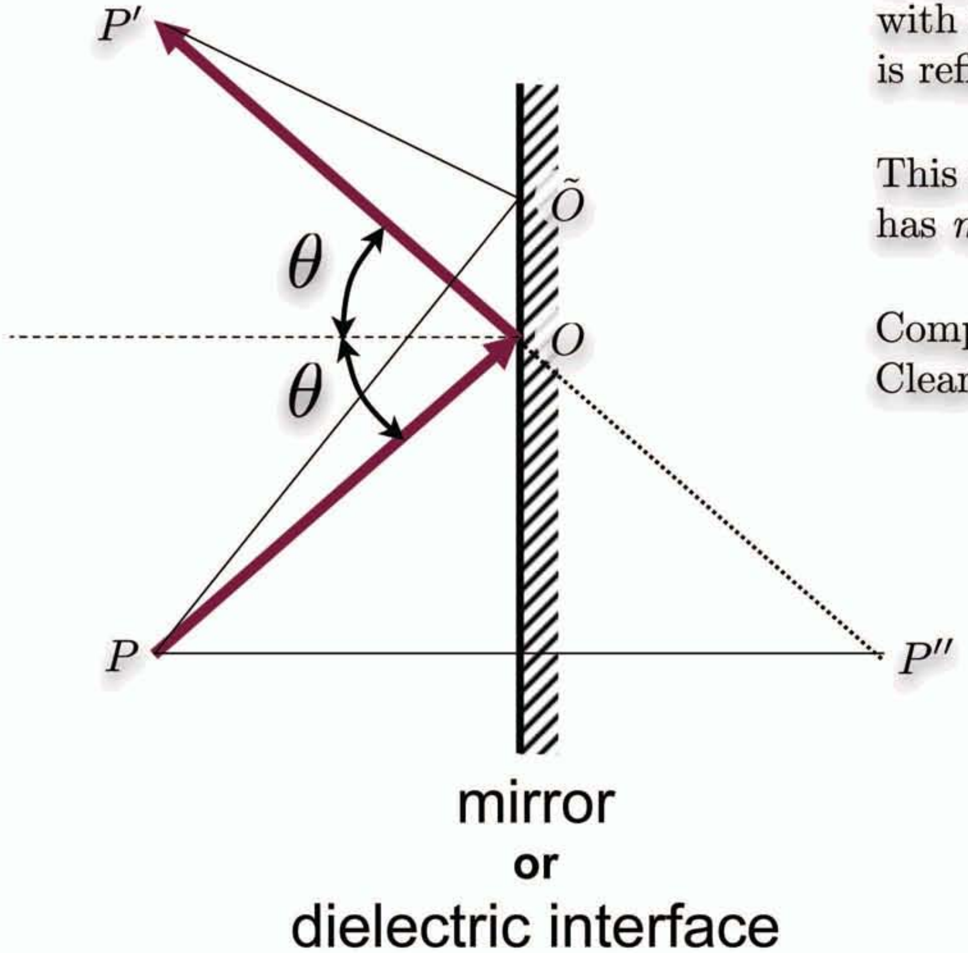
The law of reflection



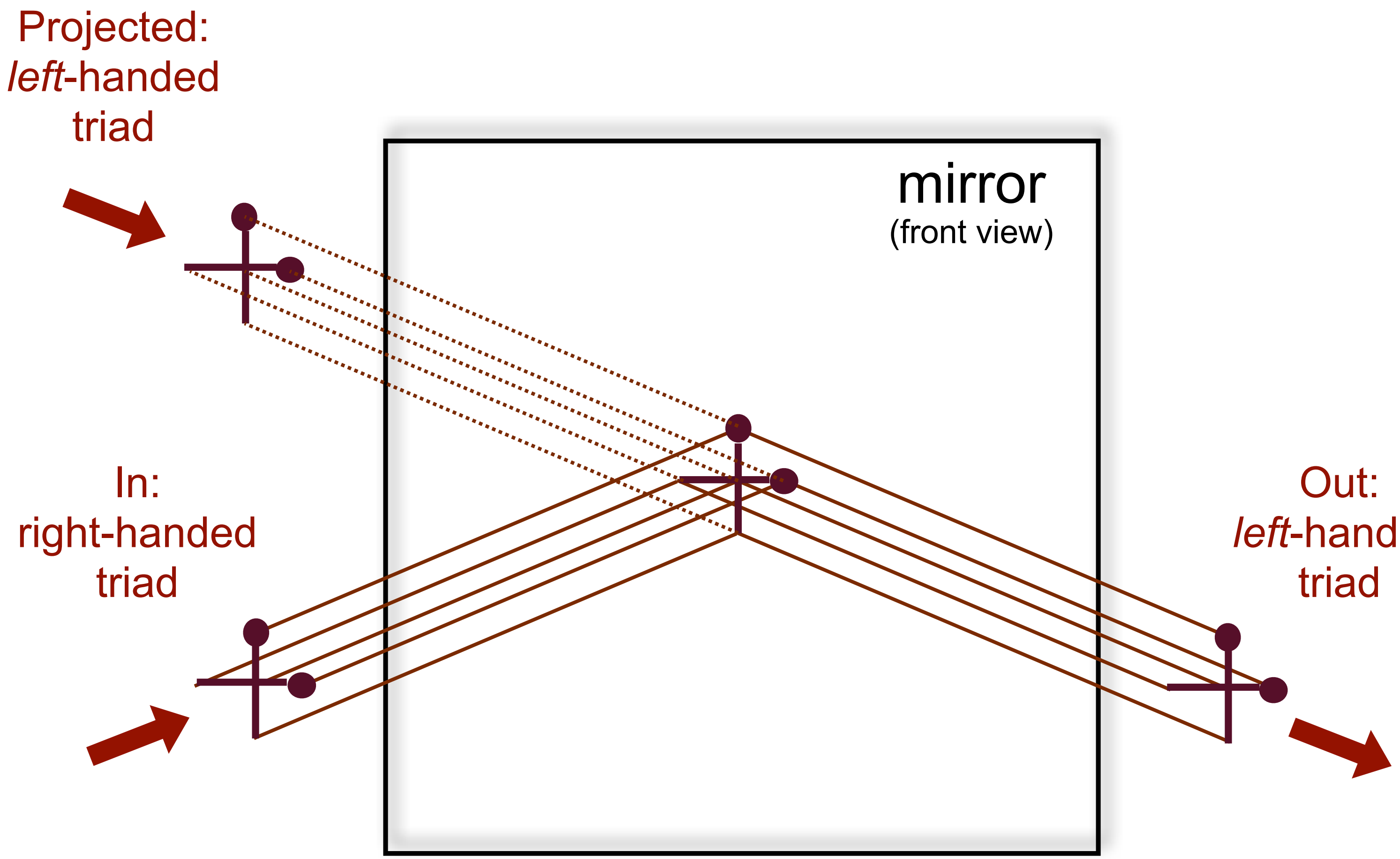
A ray departing from P in the direction θ with respect to the mirror normal is reflected symmetrically at the same angle θ.

