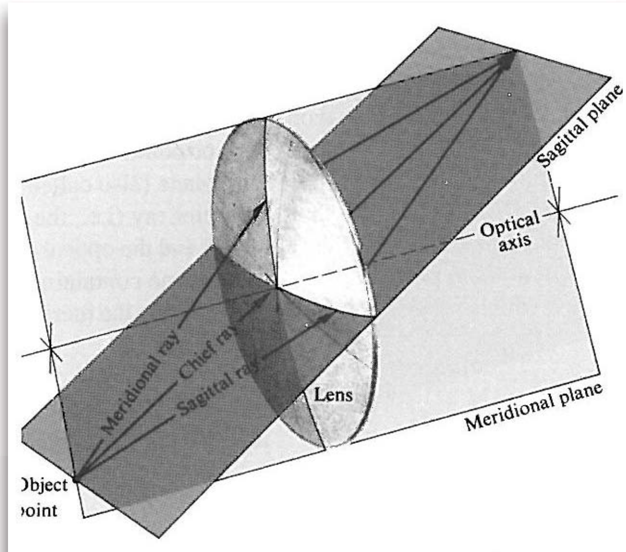
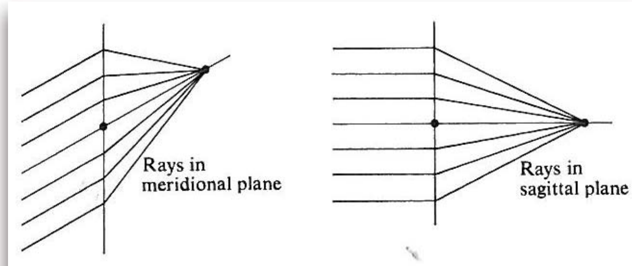
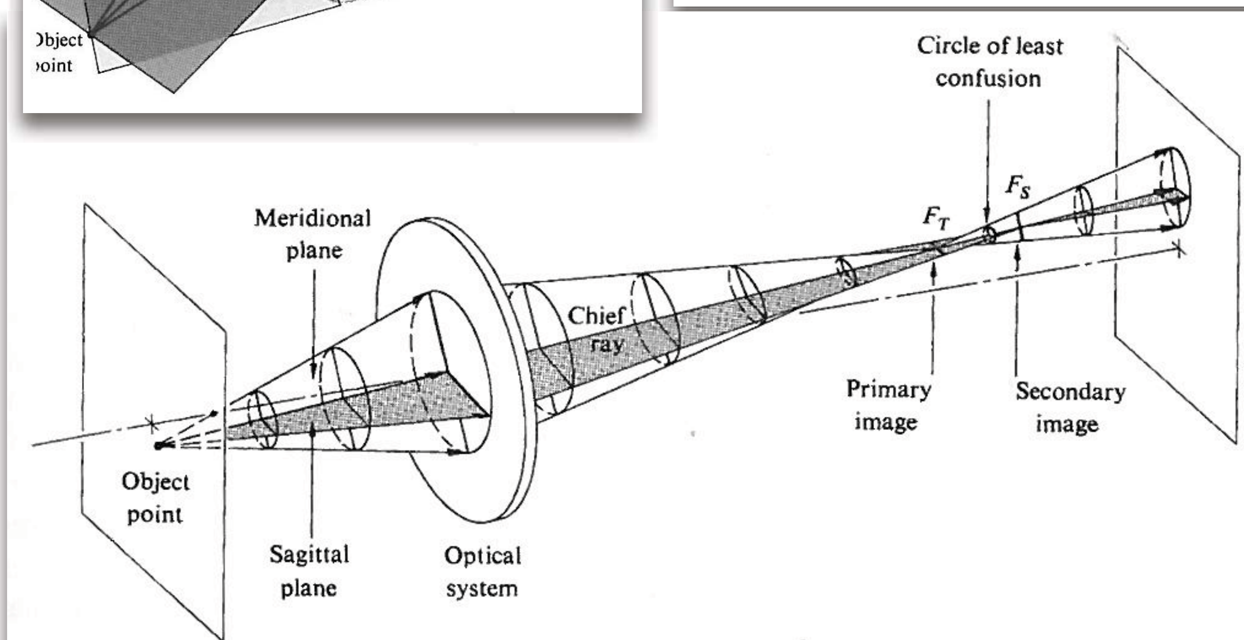
# Astigmatism

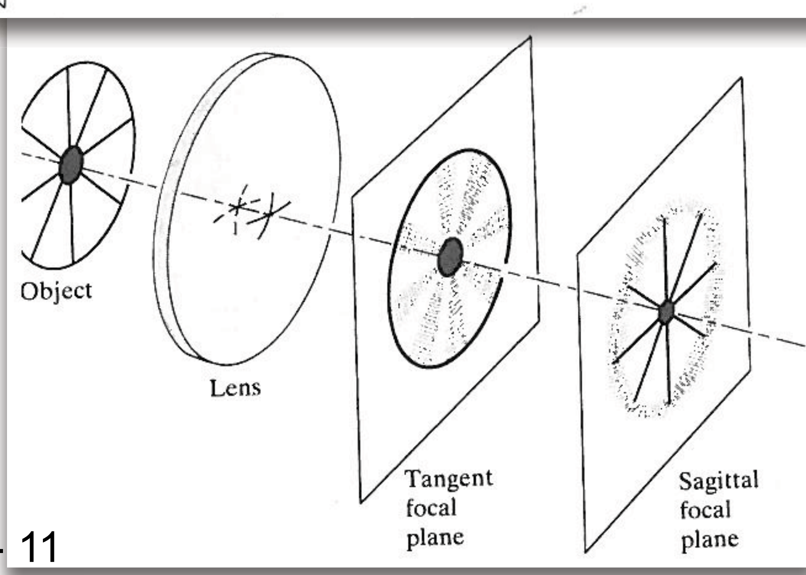





ure 6.26 The sagittal and meridional planes.



