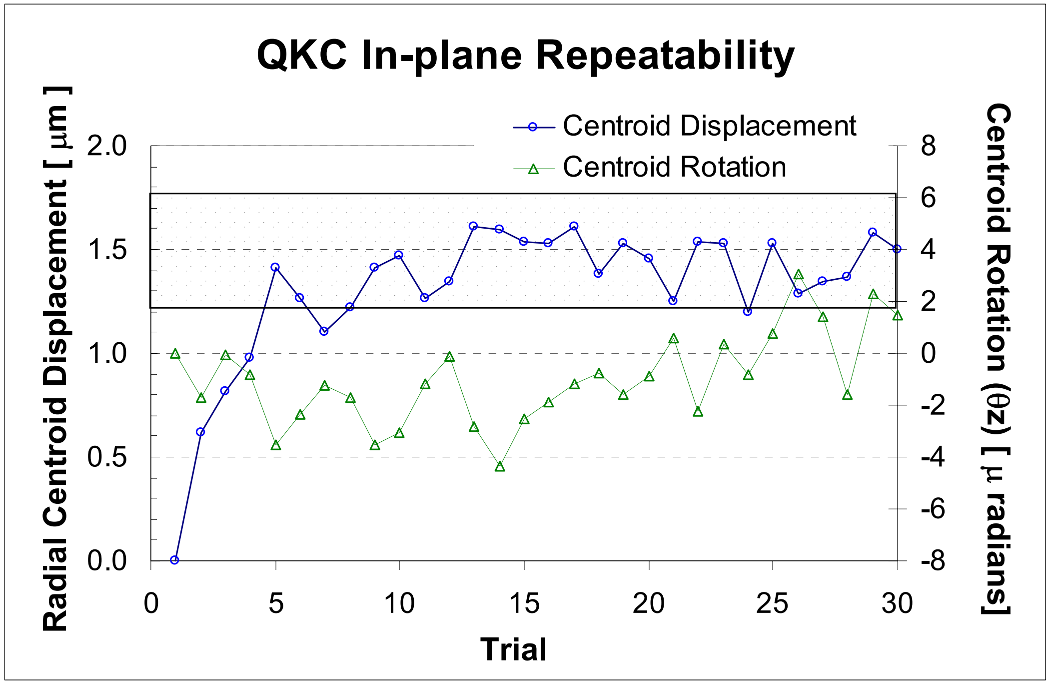


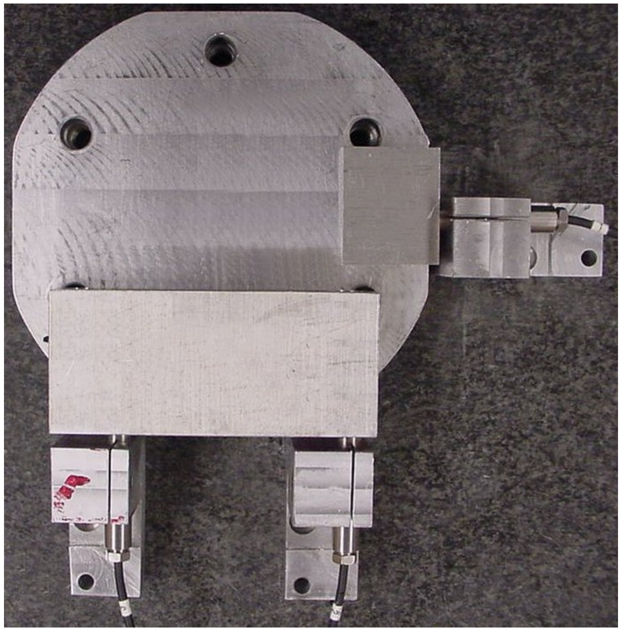


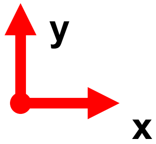


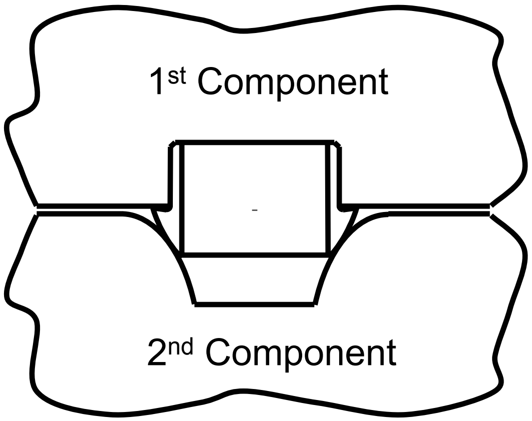


