Availability

From chapter 8 VW&S

control volume in uniform state, uniform flow process USUF, irreversible $Q_{c.v.}$ and $W_{c.v.}$

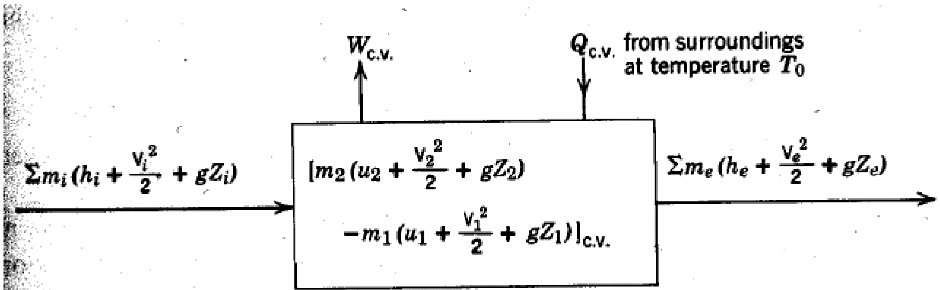
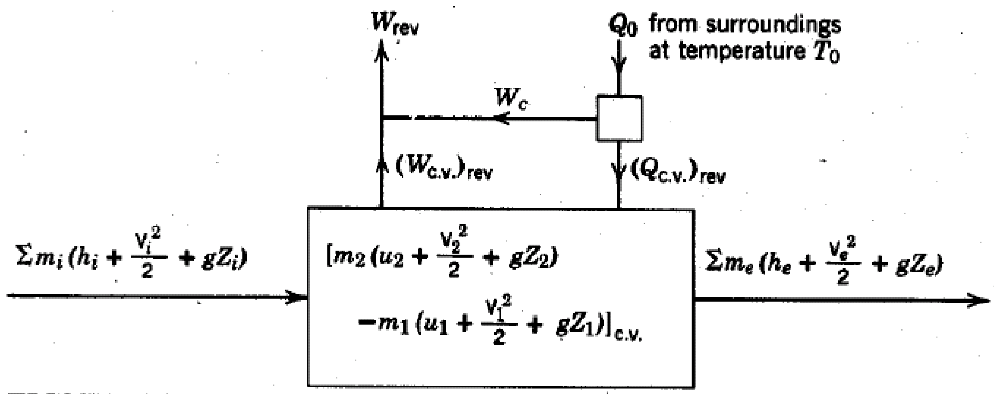


FIGURE 8.1
A uniform-state, uniform-flow process.

what if process were reversible, how much work would have been done if the process had been reversible



<= reversible heat engine

FIGURE 8.2
A reversible process for the same change of state as in Fig. 8.1.

$W_{rev} = W_{c\_v\_rev} + W_c$ reversible heat transfer takes place through reversible heat engine with output W_c (8.1)

$I = \text{irreversibility} = W_{rev} - W_{cv}$

first law for uniform state, uniform process ... from first_law.mcd

uniform state, uniform flow process (USUF)

(5.54) = (8.3)

$$Q_{c\_v\_rev} + \sum_n \left[m_{i_n} \cdot \left(h_i + \frac{V_i^2}{2} + g \cdot z_i \right) \right] = \sum_n \left[m_{e_n} \cdot \left(h_e + \frac{V_e^2}{2} + g \cdot z_e \right) \right] \dots$$

$$+ m_2 \cdot \left(u_2 + \frac{V_2^2}{2} + g \cdot z_2 \right) - m_1 \cdot \left(u_1 + \frac{V_1^2}{2} + g \cdot z_1 \right) + W_{c\_v\_rev}$$

for the reversible heat engine ... $W_c = Q_o - Q_{c\_v\_rev}$

from second law ΔS for Q_o and $Q_{c\_v\_rev}$ are the same, Q_o at constant temperature

$$\Delta S = \Delta S_{c\_v\_rev} \qquad Q_o = T_o \cdot \Delta S \qquad \Delta S_{c\_v\_rev} = \int_0^t \frac{Q\_dot_{c\_v\_rev}}{T}\,dt$$

that is express as integral of rate
uniform state => T constant in c.v.

$$\frac{Q_o}{T_o} = \int_0^t \frac{Q\_dot_{c\_v\_rev}}{T}\,dt \qquad \text{and} \qquad Q_o = T_o \cdot \int_0^t \frac{Q\_dot_{c\_v\_rev}}{T}\,dt$$

substituting into ...