This is because the symmetric path POP' has *minimum length*.

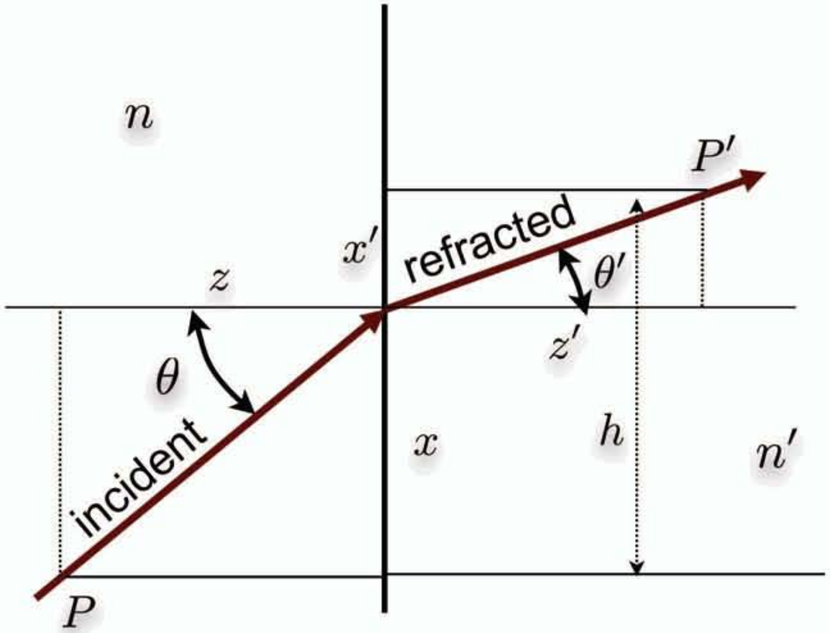
Compare, for example, the alternative $P\tilde{O}P'$. Clearly, $|PO| + |OP'| < |P\tilde{O}| + |\tilde{O}P'|$.

Consider the continuation of OP' *backwards* through the mirror. To an observer in the direction of P', the ray will appear to have originated at P''.

Reflection from mirrors



The law of refraction (Snell's Law)



Let P, P' denote two points along the ray trajectory. According to the Fermat principle, the angle θ must be such as to minimize the optical path length between PP'.

This is expressed as

$$(\mathrm{OPL}) = n\sqrt{x^2+z^2} + n'\sqrt{(h-x)^2+z'^2}.$$

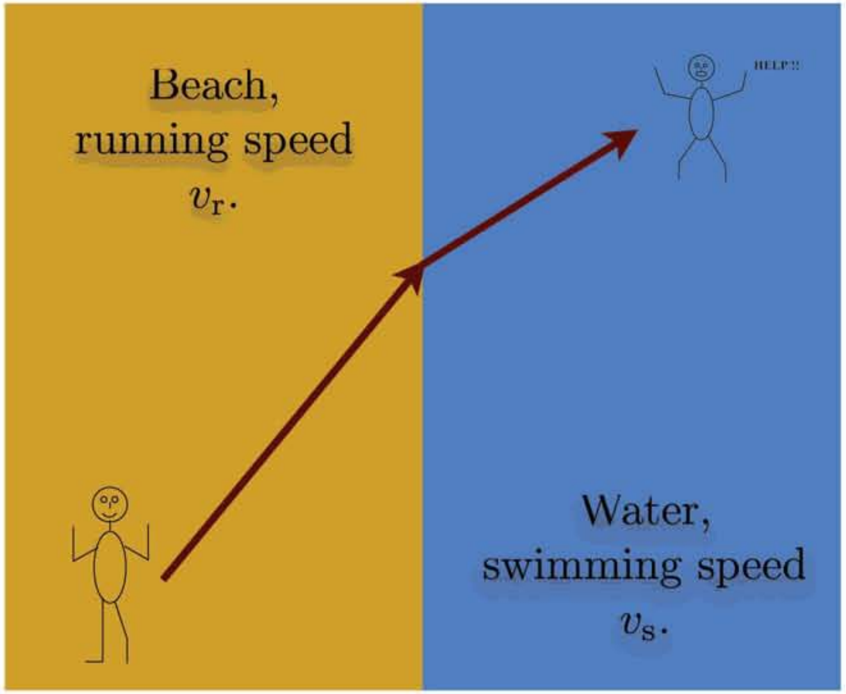
Taking derivatives with respect to x,

$$\frac{\partial(\mathrm{OPL})}{\partial x} = n\frac{x}{\sqrt{x^2+z^2}} - n'\frac{h-x}{\sqrt{(h-x)^2+z'^2}} = n\sin\theta - n'\sin\theta' = 0$$