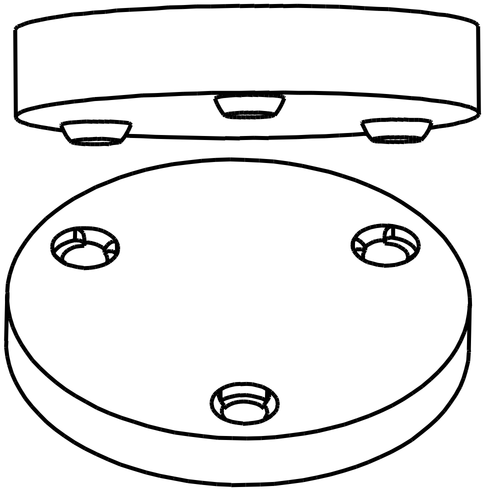
QKC Performance







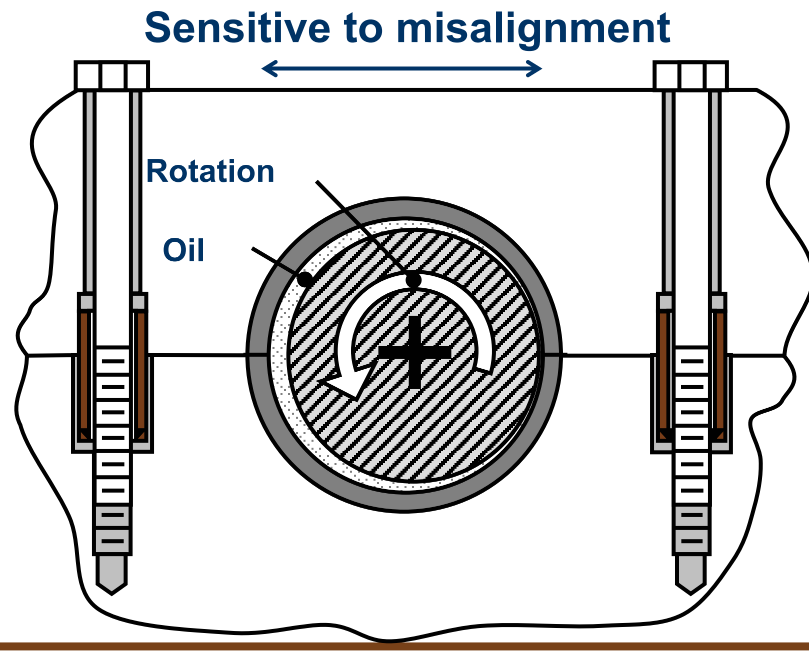


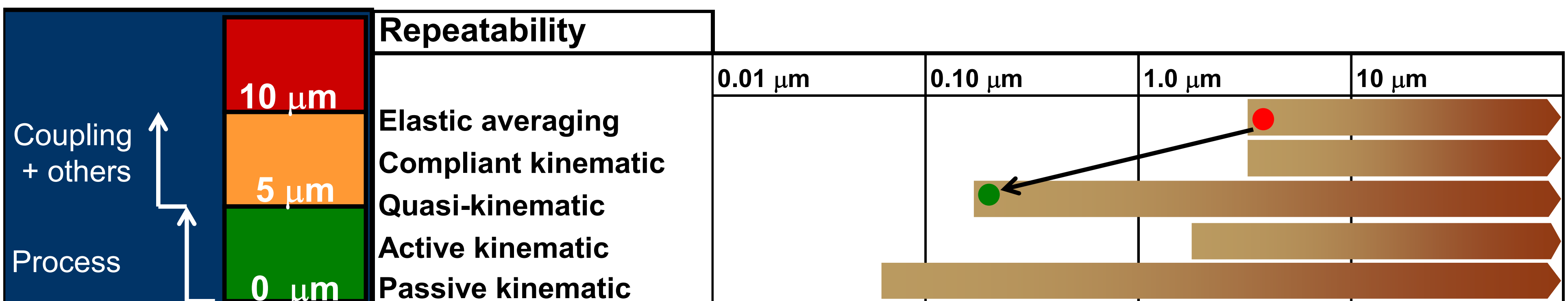


Practical use: Duratec^{TM} assembly

Characteristics:

- ❑ Ford V6 300,000 / year Cycle time: < 30 s

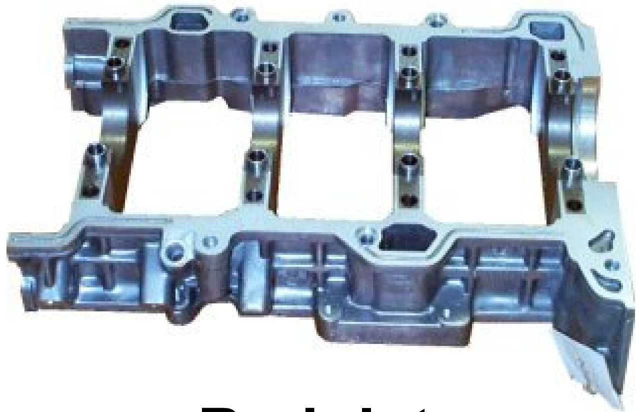




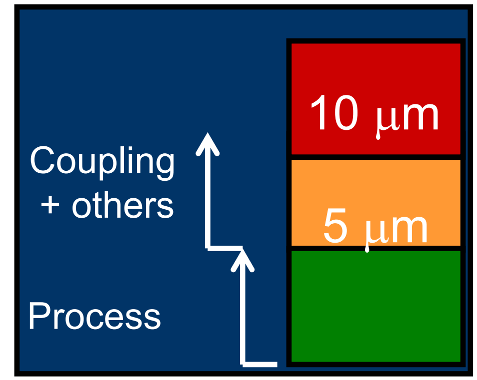
Goal: 5 micron assembly

Components

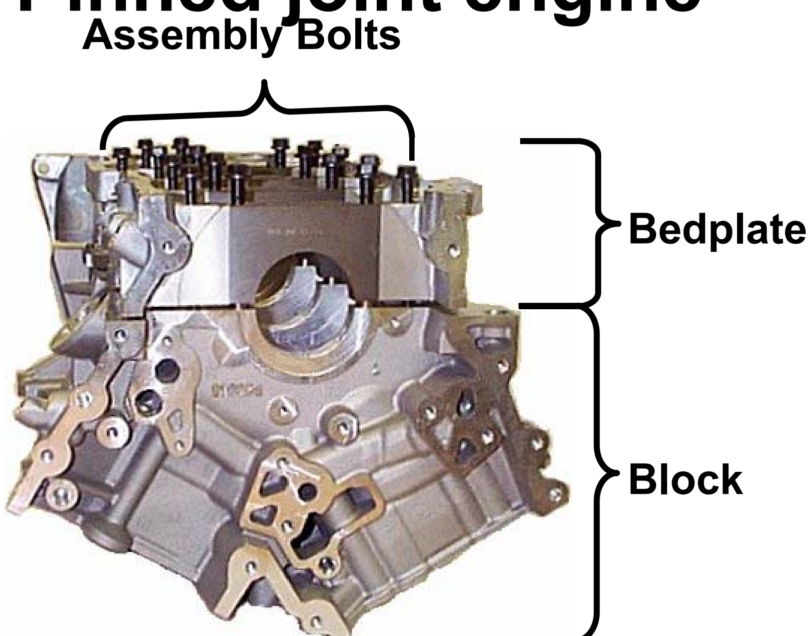
Block



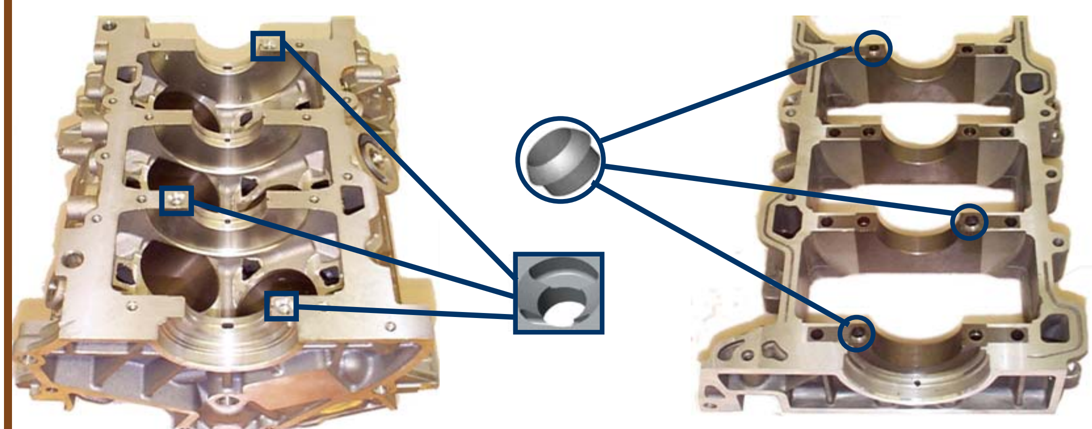
Bedplate



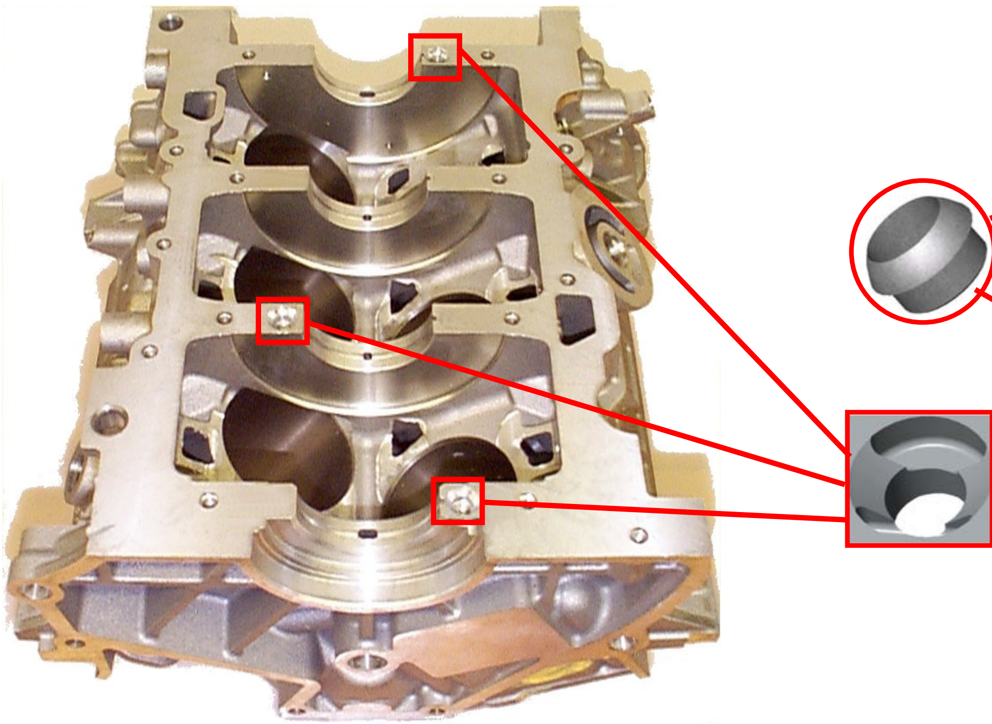