# Field curvature and Petzval condition



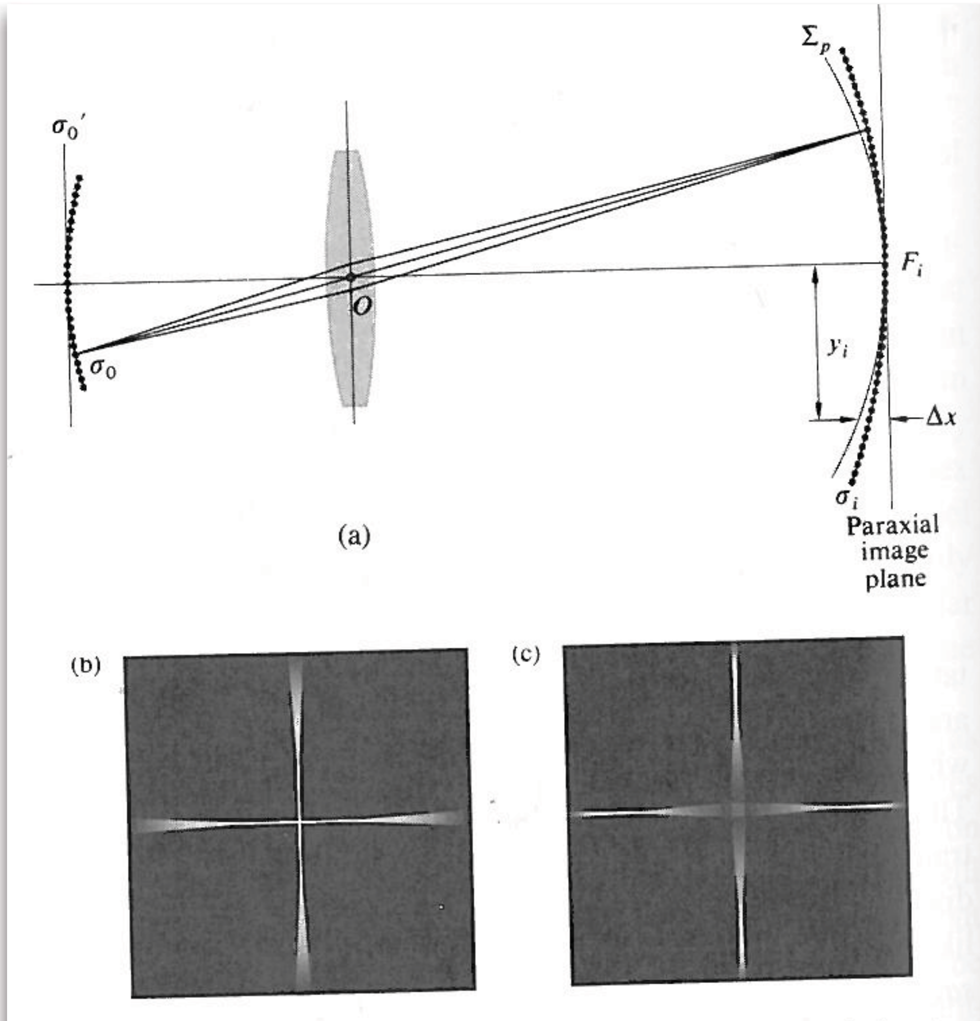



**Figure 6.29** Field curvature. (*a*) When the object corresponds to $\sigma_0'$, the image will correspond to surface $\Sigma_P$. (*b*) The image formed on a flat screen near the paraxial image plane will be in focus only at its center. (*c*) Moving the screen closer to the lens will bring the edges into focus.

Petzval condition: eliminates field curvature. For two lenses, indices of refraction $n_1$, $n_2$, focal lengths $f_1$, $f_2$, respectively:

$$\frac{1}{n_1 f_1} + \frac{1}{n_2 f_2} = 0$$

or, equivalently,

$$n_1 f_1 + n_2 f_2 = 0 \qquad (6.44)$$

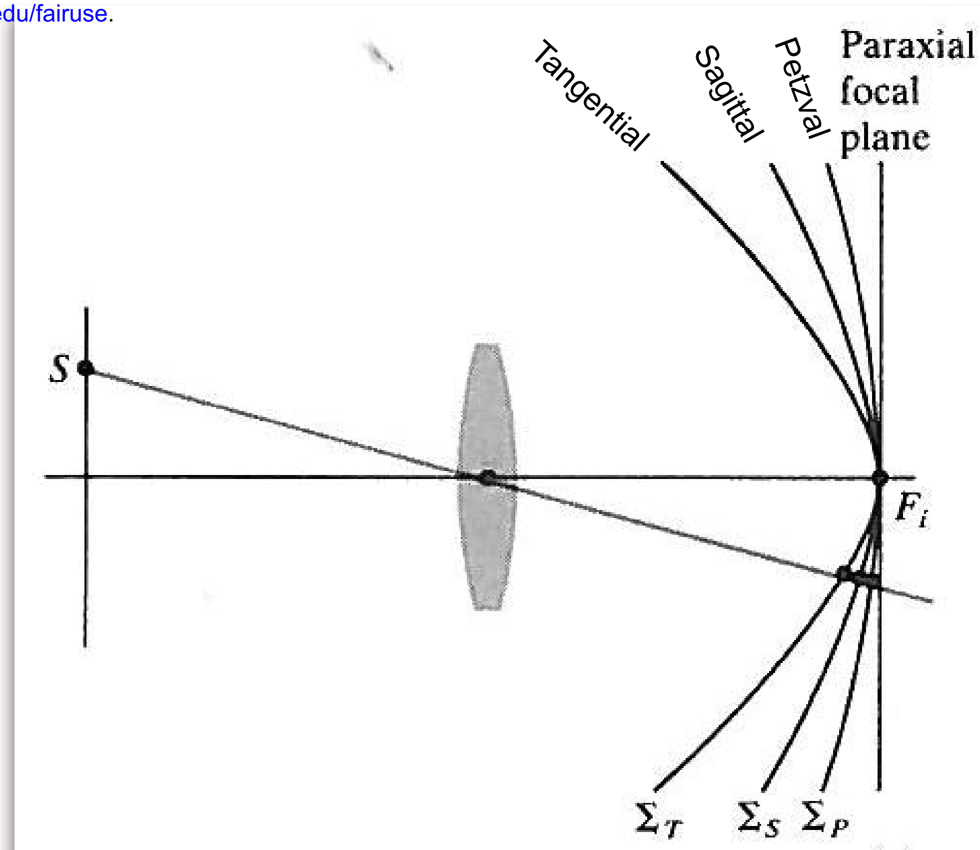
This is the so-called **Petzval condition**. As an example of its use, suppose we combine two thin lenses, one positive, the other negative, such that $f_1 = -f_2$ and $n_1 = n_2$. Since

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \qquad [6.8]$$

$$f = \frac{f_1^2}{d}$$

# Tangential, sagittal and Petzval image surfaces





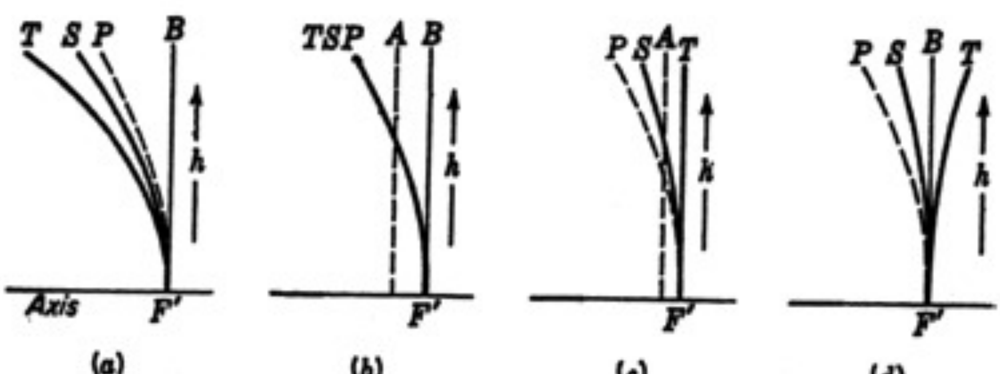
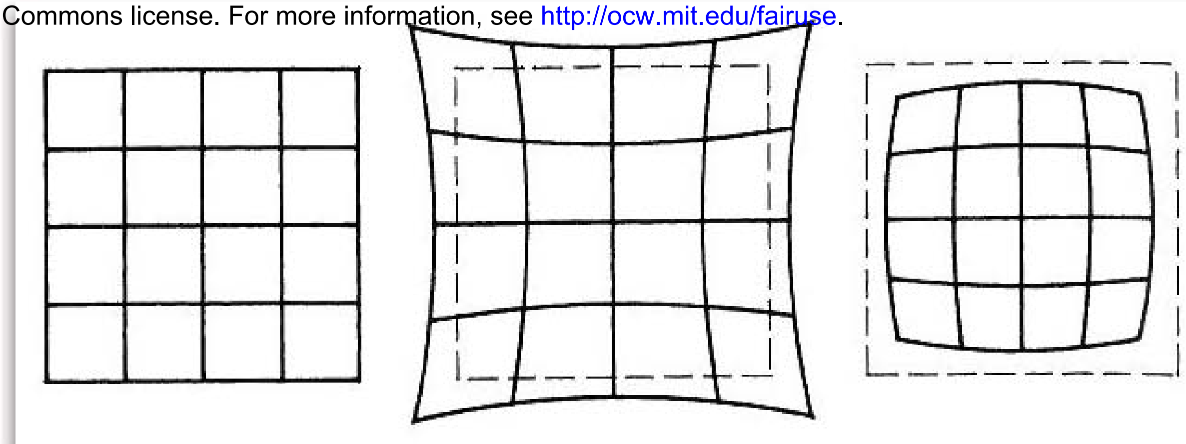




FIGURE 9R
Diagrams showing the astigmatic surfaces *T* and *S* in relation to the fixed Petzval surface *P* as the spacing between lenses (or between lens and stop) is changed.