$$W_c = Q_o - Q_{c\_v\_rev} = T_o \cdot \int_0^t \frac{Q\_dot_{c\_v\_rev}}{T}\,dt - Q_{c\_v\_rev} \qquad (8.4)$$

also for USUF
from second law

$$m_2 \cdot s_2 - m_1 \cdot s_1 + \sum_n \left(m_e \cdot s_e\right) - \sum_n \left(m_i \cdot s_i\right) = \int_0^t \frac{Q\_dot_{c\_v}}{T}\,dt$$

(7.56) = when reversible (8.5)

substituting (8.5) into (8.4) ...

$$W_c = Q_o - Q_{c\_v\_rev} = T_o \cdot \left[m_2 \cdot s_2 - m_1 \cdot s_1 + \sum_n \left(m_e \cdot s_e\right) - \sum_n \left(m_i \cdot s_i\right)\right] - Q_{c\_v\_rev}$$

so ... the bottom line, substitute (8.3) rearranged and (8.6) into (8.1) $W_{rev} = W_{c\_v\_rev} + W_c$

$$\left.\begin{aligned} W_{rev} = Q_{c\_v\_rev} &+ \sum_n \left[m_{i_n} \cdot \left(h_i + \frac{V_i^2}{2} + g \cdot z_i\right)\right] - \left[\sum_n \left[m_{e_n} \cdot \left(h_e + \frac{V_e^2}{2} + g \cdot z_e\right)\right]\right] \ldots \\ &+ -\left[m_2 \cdot \left(u_2 + \frac{V_2^2}{2} + g \cdot z_2\right) - m_1 \cdot \left(u_1 + \frac{V_1^2}{2} + g \cdot z_1\right)\right] \ldots \end{aligned}\right\} (8.3)$$

$$\left. + T_o \cdot \left[m_2 \cdot s_2 - m_1 \cdot s_1 + \sum_n \left(m_e \cdot s_e\right) - \sum_n \left(m_i \cdot s_i\right)\right] - Q_{c\_v\_rev} \right\} (8.6)$$

$Q_{c\_v\_rev}$ cancels and rearranging (moving T_o and s terms into mass flow terms) ...

reversible work (maximum) of a control volume that exchanges heat with the surroundings at To

$$\begin{aligned} W_{rev} = &\sum_n \left[m_{i_n} \cdot \left(h_i - T_o \cdot s_i + \frac{V_i^2}{2} + g \cdot z_i\right)\right] - \left[\sum_n \left[m_{e_n} \cdot \left(h_e - T_o \cdot s_e + \frac{V_e^2}{2} + g \cdot z_e\right)\right]\right] \ldots \\ &+ -\left[m_2 \cdot \left(u_2 - T_o \cdot s_2 + \frac{V_2^2}{2} + g \cdot z_2\right) - m_1 \cdot \left(u_1 - T_o \cdot s_1 + \frac{V_1^2}{2} + g \cdot z_1\right)\right] \end{aligned}$$

(8.7)

latter [..] is total for c.v.

two special cases: a system (fixed mass) and steady-state, steady flow process for a control volume

system (fixed mass)

$$\sum_n \left[m_{i_n} \cdot \left(h_i - T_o \cdot s_i + \frac{V_i^2}{2} + g \cdot z_i \right) \right] = 0 \qquad \sum_n \left[m_{e_n} \cdot \left(h_e - T_o \cdot s_e + \frac{V_e^2}{2} + g \cdot z_e \right) \right] = 0 \qquad m_1 = m_2 = m$$

system (fixed mass

$$\frac{W_{rev\_1\_2}}{m} = w_{rev\_1\_2} = \left(u_1 - T_o \cdot s_1 + \frac{V_1^2}{2} + g \cdot z_1 \right) - \left(u_2 - T_o \cdot s_2 + \frac{V_2^2}{2} + g \cdot z_2 \right) \qquad (8.8)$$

steady-state, steady flow process

$$m_2 \cdot \left(u_2 - T_o \cdot s_2 + \frac{V_2^2}{2} + g \cdot z_2 \right) - m_1 \cdot \left(u_1 - T_o \cdot s_1 + \frac{V_1^2}{2} + g \cdot z_1 \right) = 0$$

steady-state, steady flow process - rate form

$$W\_dot_{rev} = \sum_n \left[m_{i_n} \cdot \left(h_i - T_o \cdot s_i + \frac{V_i^2}{2} + g \cdot z_i \right) \right] - \left[\sum_n \left[m_{e_n} \cdot \left(h_e - T_o \cdot s_e + \frac{V_e^2}{2} + g \cdot z_e \right) \right] \right] \qquad (8.9)$$

single flow of fluid

$$\frac{W\_dot_{rev}}{m\_dot} = w_{rev} = h_i - T_o \cdot s_i + \frac{V_i^2}{2} + g \cdot z_i - \left(h_e - T_o \cdot s_e + \frac{V_e^2}{2} + g \cdot z_e \right) \qquad (8.10)$$

above represents maximum work for given change of state of a system
what is maximum work that can be done by system in a given state???