$$\Rightarrow n\sin\theta = n'\sin\theta'$$

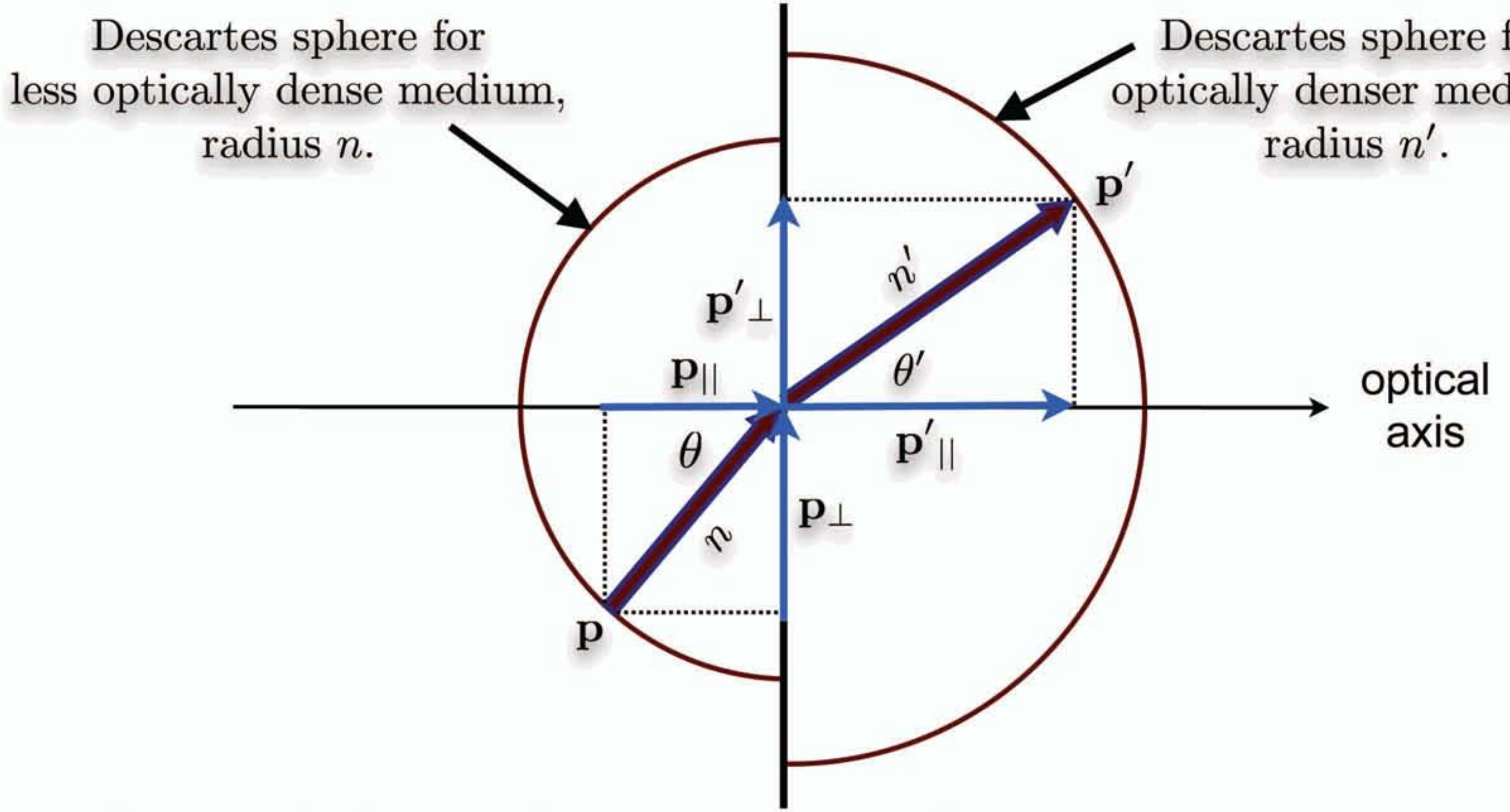
This result is known as *Snell's Law*, or *Law of Refraction*.

Snell's Law as minimum time principle



Which path should the lifeguard follow
to reach the drowning person in minimum time?

Snell's Law as momentum conservation principle



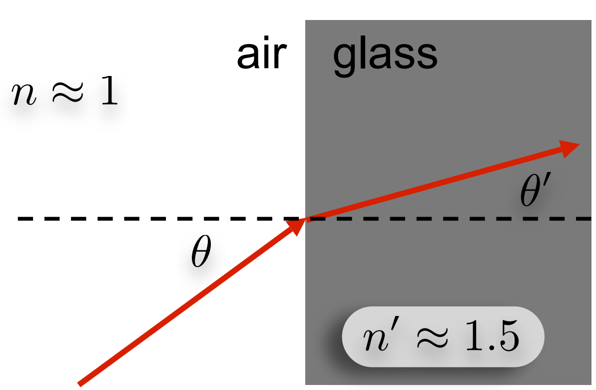
Let $\mathbf{p}$ and $\mathbf{p}'$ denote the ray momenta in the two media, respectively.

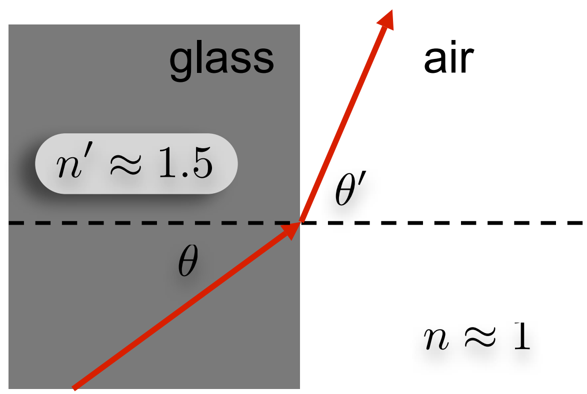
A change in the longitudinal momentum $\mathbf{p}_{||} \neq \mathbf{p}'_{||}$ is permissible, because of the medium change along the optical axis.

However, the lateral momentum must be conserved:

$$\mathbf{p}_{\perp} = \mathbf{p}'_{\perp} \Rightarrow n \sin\theta = n' \sin\theta' \text{ (i.e. Snell's law.)}$$

Two types of refraction





$$\text{n}\sin\theta = n'\sin\theta'$$

from lower to higher index
(towards *optically denser* material)

angle wrt normal decreases

the maximum angle θ'
that can enter the optically dense medium
is such that

$$n'\sin\theta' = n$$

from higher to lower index
(towards *optically less dense* material)

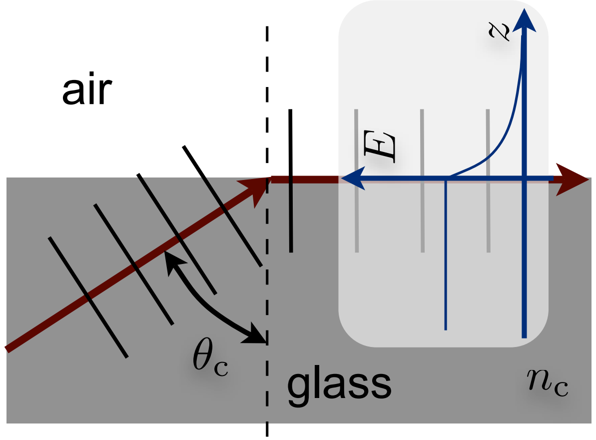
angle wrt normal increases

if the combination of n and θ
is such that

$$n\sin\theta > n'$$

then Snell's law in the less dense medium
cannot be satisfied. This situation is
known as **Total Internal Reflection (TIR)**

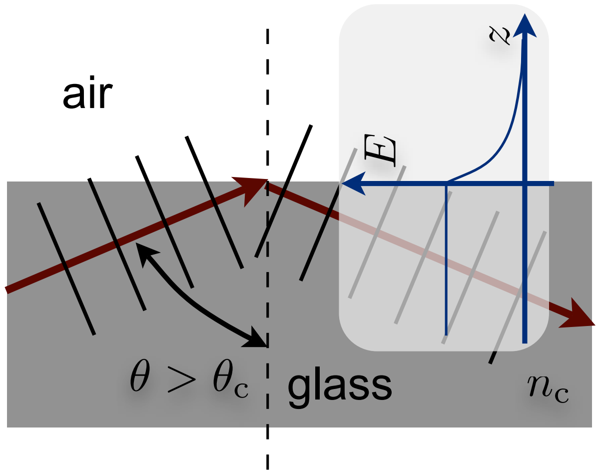
Total Internal Reflection (TIR)



$$n_c \sin\theta_c = 1$$

at critical angle, the refracted light propagates parallel to the interface

the result is called a “surface wave”



