# 13.42 Design Principles for Ocean Vehicles

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## Froude Krylov Excitation Force

### 1. Radiation and Diffraction Potentials

The total potential is a linear superposition of the incident, diffraction, and radiation potentials,

$$\phi = (\phi_I + \phi_D + \phi_R)\, e^{i\omega t}. \tag{1}$$

The **radiation** potential is comprised of six components due to the motions in the six directions, $\phi_j$ where $j = 1,2,3,4,5,6$. Each function $\phi_j$ is the potential resulting from a unit motion in $j^{th}$ direction for a body floating in a quiescent fluid. The resulting body boundary condition follows from lecture 15:

$$\frac{\partial \phi_j}{\partial n} = i\omega n_j; \quad (j = 1,2,3) \tag{2}$$

$$\frac{\partial \phi_j}{\partial n} = i\omega(\vec{r} \times \hat{n})_{j-3}; \quad (j = 4,5,6) \tag{3}$$

$$\vec{r} = (x, y, z) \tag{4}$$

$$\hat{n} = n_j (j = 1,2,3) = (n_x, n_y, n_z) \tag{5}$$

In order to meet all the boundary conditions we must have waves that radiate away from the body. Thus $\phi_j \propto e^{\mp ikx}$ as $x \to \pm\infty$.

For the **diffraction** problem we know that the derivative of the total potential (here the incident potential plus the diffraction potential without consideration of the radiation potential) normal to the body surface is zero on the body: $\frac{\partial \phi_T}{\partial n} = 0$ on $S_B$, where $\phi_T = \phi_I + \phi_D$.

$$\frac{\partial \phi_I}{\partial n} = -\frac{\partial \phi_D}{\partial n}; \, on \; S_B \tag{6}$$

We have so far talked primarily about the incident potential. The formulation of the incident potential is straight forward from the boundary value problem (BVP) setup in lecture 15. There exist several viable forms of this potential function each are essentially a phase shifted version of another. The diffraction potential can also be found in the same fashion using the BVP for the diffraction potential with the appropriate boundary condition on the body. This potential can be approximated for a *long wave* condition. This long wave approximation assumes that the incident wavelength is very long compared to the body diameter and thus the induced velocity field from the incident waves on the structure can be assumed constant over the body and approximated by the following equation:

$$\phi_D \simeq \frac{i}{\omega}\left[\frac{\partial \phi_I}{\partial x}\phi_1 + \frac{\partial \phi_I}{\partial y}\phi 2 + \frac{\partial \phi_I}{\partial z}\phi_3\right] \tag{7}$$

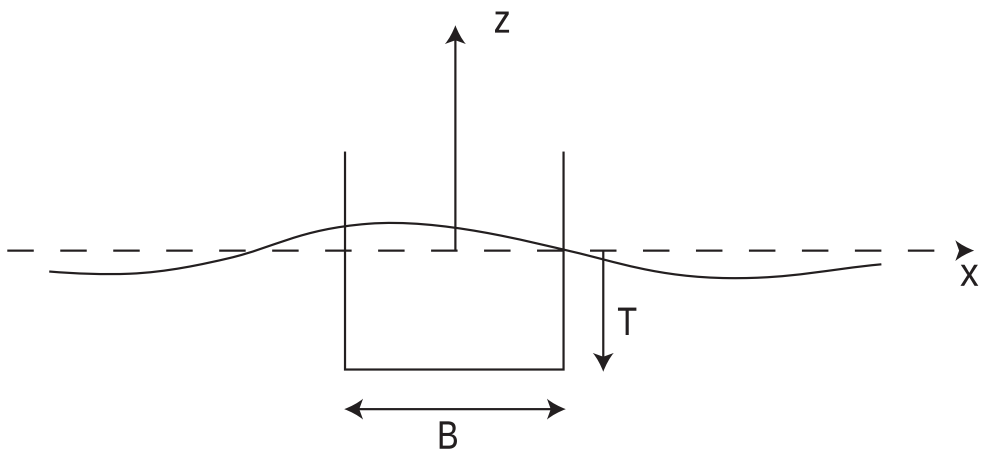
Further explanation of this approximation can be found in Newman (p. 301).