# Distortion





No distortion　　Pincushion　　Barrel

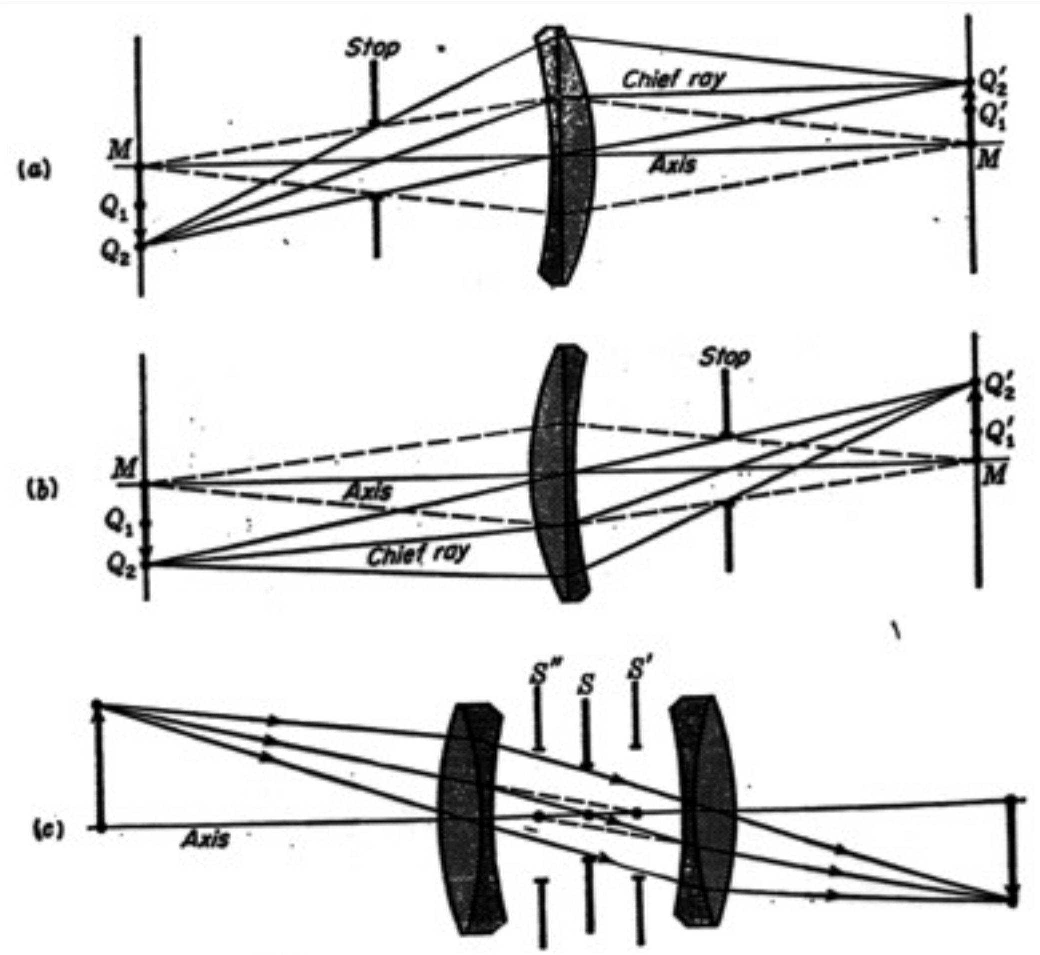
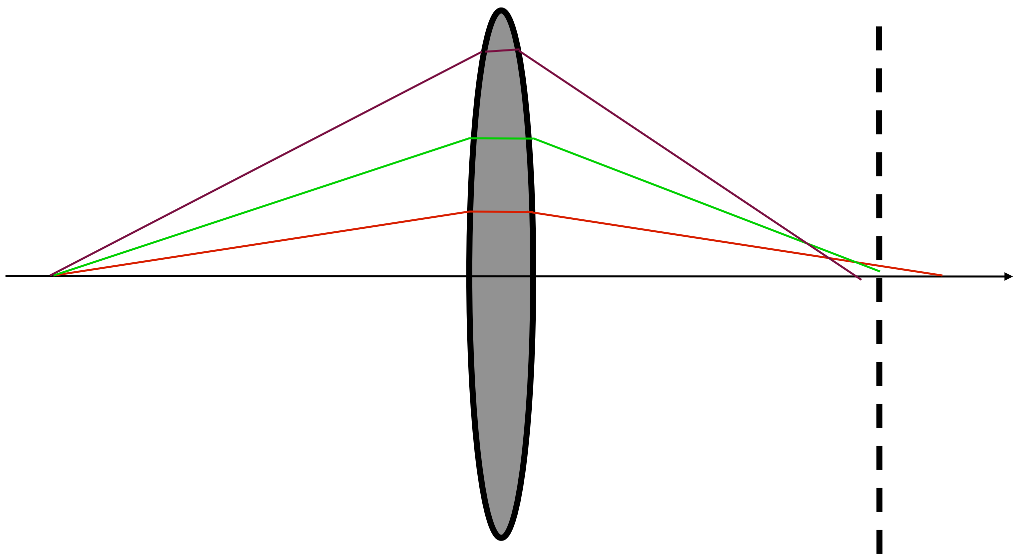
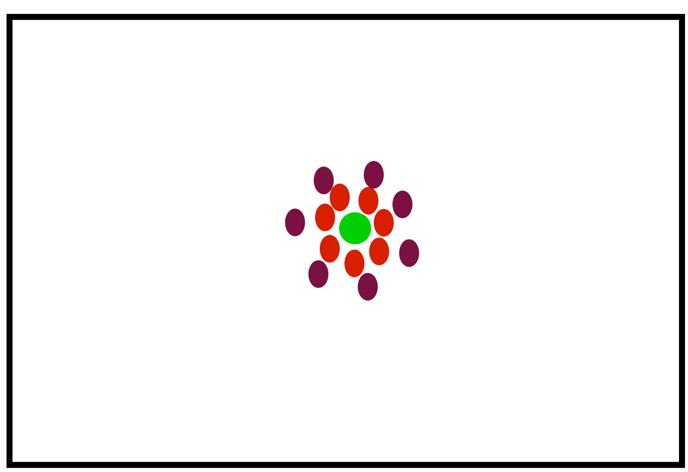



FIGURE 9U
(*a*) A stop in front of a lens giving rise to barrel distortion. (*b*) A stop behind a lens giving rise to pincushion distortion. (*c*) A symmetrical doublet with a stop between is relatively free of distortion.