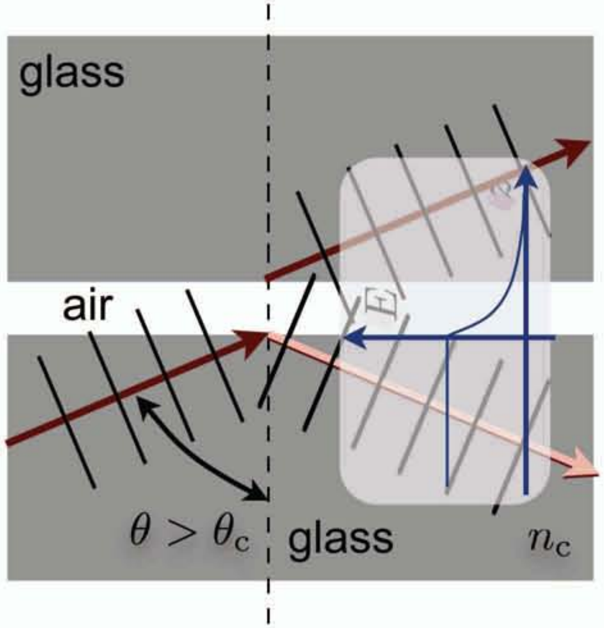
at angles of incidence higher than critical,
the light is totally internally reflected
almost as if
the glass-air interface were a mirror

in both cases of surface wave and TIR,
there is an exponential tail of electric field
leaking into the medium of lower optical density;
this is called the **evanescent wave.**

Typically, the evanescent wave is a few wavelengths long
but it can be much longer near the critical angle

We will learn more about evanescent waves after we cover the electromagnetic nature of light

Frustrated Total Internal Reflection (FTIR)



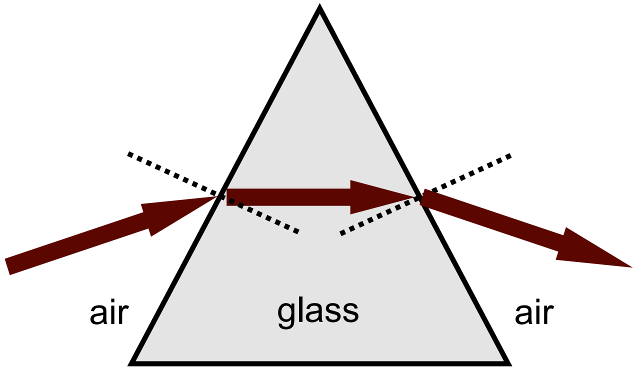
If another dielectric approaches within a few distant constants from the TIR interface, the tail of the exponentially evanescent wave becomes propagating; *i.e.* the light couples out of the medium.

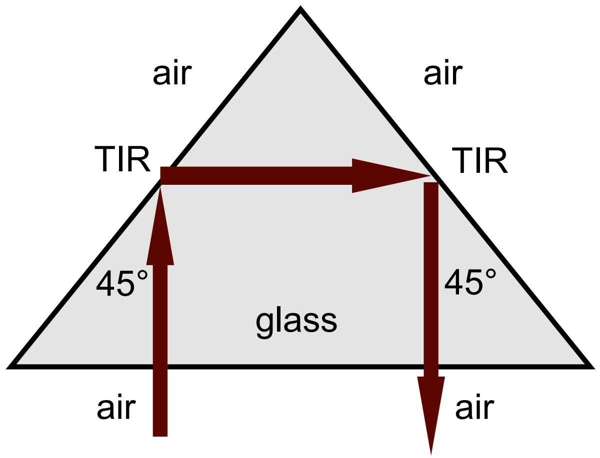
This situation is known as *Frustrated Total Internal Reflection (FTIR).*

In quantum mechanics, there is an analogous effect known as *tunneling.*

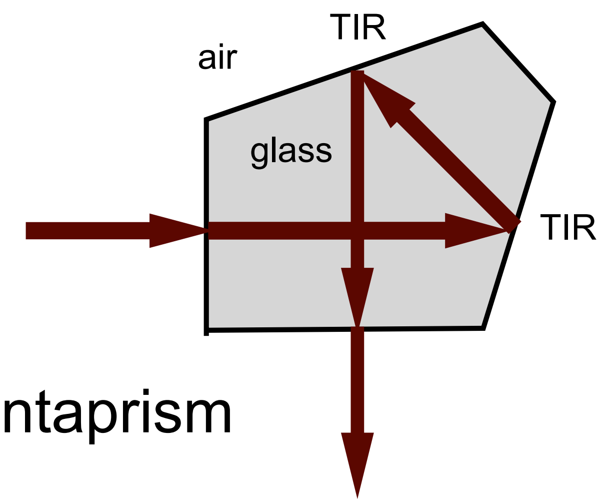
We will compute the amount of energy coupled out of the medium of incidence later, after we study the electromagnetic nature of light.