Pinned joint engine

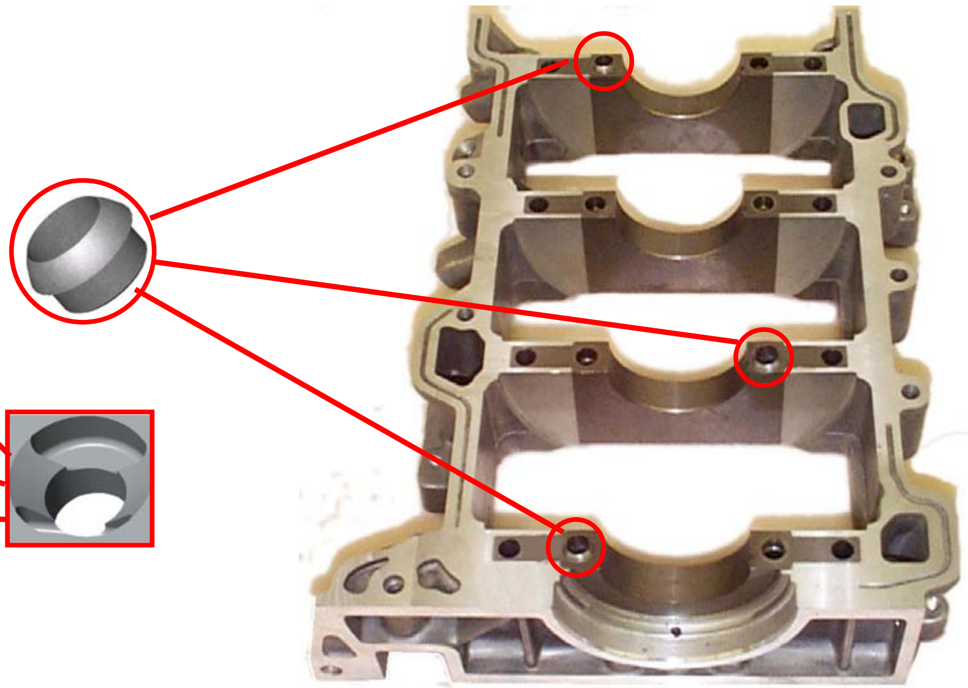


QKC equipped engine



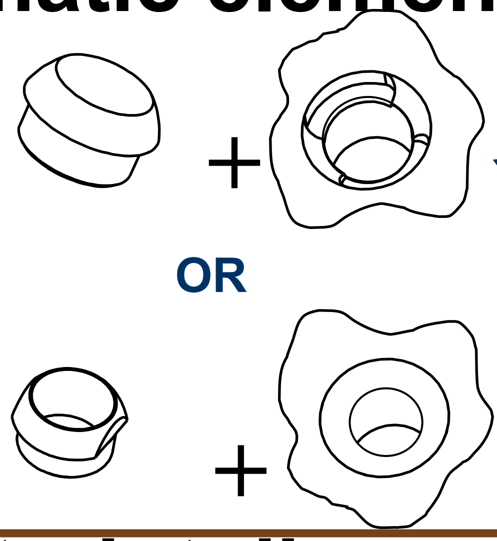
Duratec^{TM} QKC in detail





Low-cost couplings

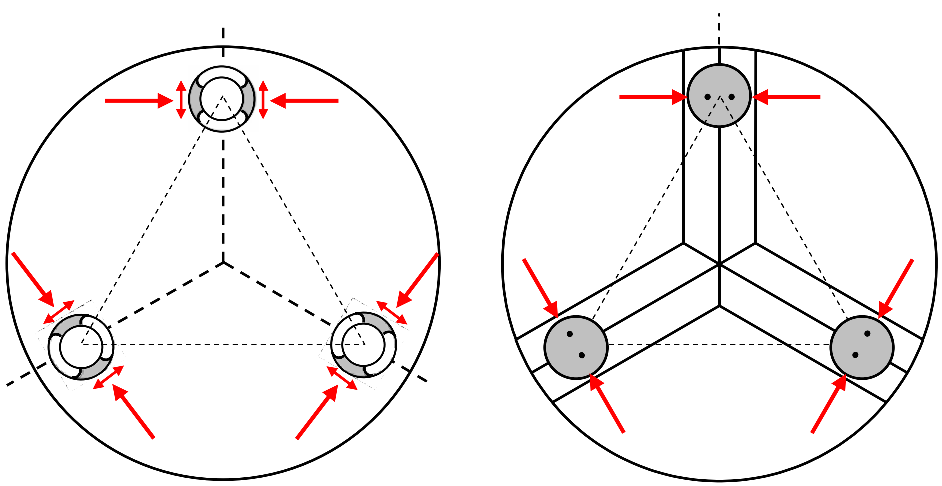
Kinematic elements



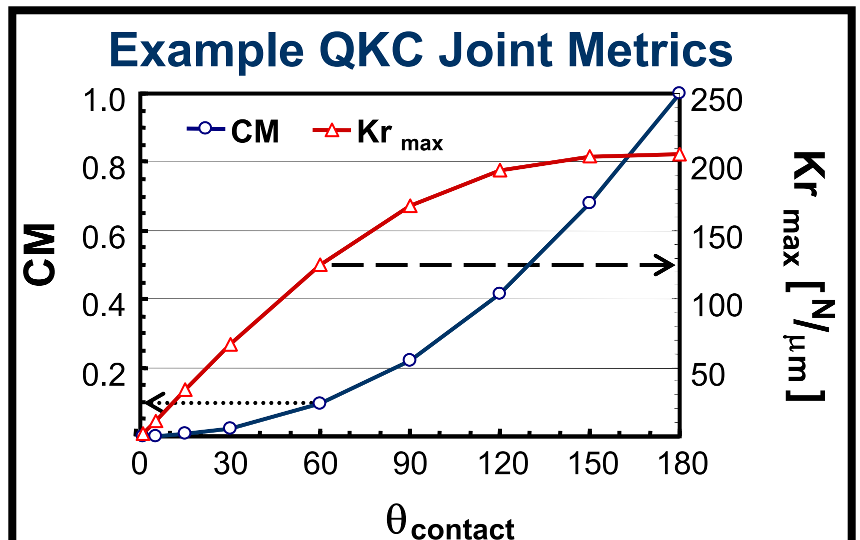
Manufacturing

