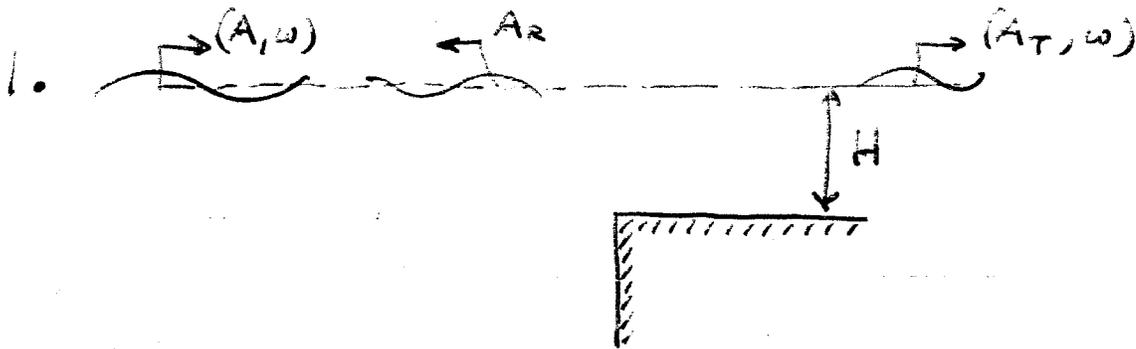13.022

SPRING 98

QUIZ 1

SOLUTIONS

1.

LET A_T BE THE COMPLEX TRANSMITTED WAVE AMPLITUDE
AND A_R THE REFLECTED AMPLITUDE

a) $P_{IN} = \frac{1}{2} \rho g A^2 V_g = \frac{1}{2} \rho g A^2 \frac{g}{2\omega} = \frac{1}{4\omega} \rho g^2 A^2 .—$

$P_{TRANSMITTED} = \frac{1}{2} P_{IN} = \frac{1}{2} \rho g |A_T|^2 V_g$

IN FINITE DEPTH : $V_g = \left(\frac{1}{2} + \frac{kH}{\sinh 2kH} \right) V_p$

$$V_p = \frac{\omega}{k}, \qquad \omega^2 = g k \tanh kH$$

$\Rightarrow |A_T|^2 = \frac{\frac{1}{2} P_{IN}}{\frac{1}{2} \rho g V_g} = \frac{\frac{1}{8\omega} \rho g^2 A^2}{\frac{1}{2} \rho g V_g} = \frac{1}{4\omega} g \frac{A^2}{V_g}$
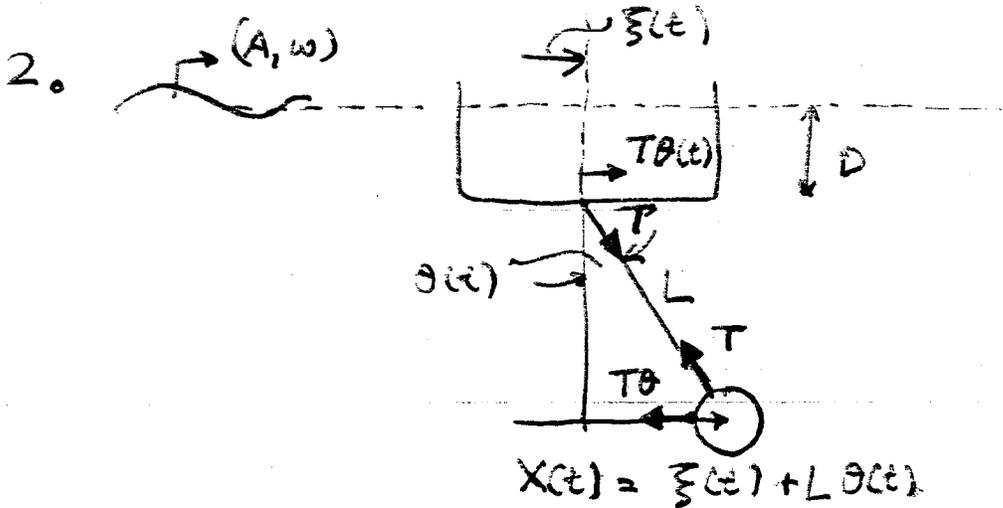
$$|A_T|^2 = \frac{1}{2} A^2 \frac{(V_g)_\infty}{(V_g)_H} \cdot -$$

b) ENERGY CONSERVATION:

$$\frac{1}{2} \rho g A^2 (V_g)_\infty = \frac{1}{2} \rho g |A_T|^2 (V_g)_H + \frac{1}{2} \rho g |A_R|^2 (V_g)_\infty$$

$$\Rightarrow |A_R|^2 = A^2 - |A_T|^2 \frac{(V_g)_H}{(V_g)_\infty} = \frac{1}{2} A^2$$

$$\Rightarrow |A_R| = \frac{1}{\sqrt{2}} A \cdot -$$

2.