Prisms



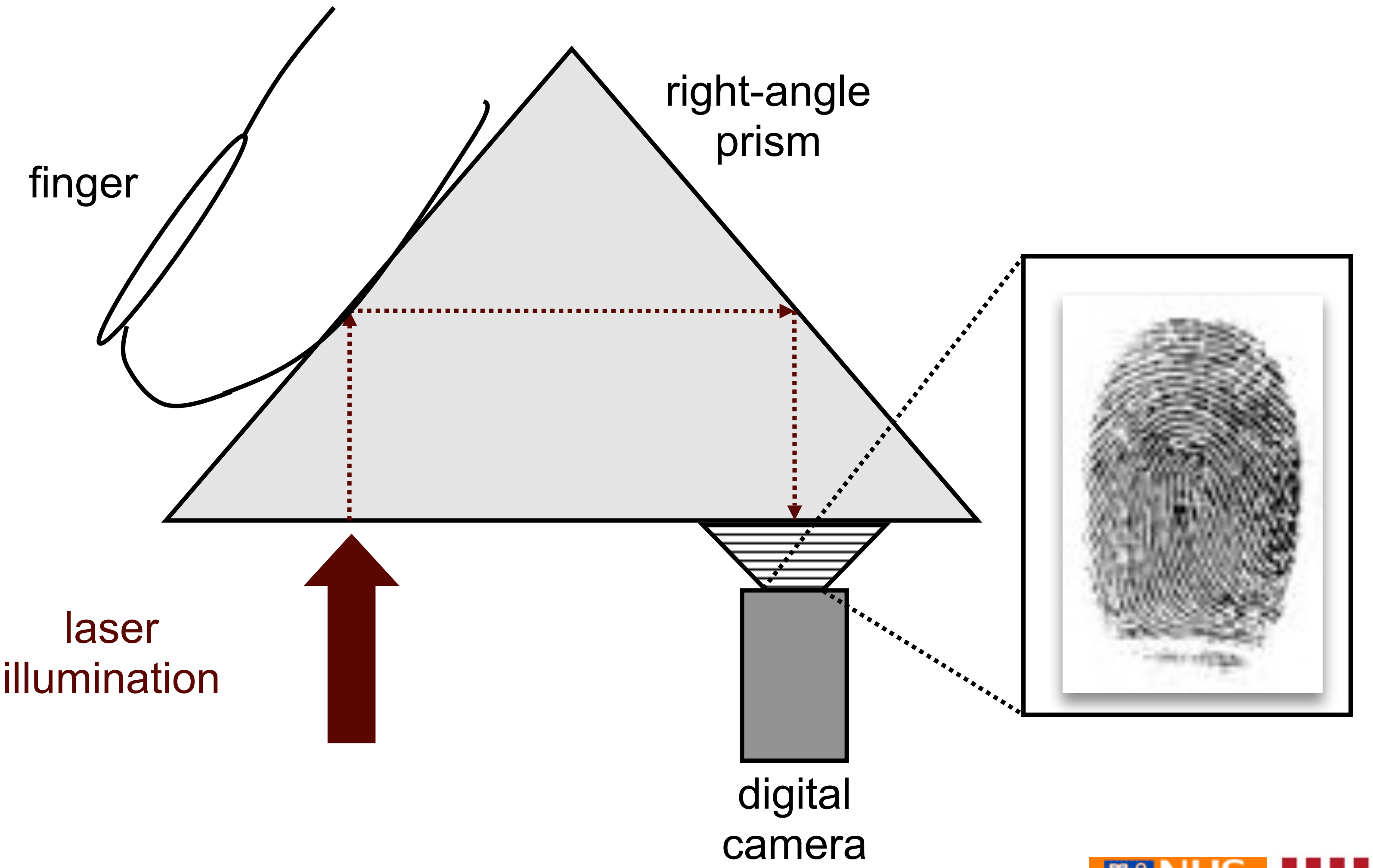


retro-reflector

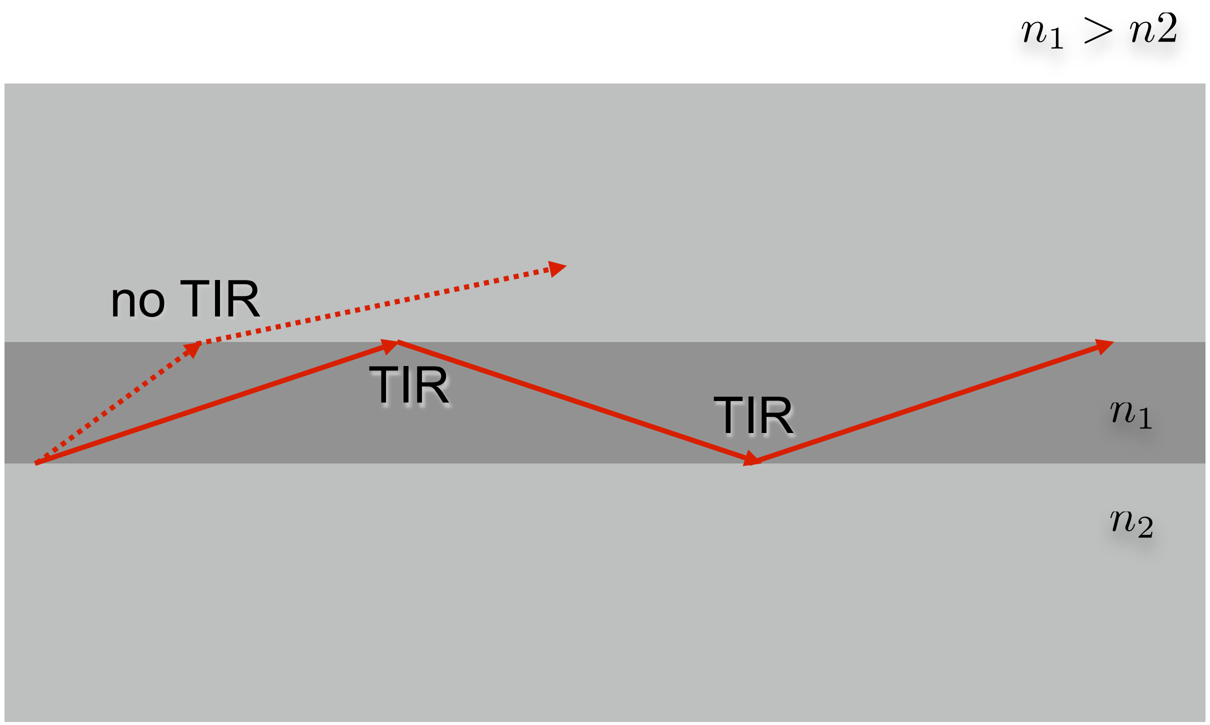


pentaprism

Fingerprint sensors



Optical waveguides

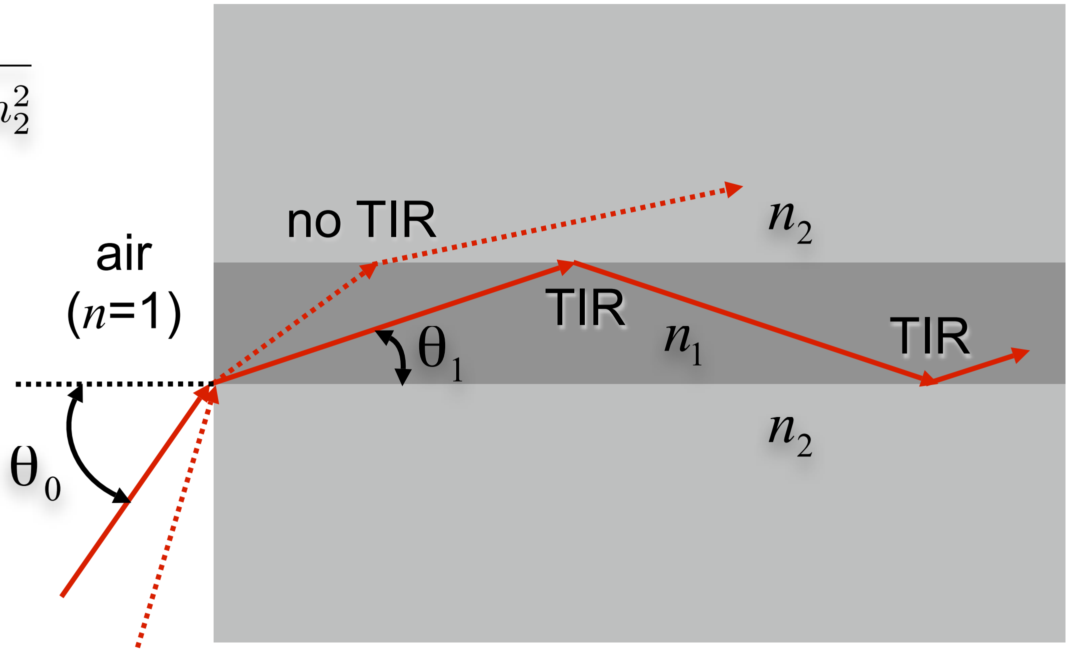


"planar" waveguide: high-index dielectric material sandwiched between lower-index dielectrics