# Optical design

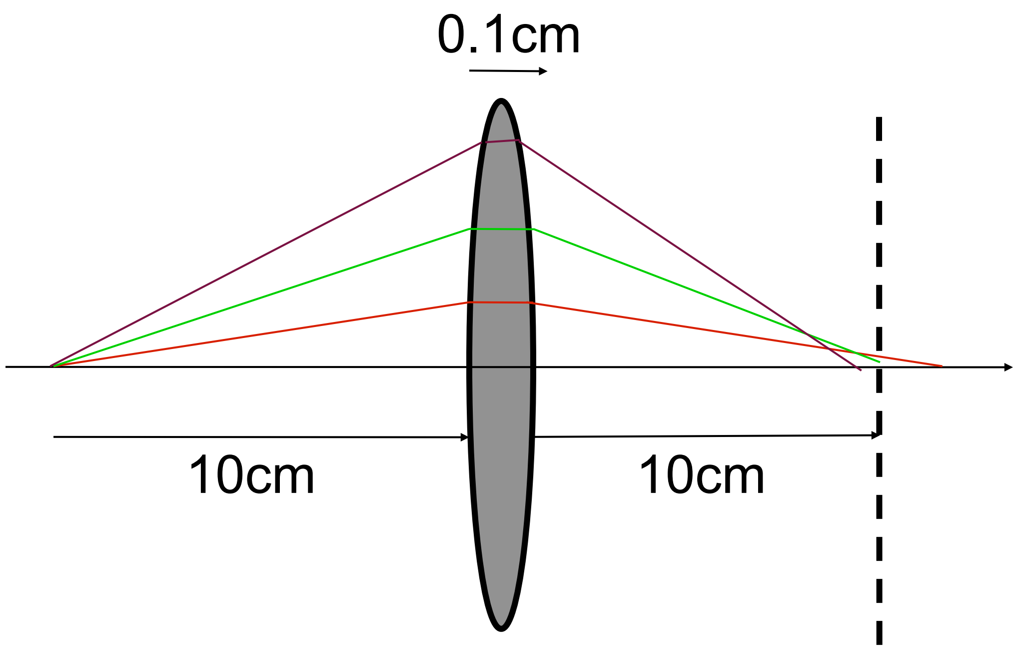


Exact ray-tracing



ray scatter diagram ( ⇔ failure to perfectly focus)

# Optical design




| Surface name | Curvature | Index to the right | Distance to next element |
|---|---|---|---|
| S0 | Inf | 1.0 | 0.1 |
| S1 | 21.6 | 1.54 | 0.001 |
| S2 | -21.6 | 1.0 | 0.1 |
| S3 | ... | ... | ... |
| S4 | ... | ... | ... |
| | | | |

# Features of optical design software

- Databases of common lenses and elements sold by vendors
- Simulate aberrations and ray scatter diagrams for various points along the field of the system
- Standard optical designs (e.g. achromatic doublet, Cooke triplet)
- Permit optimization of design parameters (e.g. curvature of a particular surface or distance between two surfaces) *vs* designated functional requirements (e.g. field curvature and astigmatism coefficients)
- Also account for diffraction by calculating the modulation transfer function (MTF) at different points along the field

# Optical design software vendors

- Zemax
- Code V
- Oslo
- acos

- specialized shops

Screenshots of optical design software removed due to copyright restrictions.

# ZEMAX® Demonstration

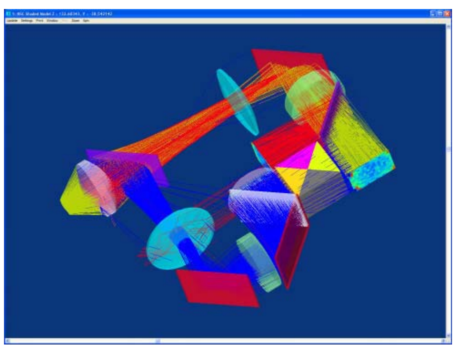
Commercial ray tracing software: Code V®, ZEMAX®, OSLO®

Ray tracing based on matrix method $$\begin{pmatrix} 1 & -(n'-n)/R \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 \\ D/n & 1 \end{pmatrix}$$

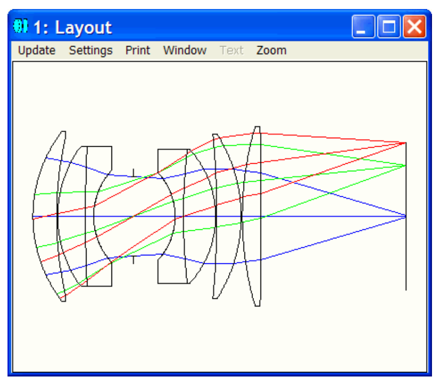
Lens, illumination, imaging system design

Various analysis including (PSF, MTF, OTF, aberration, etc)

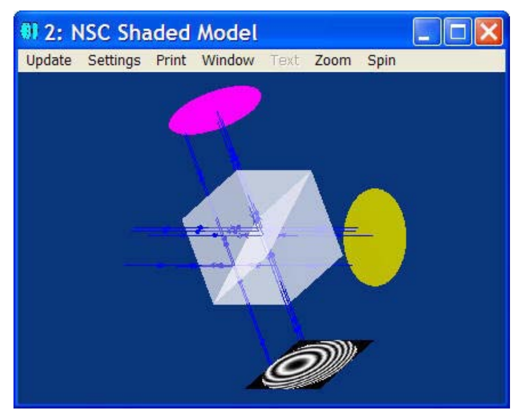
Optimization and tolerance analysis



Digital Projectors




Imaging system



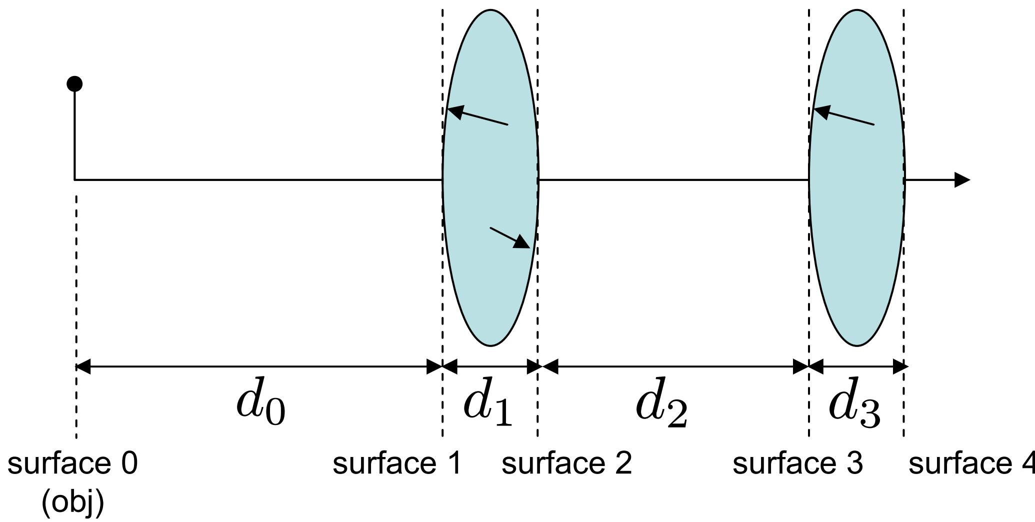

interferometer



www.zemax.com

# How to input?

Inputs: Radius, thickness, glass (air/glass/mirror), ….