answer: when system is in equilibrium with the environment, no spontaneous change of state can occur, and is incapable of doing work. therefore if system in a given state undergoes a completely reversible process until it is in equilibrium with the environment, the maximum reversible work will have been done by the system

steady state, steady flow process ...(e.g. single flow)

$$w_{rev} = h_i - T_o \cdot s_i + \frac{V_i^2}{2} + g \cdot z_i - \left(h_e - T_o \cdot s_e + \frac{V_e^2}{2} + g \cdot z_e \right) \qquad (8.10)$$

maximum when mass leaving c.v. is in equilibrium with environment. define ψ = availability (per unit mass flow)

steady state, steady flow process ...(e.g. single flow ...availability (per unit mass flow)

$$\psi = h - T_o \cdot s + \frac{V^2}{2} + g \cdot z - \left(h_o - T_o \cdot s_o + \frac{V_o^2}{2} + g \cdot z_o \right) \qquad (8.16)$$

reversible work between any two states = decrease in availablity between them

$$w_{rev} = \psi_i - \psi_e = h_1 - T_o \cdot s_1 - h_2 + T_o \cdot s_2 = h_1 - T_o \cdot s_1 - h_2 + T_o \cdot s_2 = \left(h_1 - h_2 \right) - T_o \cdot \left(s_1 - s_2 \right) \qquad (8.17) \text{ extended}$$

▸ extension

can be written for more than one flow ...

$$W-dot_{rev} = \sum_n \left(m_{i_n}\cdot\psi_{i_n}\right) - \sum_n \left(m_{e_n}\cdot\psi_{e_n}\right) \qquad (8.18)$$

for a system (no flow across the control surface)

. need to account for work done by system against the surroundings ...
... assume kinetic and potential energy changes negligible ...

$$\frac{W_{rev\_1\_2}}{m} = w_{rev\_1\_2} = \left(u_1 - T_o\cdot s_1 + \frac{V_1^{\,2}}{2} + g\cdot z_1\right) - \left(u_2 - T_o\cdot s_2 + \frac{V_2^{\,2}}{2} + g\cdot z_2\right) \qquad (8.8)$$

becomes ... $w_{rev\_1\_2} = \left(u_1 - T_o\cdot s_1\right) - \left(u_2 - T_o\cdot s_2\right)$

$$w_{rev\_max} = \left(u - T_o\cdot s\right) - \left(u_o - T_o\cdot s_o\right) \qquad (8.19)$$

availability per unit mass is then ... this maximum work - that done against the surroundings

$$W_{surr} = p_o\cdot\left(V_o - V\right) = m\cdot p_o\cdot\left(v_o - v\right) \qquad (8.20)$$

$$\phi = availability\_w\_o\_KE\_PE = w_{rev\_max} - w_{surr} = \left(u - T_o\cdot s\right) - \left(u_o - T_o\cdot s_o\right) + p_o\cdot\left(v - v_o\right)$$

availability w/o KE and PE per unit mass of system

$$\phi = \left(u + p_o\cdot v - T_o\cdot s\right) - \left(u_o + p_o\cdot v_o - T_o\cdot s_o\right) = u - u_o + p_o\cdot\left(v - v_o\right) - T_o\cdot\left(s - s_o\right) \qquad (8.21)$$

and reversible work maximum between states 1 and 2 is ...

$$w_{rev\_1\_2} = \phi_1 - \phi_2 - p_o\cdot\left(v_1 - v_2\right) + \frac{V_1^{\,2} - V_2^{\,2}}{2} + g\cdot\left(z_1 - z_2\right) \qquad (8.22)$$

check ...

$$\phi_1 := u_1 - u_o + p_o\cdot\left(v_1 - v_o\right) - T_o\cdot\left(s_1 - s_o\right) \qquad \phi_2 := u_2 - u_o + p_o\cdot\left(v_2 - v_o\right) - T_o\cdot\left(s_2 - s_o\right)$$

$$w_{rev\_1\_2} := \phi_1 - \phi_2 - p_o\cdot\left(v_1 - v_2\right) + \frac{V_1^{\,2} - V_2^{\,2}}{2} + g\cdot\left(z_1 - z_2\right)$$

$$w_{rev\_1\_2}\ \text{simplify} \ \rightarrow u_1 - T_o\cdot s_1 - u_2 + T_o\cdot s_2 + \frac{1}{2}\cdot V_1^{\,2} - \frac{1}{2}\cdot V_2^{\,2} + g\cdot z_1 - g\cdot z_2$$

matches ...