$$X(t) = \xi(t) + L\theta(t)$$

TOTAL X-DISPLACEMENT OF BODY

$$X(t) = \xi(t) + L\theta(t) \cdot -$$

• SURGE EQUATION OF MOTION OF SHIP:

$$(M+A)\ddot{\xi} + B\dot{\xi} = X(t) + T\theta(t)$$

• SURGE EQUATION OF MOTION OF BUOY:

$$(m+a)\ddot{X} = f(t) - T\theta(t)$$

$f(t)$ = EXCITING FORCE ON BUOY AT $X=0$:

$$\bullet \quad f(t) = -\left(\frac{a}{\rho} + \forall\right)\frac{\partial P_{\pm}}{\partial x}\bigg|_{x=0} = -2\pi a^2 \frac{\partial P_{\pm}}{\partial x}\bigg|_{z=-(D+L) \longrightarrow x=}$$

$$\bullet \quad a = \rho\forall = \pi\rho a^2$$

COUPLED SYSTEM:

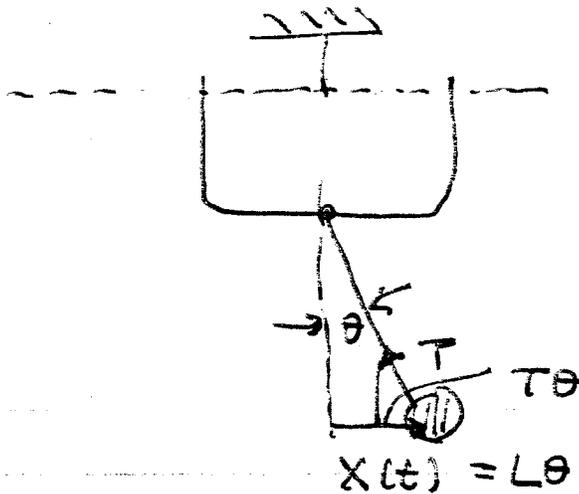$$\begin{cases} (M+A)\ddot{\xi} + B\dot{\xi} - T\theta(t) = X(t) \\ (m+a)\ddot{X} + T\theta(t) = f(t) \end{cases}$$

LET: $\quad \xi = Re\{\Xi\, e^{i\omega t}\}$

$$\theta = Re\{\Theta\, e^{i\omega t}\}. —$$

$$\begin{cases} -\omega^2(M+A)\,\Xi + i\omega B\,\Xi - T\Theta = X \\ -\omega^2(m+a)\;(\Xi + L\Theta) + T\Theta = F \end{cases} \qquad (A)$$

$$F = -2\pi\,a^2\,\frac{\partial}{\partial x}\left\{\frac{i\,\delta A}{\omega}\,e^{kz - ik x + i\omega t}\,(-\rho i\omega)\right\}_{\substack{x=0 \\ z=-(D+L)}}$$

$$= -2\pi\,a^2\left\{(-\rho i\omega)\frac{i\delta A}{\omega}\;(-ik)\,e^{-k(D+L)}\right\}$$

$$= 2\pi\rho a^2 i\,\omega^2 A\,e^{-k(D+L)}$$

● RESONANT PERIOD OF A PENDULUM

$$X(t) = L\theta$$

$$(m+a)\,\ddot{X} + T\theta = 0 \;\Rightarrow\; (m+a)L\ddot{\theta} + T\theta = 0$$

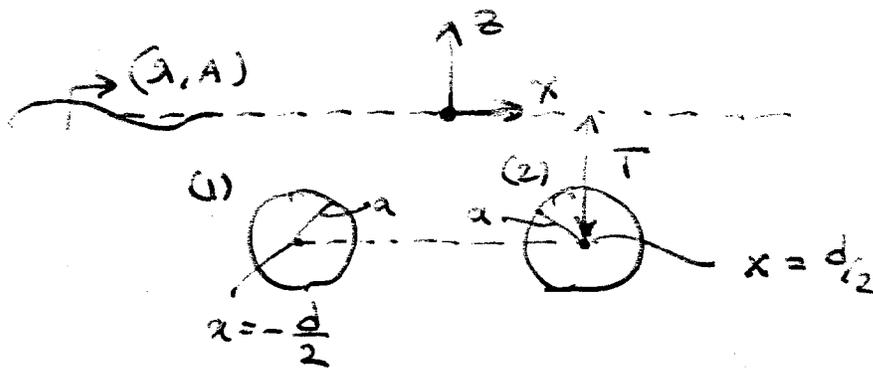$$\Rightarrow\; \omega^2 = \frac{T}{L(m+a)} \;\circ\!-$$