Diagrams removed for copyright reasons.
"Cast + Form Tool = Finished"

Constraint diagrams



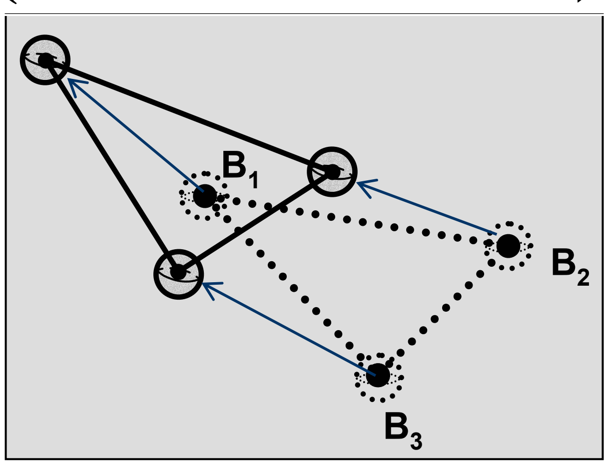
Metrics



Solving for coupling errors

Error | HTM | Amplification Arm | Centroid displacement

$$\begin{pmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & -\varepsilon_z & \varepsilon_y & \delta_x \\ \varepsilon_z & 1 & -\varepsilon_x & \delta_y \\ -\varepsilon_y & \varepsilon_x & 1 & \delta_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} X_i - x_c \\ Y_i - y_c \\ Z_i - z_c \\ 1 \end{pmatrix} - \begin{pmatrix} X_i - x_c \\ Y_i - y_c \\ Z_i - z_c \\ 1 \end{pmatrix}$$