Numerical Aperture (NA) of a waveguide

NA is the sine of the largest angle that is waveguided

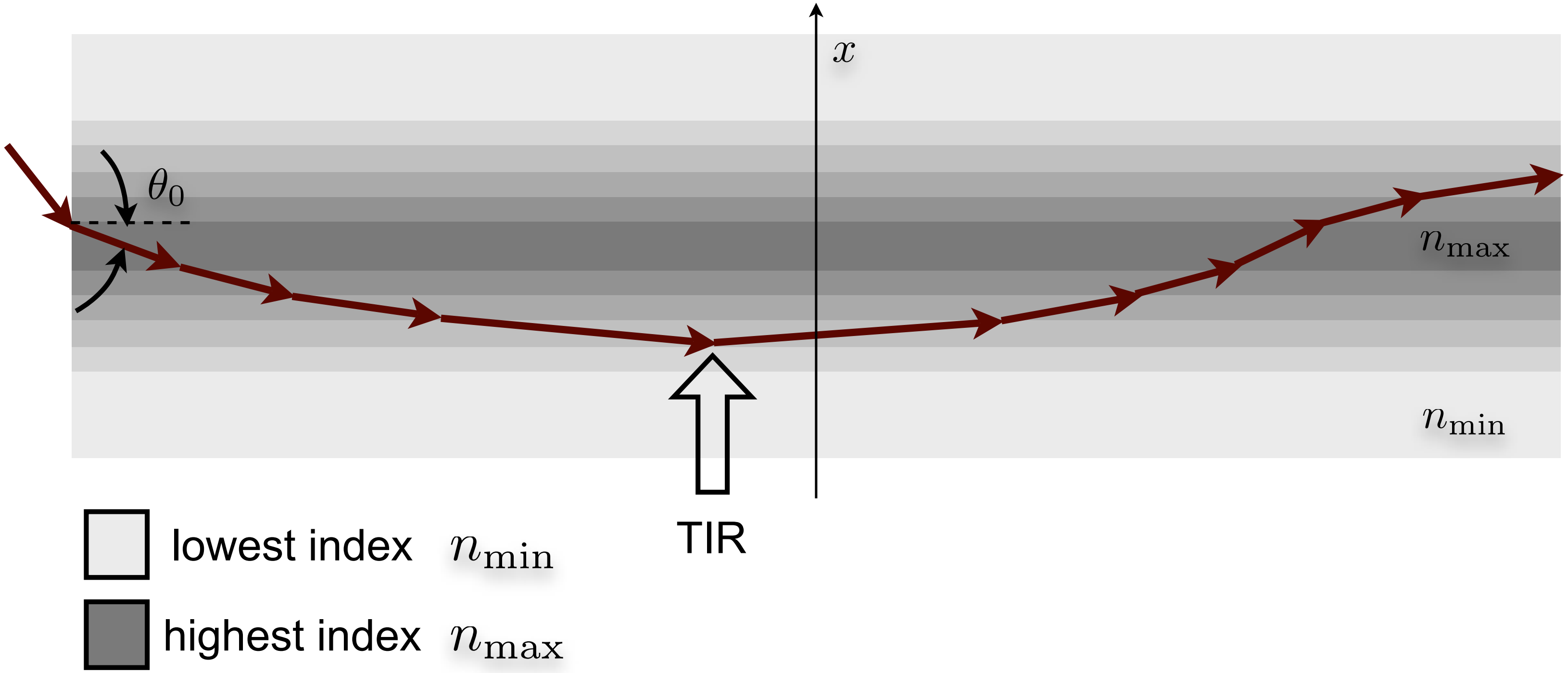
$$(\mathrm{NA}) \equiv \sin\theta_0 \leq \sqrt{n_1^2 - n_2^2}$$



i.e., NA is the incident *angle of acceptance* of the waveguide

high index contrast (n_1/n_2) ⇔ high NA

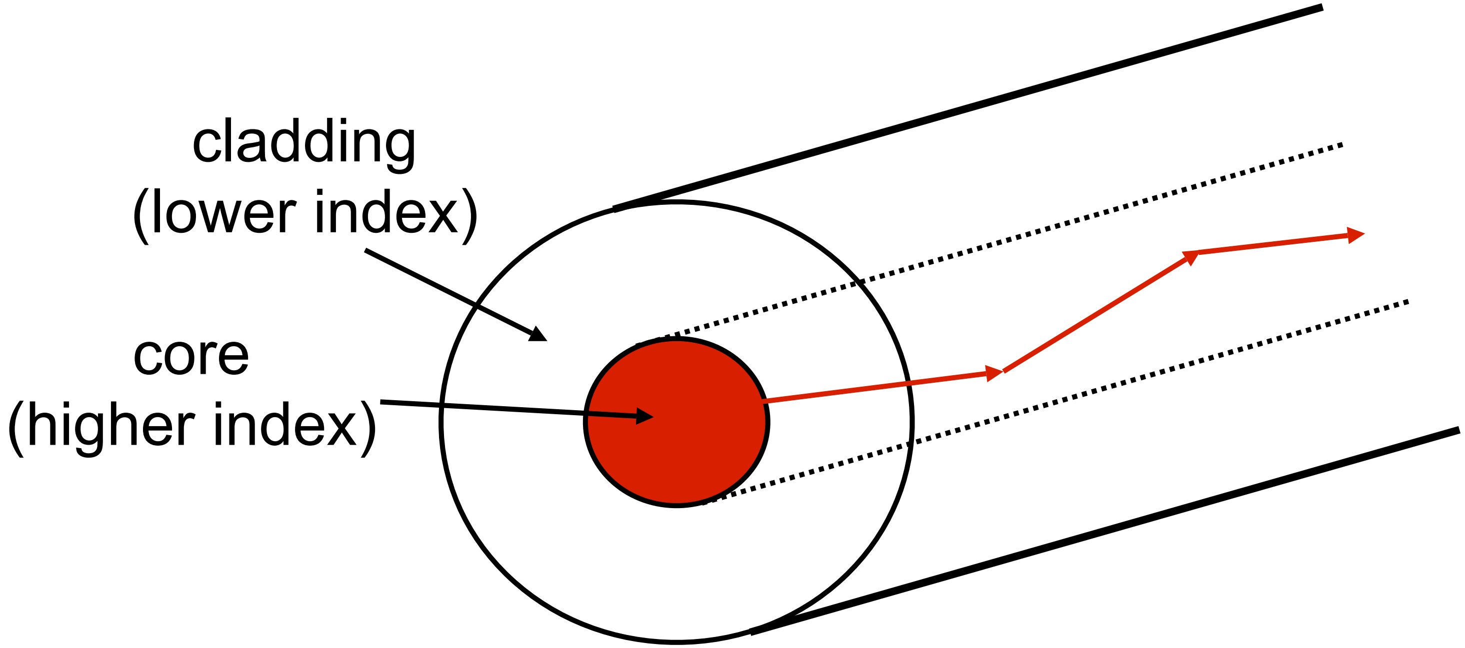
GRadient INdex (GRIN) waveguide



If the TIR condition $n_{\max} \sin \theta_0$ is satisfied,
TIR will *always* occur at one of the outer cladding interfaces;
therefore, the ray bends backwards and is guided by the GRIN structure.