Ultimately, if we assume the body to be sufficiently small as not to affect the pressure field due to an incident wave, then we can diffraction effects can be completely ignored. This assumption comes from the *Froude-Krylov hypothesis* and assures a resulting excitation force equivalent to the froude-krylov force:

$$F^{FK}(t) = -\rho \int\int \frac{\partial \phi_I}{\partial t}\, n \, dS \tag{8}$$

# 2. Vertical Froude-Krylov Force on a Single Hull Vessel




Deep water incident wave potential is:

$$\phi_I = \frac{a\omega}{k} e^{kz} \quad Re\left\{ i\, e^{i(\omega t - kx)} \right\} \tag{9}$$

The force in the vertical direction is found from the incident potential using eq. 8 along the bottom of the vessel. Here the normal in the z-direction, $n_z$, is negative: $n_z = -1$, so the force per unit length in the z-direction is

$$F_z^{FK} = Re\left\{ \int_{-B/2}^{B/2} -\rho\, i\, \omega \frac{ia\omega}{k} e^{-kT} \; e^{i(\omega t - kx)}\, dx \right\} \tag{10}$$

$$= Re\left\{ \frac{\rho o \omega^2}{k^2} a\, e^{-kT} \; e^{i\omega t} \left( e^{-ikB/2} - e^{ikB/2} \right) \right\} \tag{11}$$

$$= Re\left\{ 2\rho \frac{\omega^2}{k^2} a\, e^{-kT} \; e^{i\omega t} \;\; \sin(kB/2) \right\} \tag{12}$$

Recall that $sin\ z \ = \ \frac{e^{iz} - e^{-iz}}{2i}$.

Using the vertical velocity we can rewrite the force in terms of the velocity.

$$w(t) = Re\left\{ a\omega\, e^{kz}\; i\, e^{i(\omega t - kx)} \right\} \tag{13}$$

$$\dot{w}(t) = Re\left\{ -a\omega^2\; e^{kz}\; e^{i(\omega t - kx)} \right\} \tag{14}$$

$$\dot{w}(x = 0, z = 0, t) = Re\left\{ a\; \omega^2\; e^{i\omega t} \right\} \tag{15}$$

Now we can write the force in the vertical direction as a function of the vertical (heave) acceleration,

$$F_z = Re\left\{ \frac{2\rho}{k^2} e^{-kT} \; sin(kB/2) \; \dot{w}(0,0,t) \right\}. \tag{16}$$

Let's look at the case where $\omega \to 0$ the wavenumber, $k = \omega^2/g \to 0$, also goes to zero and the following simplifications can be made:

$$e^{kt} \simeq 1 - kT \tag{17}$$

$$\sin(kB/2) \simeq kB/2 \tag{18}$$

to yield a simplified heave force.

$$F_z^{FK} \simeq Re\left\{ 2\rho \frac{\omega^2}{k^2} \, a \, (1-kT) \, (kB/2) \, e^{i\omega t} \right\} \tag{19}$$

$$\simeq Re\left\{ \rho \; g \; aB \left( 1 - \frac{\omega^2}{g} T \right) e^{i\omega t} \right\} \tag{20}$$

If we look at the case where $\omega \to 0$ and consider the heave restoring coefficient, $C_{33} = \rho \; g \; B$, and the free surface elevation, $\eta(x,t) = Re\left\{ a \; e^{i(\omega t - kx)} \right\}$ we can rewrite this force as

$$F_z^{FK} \simeq Re\left\{ C_{33} \; \eta(x=0,t) \right\} \tag{21}$$

# 3. Horizontal Froude-Krylov Force on a Single Hull Vessel

The horizontal force on the vessel above can be found in a similar fashion to the vertical force.

$$F_x = \iint_{S_B} \rho \frac{\partial \phi_I}{\partial t} n_x \; dS \tag{22}$$

$$= Re\left\{ \rho \; i \; \omega \frac{i\omega a}{k} \int_{-T}^{0} e^{kz} \; dz \left[ e^{i(\omega t - kB/2)} - e^{i(\omega t + kB/2)} \right] \right\} \tag{23}$$

$$= Re\left\{ i\rho \frac{a\omega^2}{k} \left( 1 - e^{-kT} \right) e^{i\omega t} \; 2\sin(kB/2) \right\} \tag{24}$$

As frequency approaches zero similar simplifications can be made like above for the vertical force:

$$F_x(t) \simeq Re\left\{ i\rho \frac{a\omega^2}{k} (KT)\, e^{i\omega t}\ 2\, kB/2 \right\} \tag{25}$$

$$u(t) = Re\left\{ a\omega\, e^{kz}\ e^{i(\omega t - kx)} \right\} \tag{26}$$

$$\dot{u}(t) = Re\left\{ i\ a\ \omega^2\ e^{kz}\ e^{i(\omega t - kx)} \right\} \tag{27}$$