$$\begin{pmatrix} 1 & -(n'-n)/R \\ 0 & 1 \end{pmatrix},$$

$$\begin{pmatrix} 1 & 0 \\ D/n & 1 \end{pmatrix}$$

| surface | radius | thickness | glass |
|---|---|---|---|
| 0 | infinity | $d_0$ | |
| 1 | $R_1$ | $d_1$ | BK7 |
| 2 | $R_2$ | $d_2$ | |
| 3 | $R_3$ | $d_3$ | BK7 |

Sometimes $d_0 \approx$ infinity

⋮

System parameters: wavelength, field (object height/angle), …

1. supp. Lecture 3-b example

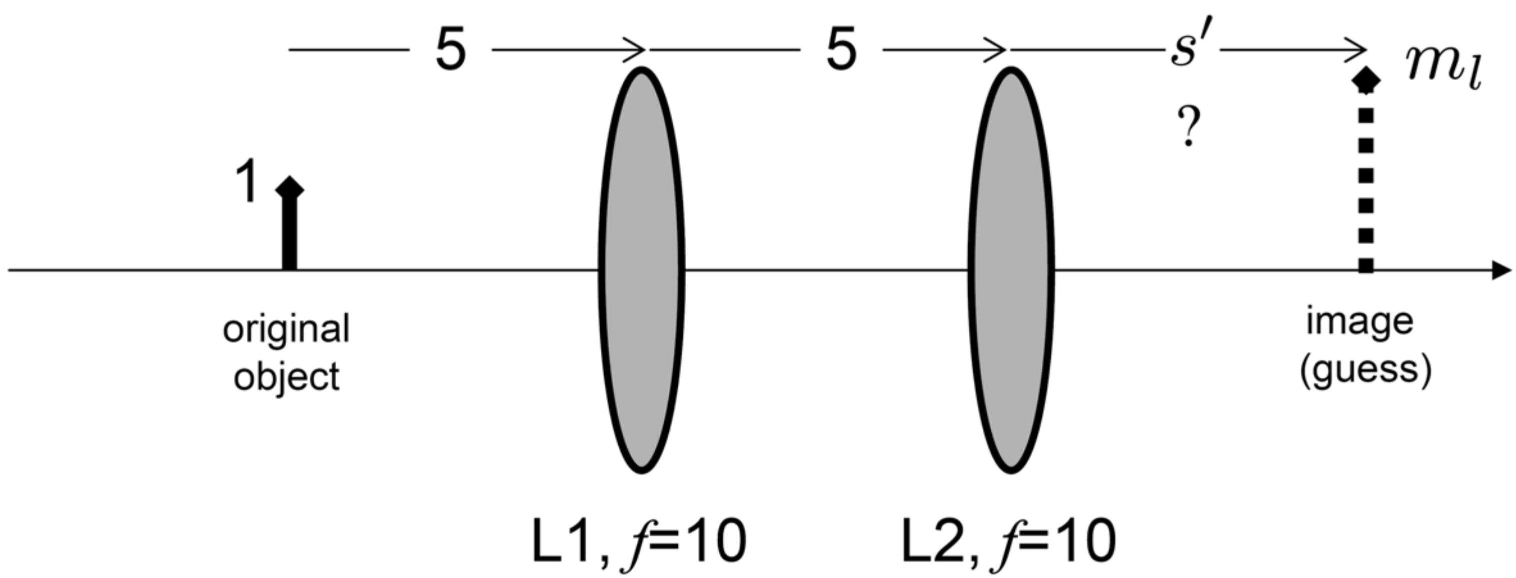
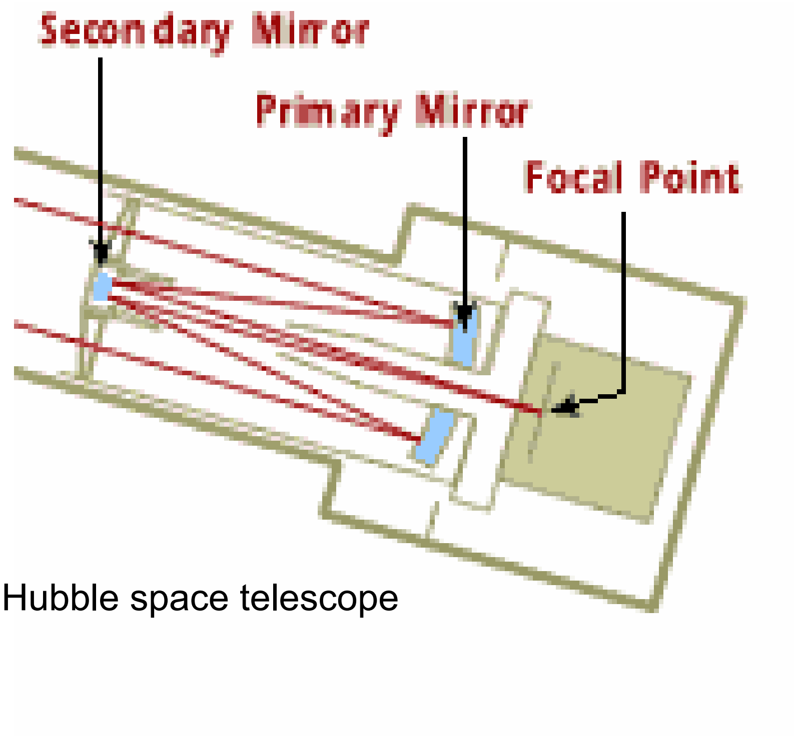