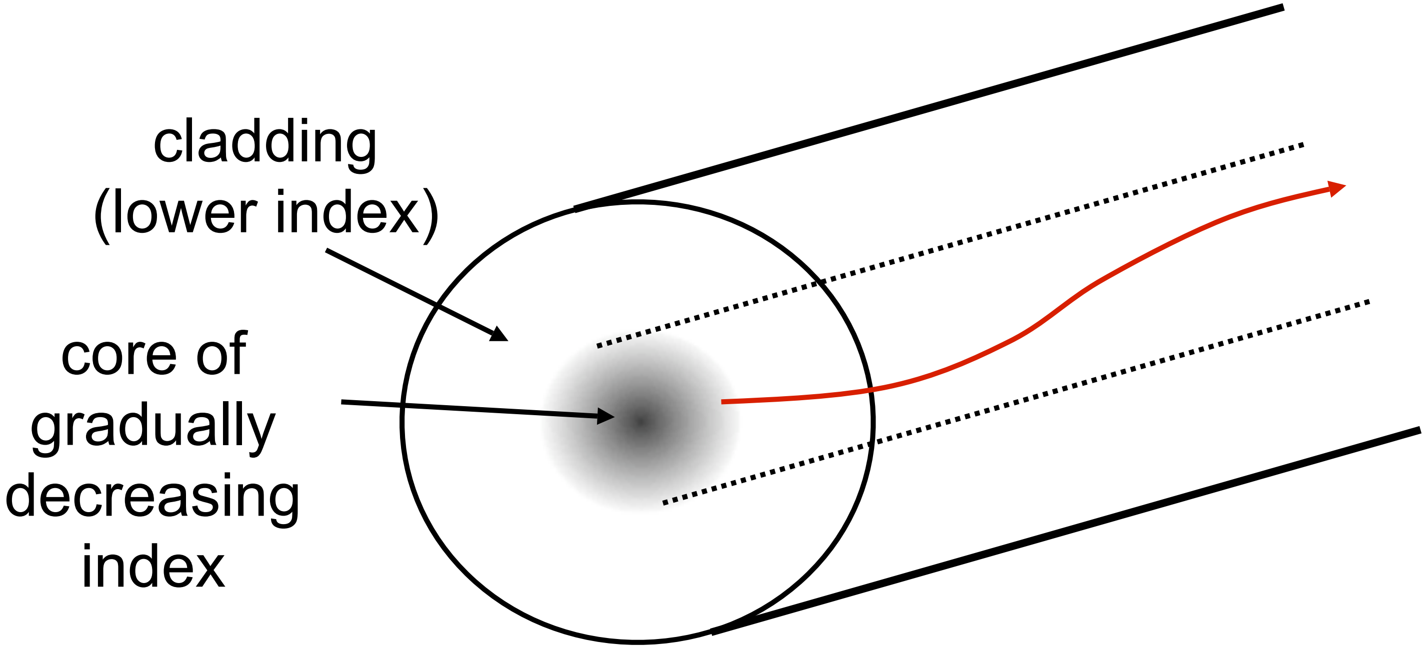
In the limit of infinitesimally small layer thicknesses and continuous index variation, the ray path becomes a smooth periodic trajectory.
In the special case $n(x) = n_{\max} - \kappa x^2$, the ray trajectory becomes *sinusoidal*.

Optical fibers: step cladding



Core diameter = 8-10μm (commercial grade)
Cladding diameter = 250μm (commercial grade)
Index contrast Δn = 0.007 (*very* low NA)
attenuation = 0.25dB/km

Optical fibers: GRIN



optical rays "swirl" in a helical trajectory
around the axis of the core

GRIN waveguides in nature: insect eyes

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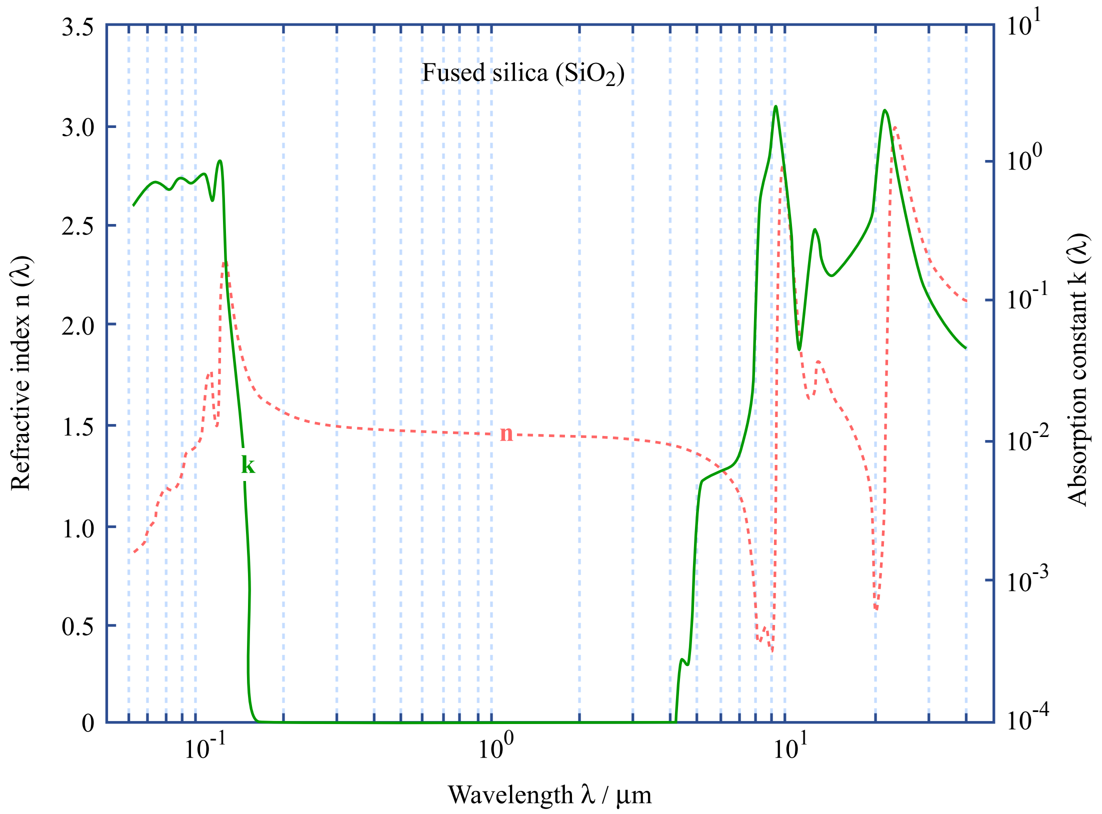
Please see
http://commons.wikimedia.org/wiki/File:Superposkils.gif

Image by Thomas Bresson at Wikimedia Commons.