● AT RESONANCE EQUATIONS (A) BECOME:

$$-\omega^2(M+A)\,\Xi + i\omega B\,\Xi - T\Theta = X$$

$$-\omega^2(m+a)\,\Xi + \underbrace{[-\omega^2(m+a)L + T]}_{\equiv 0}\,\Theta = F$$

$$\Rightarrow \quad \Xi = \frac{F}{\omega^2(m+a)} \equiv \text{COMPLEX SPHERE AMPLITUDE}$$

$$= \frac{2\pi\rho a^2\, iA\, e^{-k(D+L)}}{(m + \pi\rho a^2)} \cdot \underline{\quad}$$

$$= \frac{2i A\, e^{-k(D+L)}}{\left(\dfrac{m}{\pi\rho a^2} + 1\right)} \cdot \underline{\quad}$$

3.

USE G.I. TAYLOR FOR HEAVE AND SURGE EXCITING FORCES SINCE $\lambda \gg a$. ASSUME $\lambda \sim d$ & T. __

u) HEAVE :

$$\ddot{X} = -\left(\frac{a}{\rho} + \forall\right) \frac{\partial P_I}{\partial z}\bigg|_{\substack{z=-T \\ X = \pm d/2}}$$

$$\frac{a}{\rho} + \forall = 2\pi a^2$$

$$\frac{\partial P_I}{\partial z} = -\rho \, Re\left\{ \frac{isA}{\omega} k \, e^{-kT - ikX + i\omega t} \right\}$$

CYLINDER 1:

$$\ddot{X}_3^{(1)} = 2\pi \rho a^2 \, Re\left\{ \frac{isA}{\omega} k \, e^{-kT - ik\frac{d}{2} + i\omega t} \right\}$$

CYLINDER 2:

$$\ddot{X}_3^{(2)} = 2\pi \rho a^2 \, Re\left\{ \frac{isA}{\omega} k \, e^{-kT + ik\frac{d}{2} + i\omega t} \right\}. —$$

SURGE:

$$\ddot{X} = -\left(\frac{a}{\rho} + \forall\right) \frac{\partial P_I}{\partial x}\bigg|_{\substack{z=-T \\ X = \pm d/2}}$$

$$\frac{\partial P_I}{\partial x} = -\rho \, Re\left\{ \frac{isA}{\omega} e^{kz - ikX + i\omega t} (-ik) \right\}$$

CYLINDER 1: $z = -T, \quad X = -d/2$

CYLINDER 2: $z = -T \quad X = +d/2$

o) SWAY EXCITING FORCES IN PHASE:

$$ph\left(e^{-ik\frac{d}{2}}\right) = ph\left(e^{+ik\frac{d}{2}}\right)$$

OR $\quad e^{-ik\frac{d}{2}} = e^{ik\frac{d}{2}} \quad$ (MODULE EQUAL)

$$e^{2ik\frac{d}{2}} = 1 \Rightarrow e^{ikd} = 1$$

$$\Rightarrow kd = \left(0, 2\pi, 4\pi, \dots\right) = \frac{2\pi d}{\lambda}$$

$$\Rightarrow \frac{d}{\lambda} = \left(0, 1, 2, \dots\right) = n, \quad n = 0, 1, 2, \dots$$

SWAY EXCITING FORCES OUT OF PHASE

$$e^{-ik\frac{d}{2}} = - e^{ik\frac{d}{2}}$$

$$\Rightarrow e^{ikd} = -1 \Rightarrow kd = \left(\pi, 3\pi, 5\pi, \dots\right)$$

$$\frac{2\pi d}{\lambda} = \left(\pi, 3\pi, 5\pi, \dots\right)$$

$$\Rightarrow \frac{d}{\lambda} = \frac{1}{2}\left(1, 3, 5, \dots\right) = \frac{1}{2}(2n+1), \quad n = 0, 1, 2, \dots$$