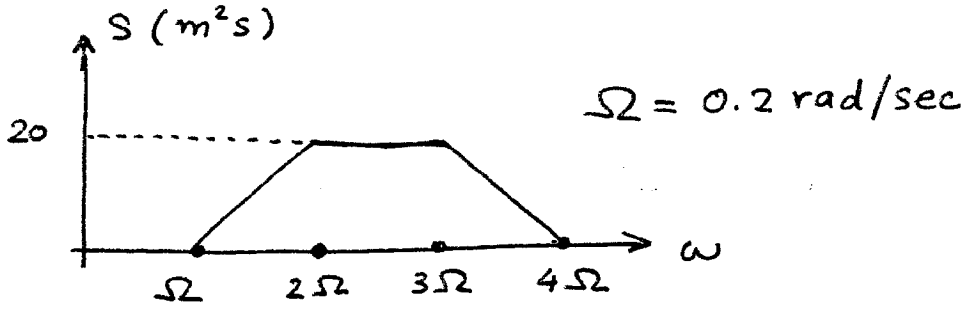
13.022

SURFACE WAVES AND THEIR INTERACTION WITH FLOATING BODIES

Quiz 1
Wednesday, October 6, 1999
1:30 Hours - Open Book

1. (50%) The wave spectrum of a unidirectional sea state has the trapezoidal form shown in the Figure



a) Determine the variance in meters of the wave elevation corresponding to the above spectrum.

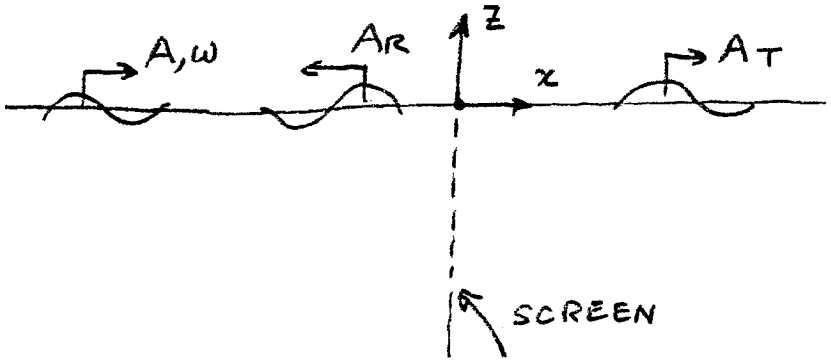
b) Derive an approximate expression for a wave record consisting of three (3) wave components, which conforms to the spectral density determined above. Explain the origin and definition of the parameters that enter this expression.

c) Use your result in b) to derive the corresponding expression for the unsteady linear pressure at z=-1m, assuming that z=0 is the calm water surface. What is the value of the variance of this pressure corresponding to the exact definition of the spectrum?

d) Use your result in b) to derive an expression for the energy flux in the positive x-direction due to this sea state.

e) The sea state described by the spectrum defined in the Figure, propagates in water of finite and uniform depth H. Determine the new form of the wave spectrum and draw an qualitative graph of it.

2. (50%) A deep-water monochromatic plane progressive wave with amplitude A and frequency ω is incident upon a vertical porous screen which dissipates 10% of the ambient energy flux. The resulting amplitudes of the reflected and transmitted waves are A_R and A_T respectively



a) Determine the modulus of A_T

b) Determine the modulus of A_R

c) Assuming that the three plane progressive waves with amplitudes A, A_R and A_T describe accurately the flow near the screen, determine the modulus and phase of the horizontal linear force exerted on the screen by the ambient waves. Assume that 10% of the cross sectional area of the screen is water and that the phases of A_R and A_T are known.

d) It is well known that according to linear theory, the trajectory of the particles under a plane progressive wave is circular. Estimate the trajectory of the particles upstream of the screen due under the incident and reflected waves with amplitudes A and A_R, respectively.

e) Assume water of finite depth H and with the amplitude and frequency of the incident wave equal to their deep water values. Are your answers about the modulus of A_T, A_R and the horizontal force on the screen going to be greater or smaller than their deep water values?