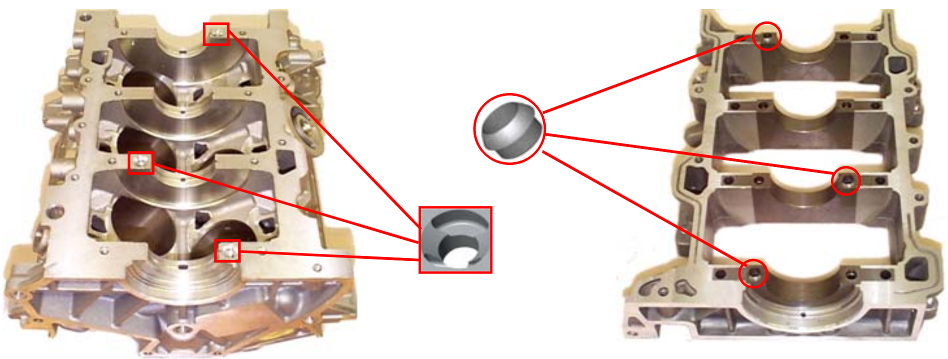


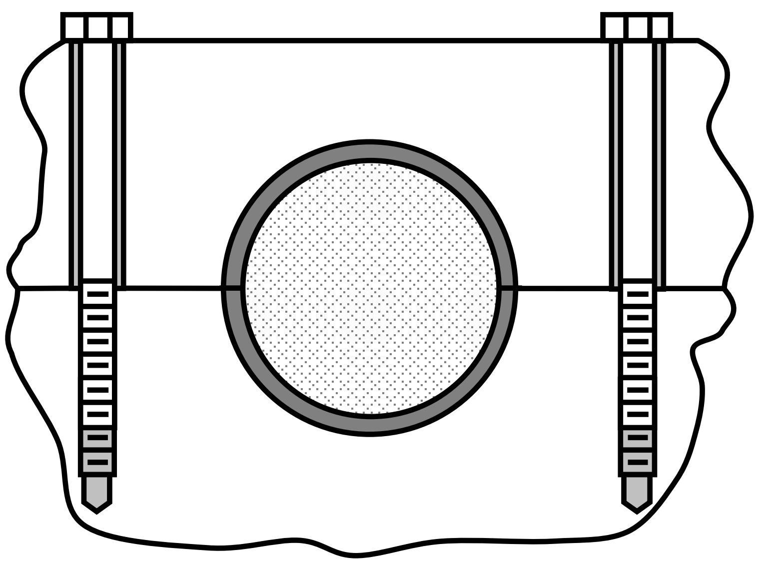
Error at i | Centroid errors | Abbe arm

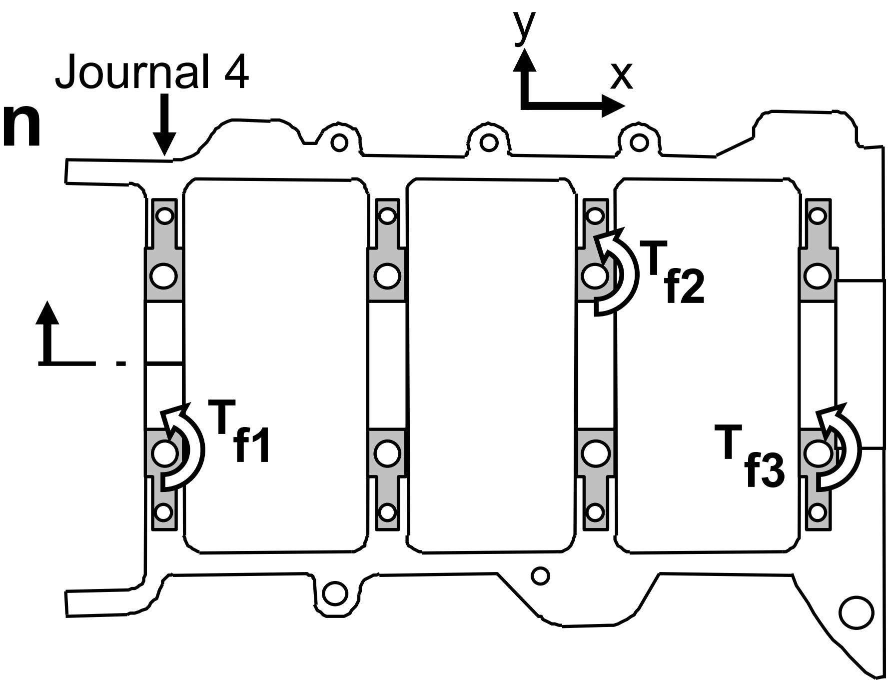
Load sources: Alignment

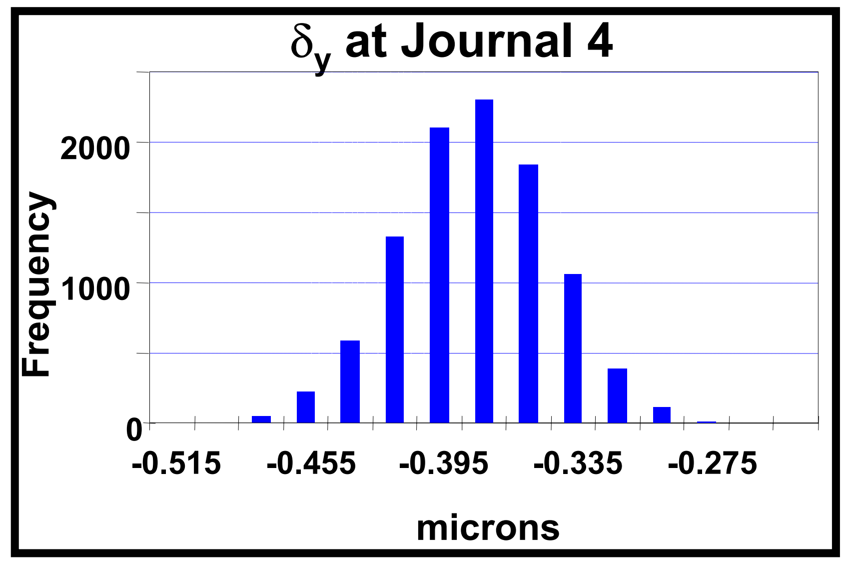
Forces/loads can vary