$$\frac{W_{rev\_1\_2}}{m} = w_{rev\_1\_2} = \left(u_1 - T_o\cdot s_1 + \frac{V_1^{\,2}}{2} + g\cdot z_1\right) - \left(u_2 - T_o\cdot s_2 + \frac{V_2^{\,2}}{2} + g\cdot z_2\right) \qquad \text{(8.8) from above}$$

define some units ... $kPa := 10^3 Pa$ $kJ := 10^3 J$

example ... geothermal well

water as saturated liquid issues from a process at 200 deg C.
What is maximum power if the environment is at 10^5 N/m^2 at 30 deg C $T_{1\_C} := 200$

$$\psi = h - T_o \cdot s + \frac{V^2}{2} + g \cdot z - \left(h_o - T_o \cdot s_o + \frac{V_o{}^2}{2} + g \cdot z_o\right) \quad (8.16)$$

1 saturated $T_1 := (273 + T_{1\_C}) \cdot K$ $T_1 = 473\ K$ $p_1 := 1.5538 MPa$ $h_1 := 852.45 \frac{kJ}{kg}$ $s_1 := 2.3309 \frac{kJ}{kg \cdot K}$

environment (dead state) $p_0 := 10^5 \frac{N}{m^2}$ $T_{0\_C} := 30$ $T_0 := (273.16 + T_{0\_C}) \cdot K$ $T_0 = 303.16\ K$ $p_0 = 100\ kPa$

water at this state is "compressed liquid" as pressure exceeds saturation pressure at 30 deg C

ref: water saturated liquid at 30 deg C $p_{sat\_30} := 4.246 kPa$

$v_{f\_30} := 1.004 \times 10^{-3} \frac{m^3}{kg}$ $h_{f\_30} := 125.79 \frac{kJ}{kg}$ $s_{f\_30} := 0.437 \frac{kJ}{kg \cdot K}$

limited values for compressed liquid are in Table A.1.4 well beyond this pressure
water is ~ incompessible
values for u, v and s can be estimated to be the saturation values at the T so... (see example validation below)

$v_0 := v_{f\_30}$ $s_0 := s_{f\_30}$

but work must be done to compress to higher pressure than saturated
estimate from definition of enthalpy:

$h = u + p \cdot v$ $dh = du + p \cdot dv + v \cdot dp$ $\int_1^2 1\, dh = h_2 - h_1 = \int_1^2 1\, du + \int_1^2 p\, dv + \int_1^2 v\, dp$

we'll see later $\int_1^2 1\, du = c_v \cdot (T_2 - T_1)$ dv ~ 0 <<<1 => v = constant so ...

$$h_2 - h_1 = v \int_1 1\, dp = v_f(T) \cdot (p_2 - p_1)$$

$h_0 := h_{f\_30} + v_{f\_30} \cdot (p_0 - p_{sat\_30})$ $h_0 = 125.886 \frac{kJ}{kg}$

$\psi := h_1 - T_0 \cdot s_1 - (h_0 - T_0 \cdot s_0)$ $\psi = 152.409 \frac{kJ}{kg}$

to show example of estimates u, v, and s of compressed liquid = saturation, but not h consider value in Table vs stauration at T

$i := 1..4$

Table A.1.1.4
p=10MPa
T=40 deg C

Table A.1.1.1
saturation
t=40 deg C

difference
(col 2 - col 3)
----
col2

$$data := \begin{pmatrix} p & 10\cdot 10^{6} & 7.384\cdot 10^{3} \\ v & 0.0010034 & 0.001008 \\ u & 166.35 & 167.56 \\ h & 176.38 & 167.57 \\ s & 0.5686 & 0.5725 \end{pmatrix}$$

$$\frac{data_{i,1} - data_{i,2}}{data_{i,1}}\cdot 100 =$$

-0.5
-0.7
5
-0.7

differences all < 1% for 10^6 pressure difference
h differs by 5%

using estimate from definition of enthalpy

$$h := data_{3,2}\frac{kJ}{kg} + data_{1,2}\frac{m^3}{kg}\cdot\left(data_{0,1}\,Pa - data_{0,2}\,Pa\right) \qquad h = 177.643\,\frac{kJ}{kg}$$

and difference is ... $$\frac{data_{3,1}\frac{kJ}{kg} - h}{data_{3,1}\frac{kJ}{kg}}\cdot 100 = -0.716$$ difference now < 1%

geothermal well example