Figure 1

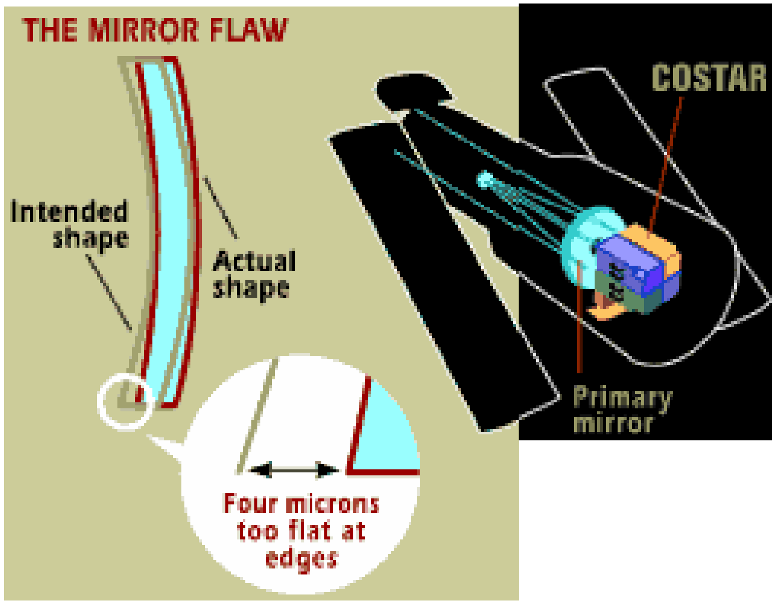
$$s' = 30\,\mathrm{mm}$$

$$M_T = -4$$

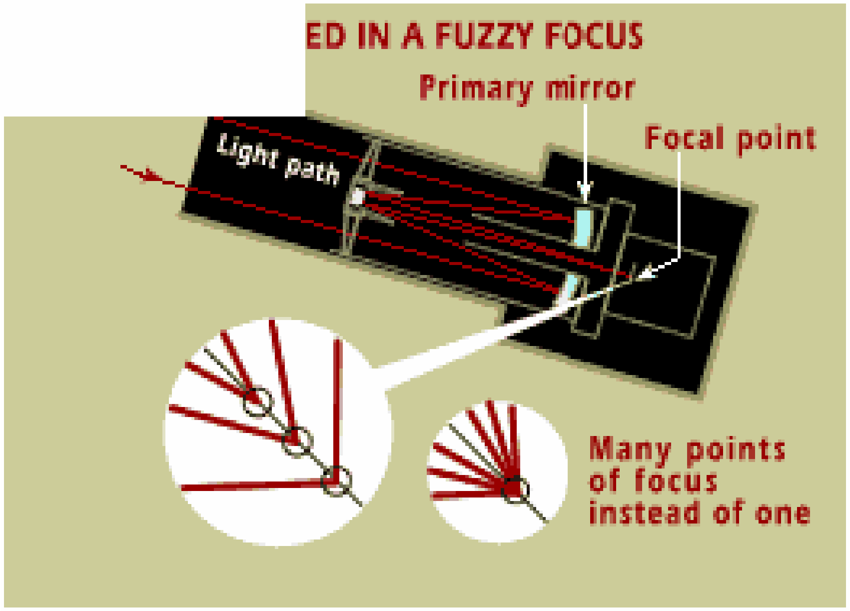
# 2. Telescope




Hubble space telescope




Primary mirror: 2.5 m diameter




Spherical aberrations!!

Courtesy of STScI/NASA.

Photo courtesy NASA

**Image of the galaxy M100 before (left) and after (right) the Hubble Space Telescope's corrective optics were installed.**

3. Cooke triplet optimization

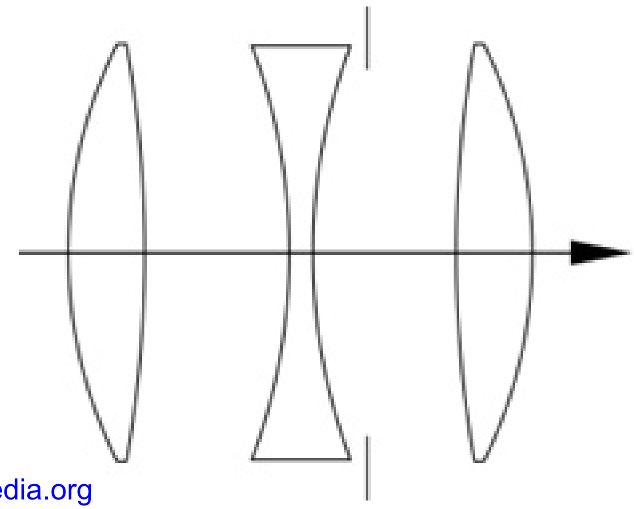


Image from Wikimedia Commons, http://commons.wikimedia.org

a negative flint glass lens in the centre
tow positive crown glass lenses on both sides

- photographic lens design designed and patented in 1893 by Dennis Taylor
- the first lens system that allows elimination of most of the optical distortion/ aberration at the outer edge of lenses.

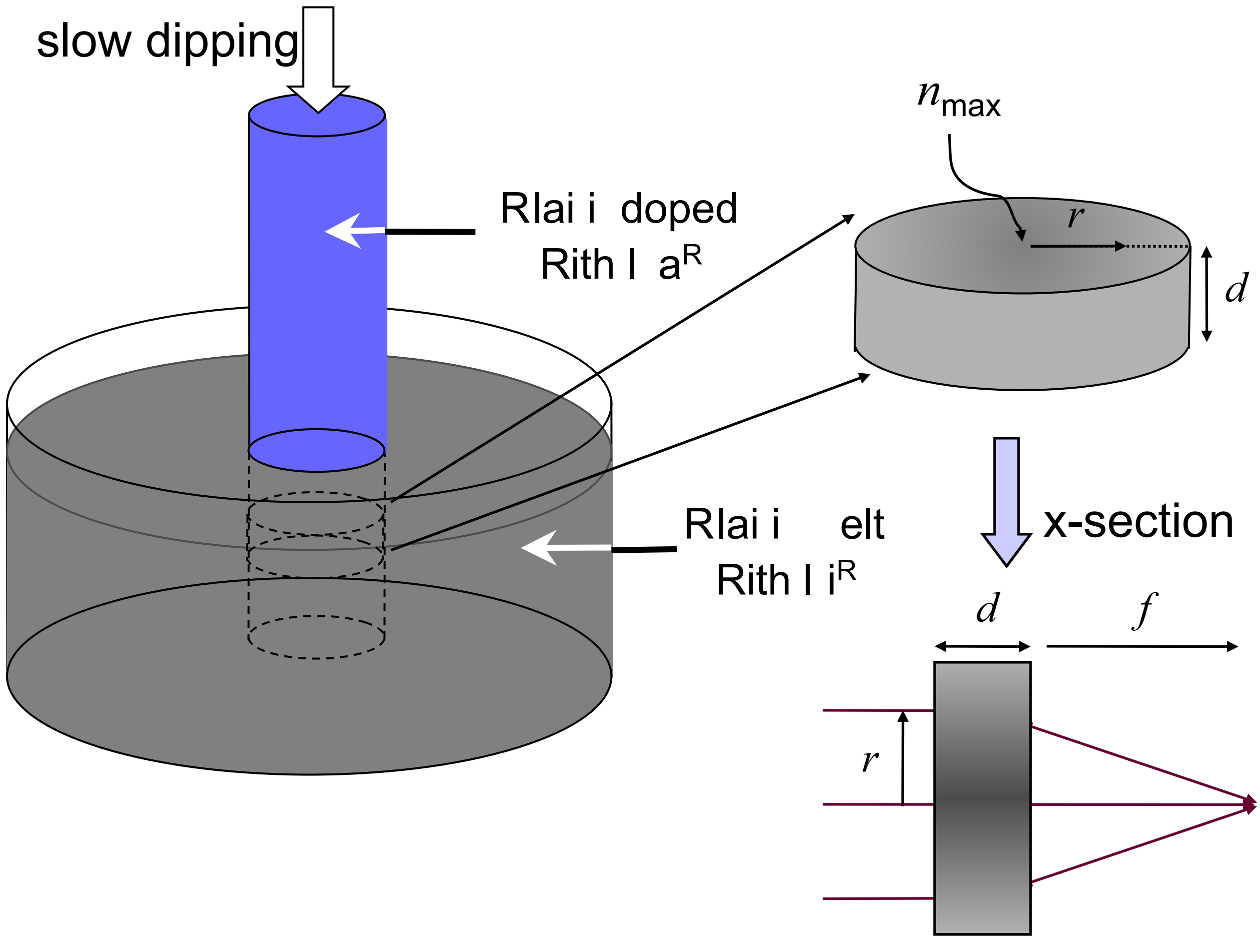
- Least square optimization

  1. Select variable (thickness, radius, etc.)
  2. Define Merit (or object or cost) function with constraints
     e.g. minimize aberrations,
     e.g. total length of the system, thickness of lenses, etc.
  3. Find parameters that minimize merit function

# Gradient Index (GRIN) optics: radial quadratic

Radial index profile:

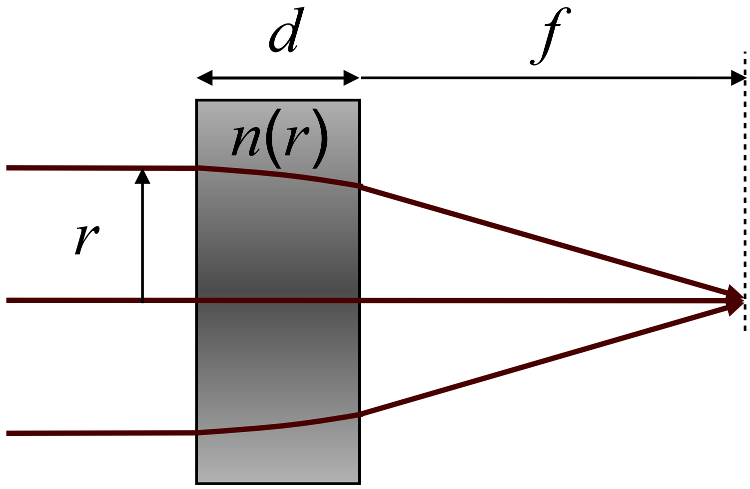
fadril ation dp ion exl hange




- Diffusion driven → parabolic index profile $n(r) = n_{\max}\left(1 - \frac{\alpha r^2}{2}\right)$
- Index contrast $n_{max}$-$n_{min} \equiv \Delta n \sim 0.1$ (commercial)

- Focal length $f = \frac{1}{n_{\max}\alpha d}$

# Paraxial focusing by a thin quadratic GRIN lens




Consider a ray from infinity entering the GRIN at elevation $r$. If focusing is to be achieved, this ray must meet the on–axis ray at a distance $f$ from the exit face; therefore, their optical paths must be equal according to Fermat's principle.

For the on–axis ray,

$$\mathrm{OPL}\,(r=0) = n_{\max} d + f.$$

For the ray at elevation $r$,

$$\mathrm{OPL}\,(r) \approx n_{\max}\left(1-\frac{\alpha r^2}{2}\right)d + \sqrt{r^2+f^2},$$

where we have neglected the small elevation decline due to the bending of the ray inside the GRIN.

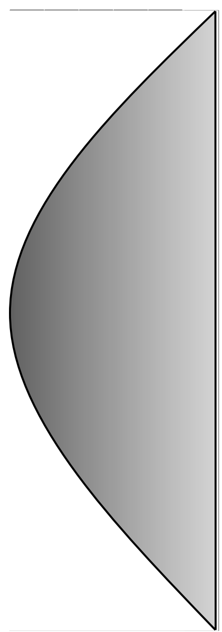
Applying Fermat's principle in the paraxial approximation,

$$\begin{aligned}
n_{\max} d + f &\approx n_{\max}\left(1-\frac{\alpha r^2}{2}\right)d + \sqrt{r^2+f^2} \Rightarrow \\
f + \frac{n_{\max}\alpha d}{2} r^2 &\approx \sqrt{r^2+f^2} \approx f\left(1+\frac{r^2}{2f^2}\right) \Rightarrow \\
\frac{n_{\max}\alpha d}{2} r^2 &\approx \frac{r^2}{2f} \Rightarrow \\
f &\approx \frac{1}{n_{\max}\alpha d}.
\end{aligned}$$