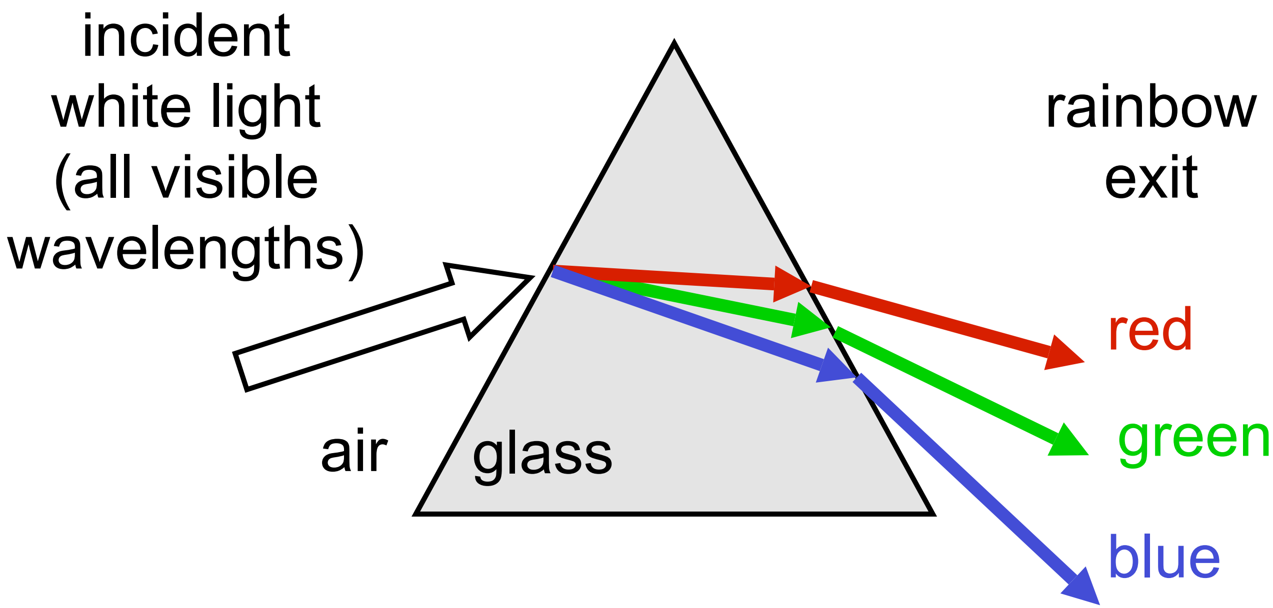
Dispersion



Dispersion = the dependence of refractive index *n* on wavelength λ.

Figure by MIT OpenCourseWare. Adapted from Fig. 2.4 in Bach, Hans, and Norbert Neuroth. *The Properties of Optical Glass*. New York, NY: Springer, 2004.

Dispersion from a prism



Newton's prism

Dispersion measures

Reference color lines
C (H- λ=656.3nm, red),
D (Na- λ=589.2nm, yellow),
F (H- λ=486.1nm, blue)

e.g., crown glass has

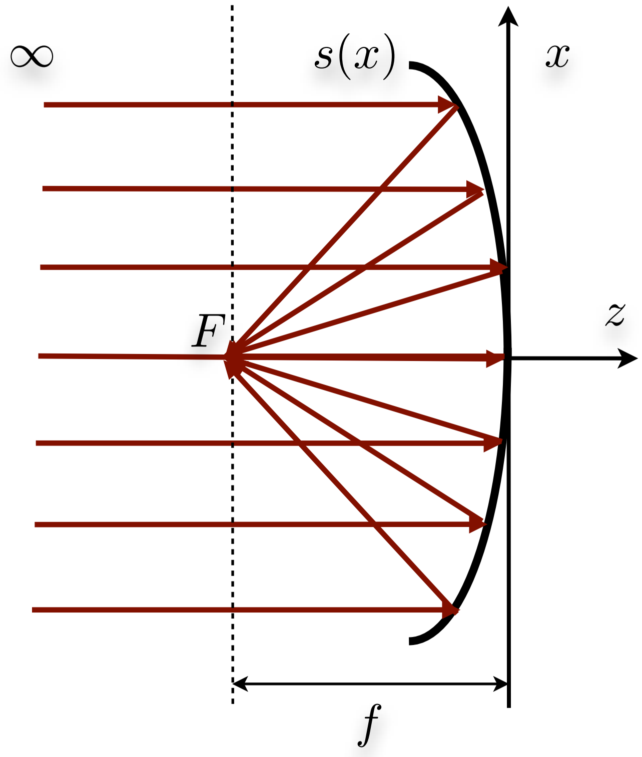
$$n_F = 1.52933 \qquad n_D = 1.52300 \qquad n_C = 1.52042$$

Dispersive power $$V = \frac{n_F - n_C}{n_D - 1}$$

Dispersive index $$\nu = \frac{1}{V} = \frac{n_D - 1}{n_F - n_C}$$ aka **Abbe's number**

e.g. for crown glass: V=0.01704 or ν=58.698

Paraboloidal reflector: *perfect* focusing



focus at F

What should the shape function $s(x)$ be in order for the incoming parallel ray bundle to come to perfect focus?

The easiest way to find the answer is to invoke Fermat's principle: since the rays from infinity follow the *minimum* path before they meet at P, it follows that they must follow the *same* path.

$$2f = f - s + \sqrt{x^2 + (f-s)^2}$$

$$f + s = \sqrt{x^2 + (f-s)^2}$$

$$\begin{aligned} x^2 &= (f+s)^2 - (f-s)^2 \Rightarrow \\ &= 4sf \Rightarrow \end{aligned}$$

A paraboloidal reflector focuses a normally incident plane wave to a point

$$s(x) = \frac{x^2}{4f}$$

2.71 / 2.710 Optics
Spring 2009