Result is variation in position



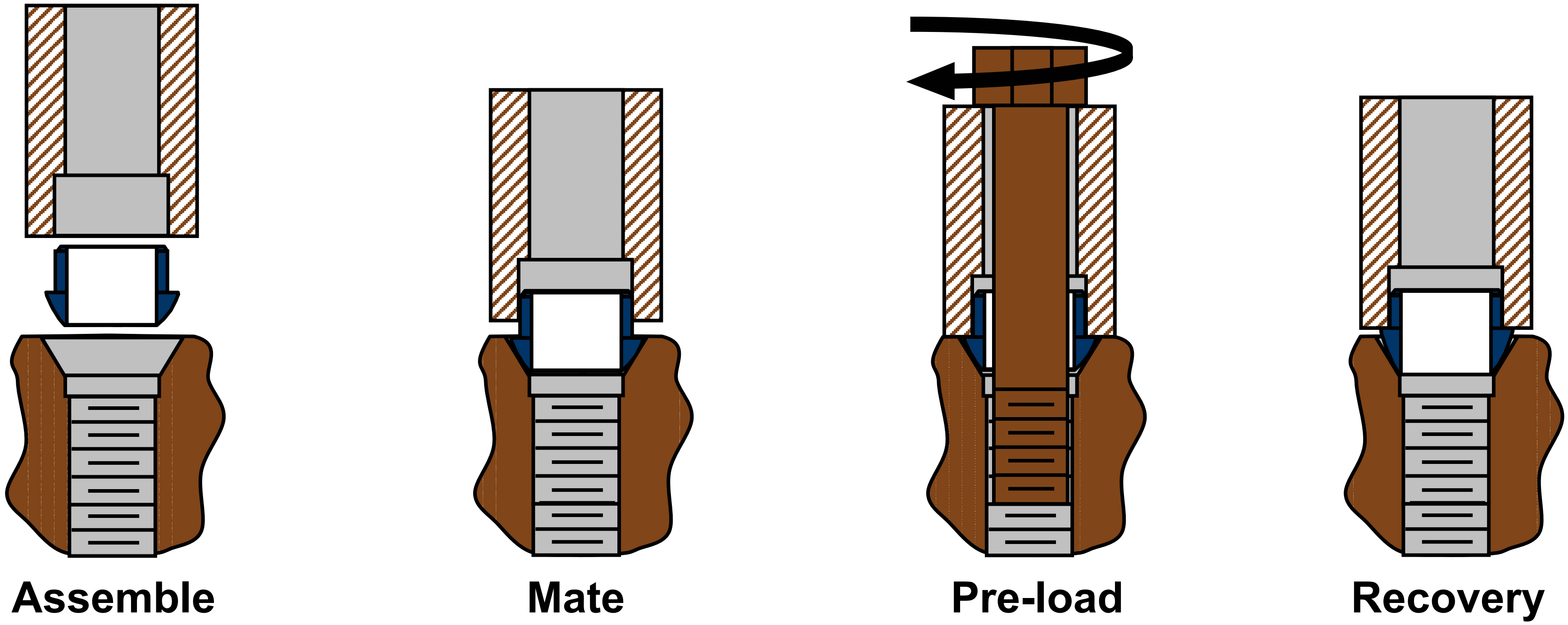






Photos by Prof. Martin Culpepper, courtesy of Ford Motor Company. Used with permission.

Form-in-place process



Groove surface after burnishing process

- Elastic recovery restores gap