13.42 Design Principles for Ocean Vehicles
Homework #3 – Basic probabilities

Out: Thursday, 19 February 2004
Due: Thursday, 26 February 2004

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1. Find the probability of drawing a five-card hand in which every card is a heart. (Assume that every possible five-card hand drawn from a standard deck of 52 cards has the same probability of being selected.)

2. Given a standard deck of 52 cards, let

   A = {x | x is an ace}
   B = {x | x is a heart}
   C = {x | x is a club, diamond, or spade}
   D = {x | x is black}

   Find the following probabilities:

   a. p($A \cup B$)
   b. p($B \cap D$)
   c. p($A \cap C$)
   d. p($B \cup D$)
   e. p(C$\cup D$)

3. Let the random variable X be the sum of the outcomes of two dice that are rolled simultaneously.

   a. Find the probability density function – *i.e.*, $\wp(x) = p(X = x)$.
   b. Find the cumulative distribution function – *i.e.*, $P(x) = p(X \le x)$.
   c. Using the probability function, find the mean, variance, and standard deviation.

4. Let the random variable x have a cumulative distribution function $P(x)$ defined by

$$P(x) = \begin{cases} 0, & x < 0 \\ x^2, & 0 \le x < \frac{1}{2} \\ \frac{1}{2}x, & \frac{1}{2} \le x < 2 \\ 1, & 2 \le x \end{cases}$$

   find the following probabilities:

   a. p($1 \le X < 1.5$)
   b. p($X \le$ ¼)
   c. p($X \ge 4$)

5. Let X be a random variable with the following probability density function:

$$\wp(x) = \begin{cases} 2x, & 0 \le x < 1 \\ 0, & \text{otherwise} \end{cases}$$

   a. Find the expected value, μ_X
   b. Find the standard deviation, σ_X