# Gradient Index (GRIN) optics: axial

<u>Rxial index profile:</u>

fadril ation dp elding & grinding



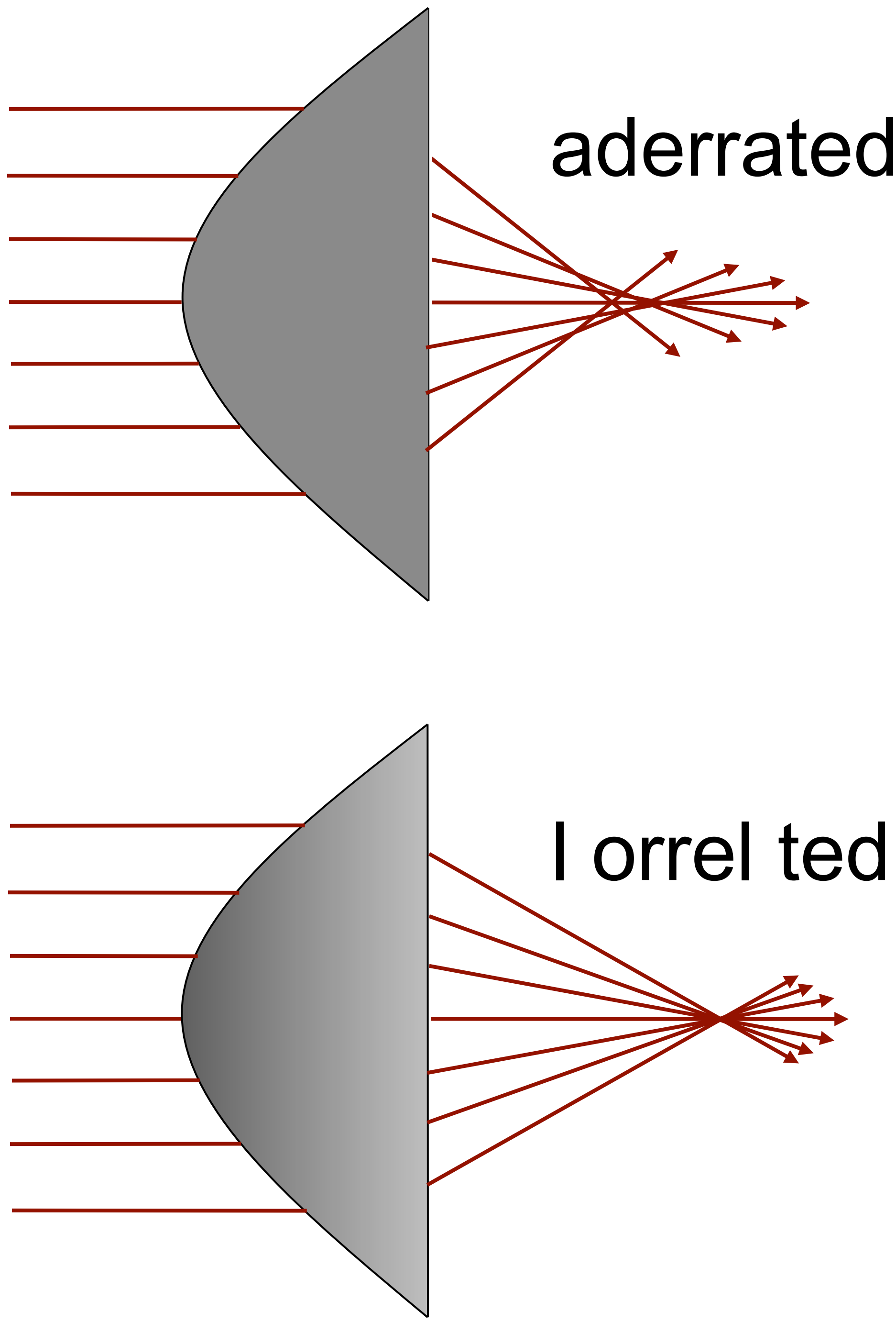
R Stack

R Meld

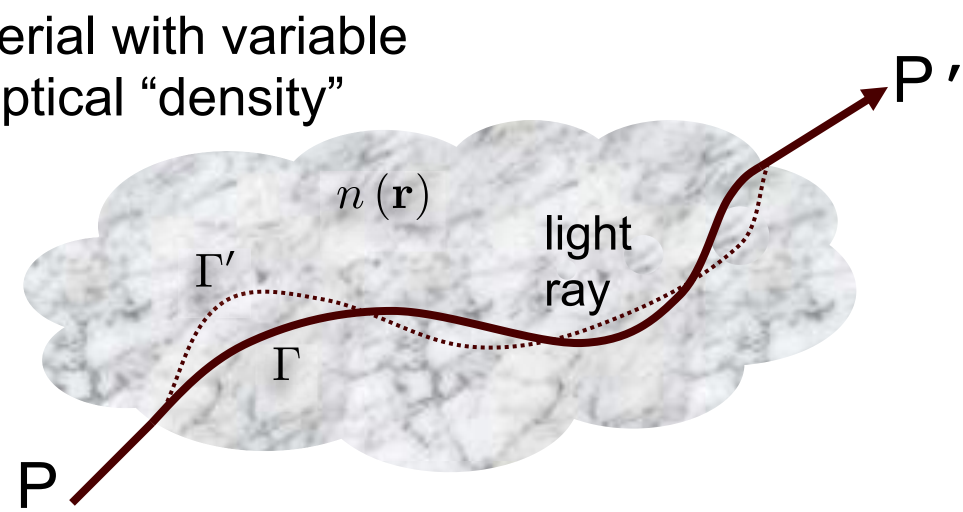
R Grind & polish to a sphere

- <u>Result:</u>
  Spherical refractive surface with axial index profile $n(z)$

# Correction of spherical aberration by axial GRIN lenses



# Generalized GRIN: what is the ray path through arbitrary *n*(*r*)?




"optical path length"

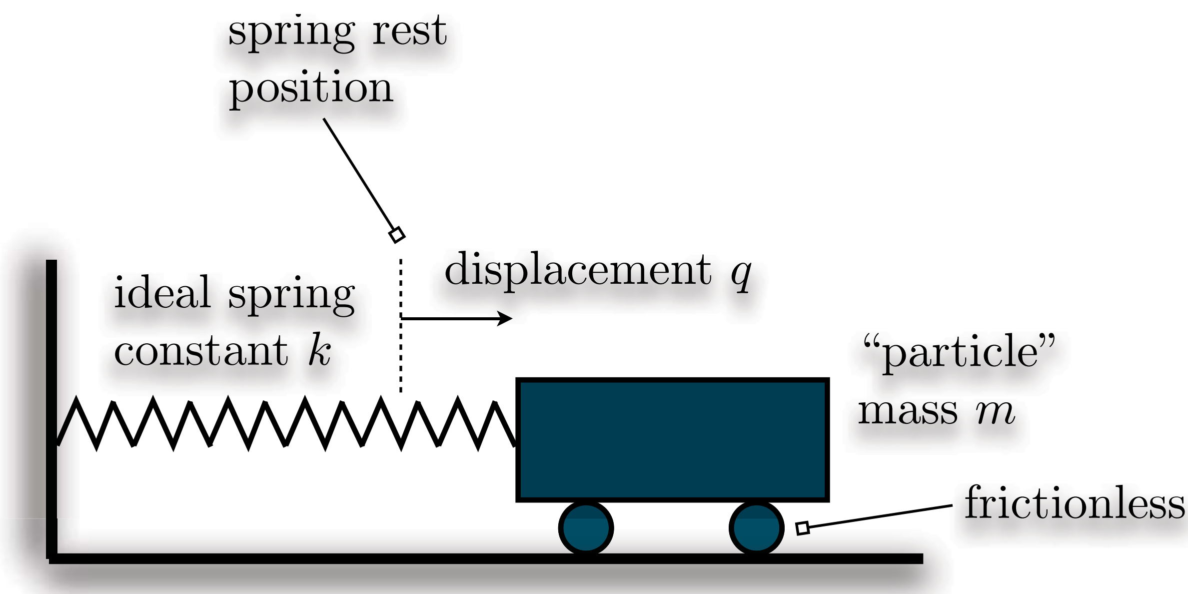
$$\int_{\Gamma} n(\mathbf{r})\, \mathrm{d}l$$

**Fermat's principle:**

The path $\Gamma$ that the ray follows is such that the value of the path integral of refractive index $n(\mathbf{r})$ along $\Gamma$ is smaller than all other possible paths $\Gamma'$.

Let's take a break from optics ...

# Mechanical oscillator




Potential energy: $V = \frac{1}{2}kq^2$.

Kinetic energy: $T = \frac{1}{2}m\dot{q}^2 = \frac{1}{2}\frac{p^2}{m}$,

where $p \equiv m\dot{q}$ is the momentum.

Since there is no dissipation,
the total energy

$$H = T + V$$

must be conserved.

**Introduction to the Hamiltonian formulation of dynamics**

The Hamiltonian formulation is a set of differential equations describing the trajectories of particles that are subject to a potential (force.) The trajectory is described in terms of the particle position $\mathbf{q}(t)$ and momentum $\mathbf{p}(t)$. The Hamiltonian is the total energy, *i.e.* the sum of kinetic and potential energies, and it is conserved if there is no dissipation in the system. For example, for a harmonic oscillator the Hamiltonian is expressed as

$$H(\mathbf{q},\mathbf{p}) = \frac{\mathbf{p}^2}{2m} + \frac{k\mathbf{q}^2}{2}. \tag{1}$$

The first term is the kinetic energy for a particle of mass $m$, and the second term is the potential energy for linear spring constant $k$.

The Hamiltonian equations in general are

$$\frac{\mathrm{d}\mathbf{q}}{\mathrm{d}t} = \frac{\partial H}{\partial \mathbf{p}}, \tag{2}$$

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = -\frac{\partial H}{\partial \mathbf{q}}. \tag{3}$$

The expressions on the right–hand side are the gradients of the Hamiltonian with respect to the vectors $\mathbf{p}$ and $\mathbf{q}$, respectively.

Let us consider the simplest case of a one–dimensional harmonic oscillator. In this case the position and momentum are scalars $q$, $p$. The Hamiltonian equations become

$$\left.\begin{aligned}\frac{\mathrm{d}q}{\mathrm{d}t} &= \frac{p}{m}\\ \frac{\mathrm{d}p}{\mathrm{d}t} &= -kq.\end{aligned}\right\} \Rightarrow \frac{\mathrm{d}^2q}{\mathrm{d}t^2} = \frac{1}{m}\frac{\mathrm{d}p}{\mathrm{d}t} = -\frac{k}{m}q \Rightarrow \frac{\mathrm{d}^2q}{\mathrm{d}t^2} + \frac{k}{m}q = 0. \tag{4}$$

We have arrived at the familiar 2$^{\text{nd}}$–order harmonic differential equation. For example, assuming a particle that is initially at position $q(t=0) = q_0$ and at rest, $p(t=0) = 0$, the solution to the Hamiltonian equations is

$$q(t) = q_0 \cos\left(\sqrt{\frac{k}{m}}t\right), \tag{5}$$

$$p(t) = -q_0\sqrt{km}\sin\left(\sqrt{\frac{k}{m}}t\right). \tag{6}$$

The solution set $\left\{q(t), p(t)\right\}$ is the trajectory of the particle. The motion represented by the trajectory that we found is clearly a harmonic oscillation.

2.71 / 2.710 Optics
Spring 2009