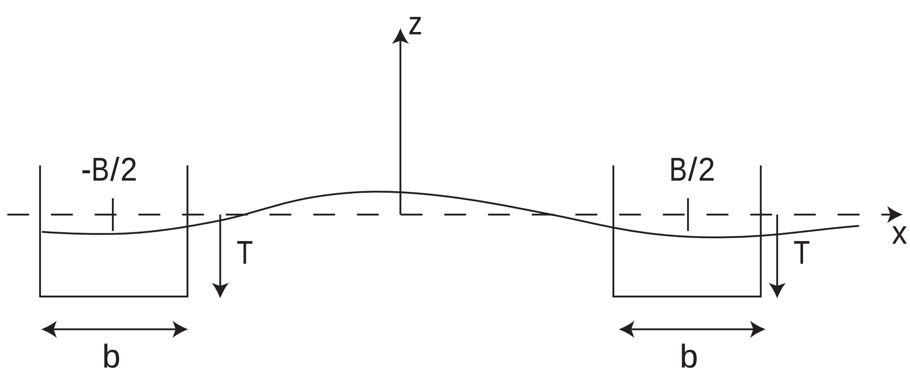
$$F_x(t) \simeq Re\left\{ \rho TB\, \dot{u}(x=0, z=0, t) \right\} \tag{28}$$

Where $\rho TB = \rho\forall$ , and $\forall$ is the vessel volume such that we are left with the surge force

$$F_x \simeq \rho\forall\, \dot{u} \tag{29}$$

$$F_z \simeq C_{33}\eta + \rho\forall\dot{w} \tag{30}$$

# 4. Multi Hulled Vessel




Again, let's make a few basic assumptions: ( $b/\lambda << 1$ ), ( $B/\lambda \sim 1$ ), ( $a < b$ ), and ( $b \sim T$ ).

Let's look at the force in the x-direction:

$$F_x^{FK} \simeq \rho\ bT\ \dot{u}(x=-B/2, z=0, t) + \rho\ bT\ \dot{u}(x=B/2, z=0, t) \tag{31}$$

$$\eta(x,t) = a\cos(\omega t - kx) \tag{32}$$

$$\dot{u}(x,z,t) = -a\ \omega^2\ e^{kz}\ \sin(\omega t - kx) \tag{33}$$

(34)

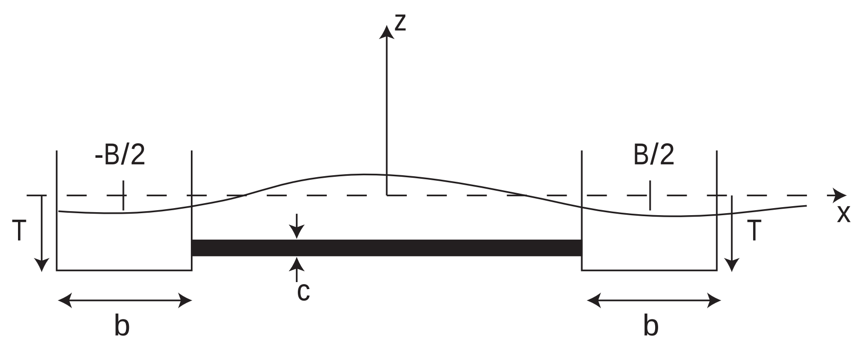
 

$$F_x^{FK} \simeq \rho bT\,(-a\,\omega^2)\left\{\sin(\omega t + kB/2) + \sin(\omega t - kB/2)\right\} \quad (35)$$

$$\simeq -2\rho\, b\, T\,(a\,\omega^2)\cos(kB/2)\sin(\omega t) \quad (36)$$

Note that when $kB/2 = \pi/2$ (or $B = \lambda/2$) then $F_x^{FK}(t) = 0$.

## *4.1. Multi Hulled Vessel with additional pontoon*




Use the same assumptions from above to find the x-force adjusted for the additional pontoon between the two hulls.

$$\begin{aligned} F_x^{FK} \simeq\;& -2\,\rho\, b\, T\,(a\,\omega^2)\cos(kB/2)\sin(\omega t) \\ & +c\; p(x=-B/2+b/2, z=0, t) \\ & -c\; p(x=B/2-b/2, z=0, t) \end{aligned}$$

The last two terms are the adjustment to the force for the addition of the pontoon, $\delta F_x^{FK}(t)$.

Pressure is found from the incident potential: $p(x,z,t) = \rho g\, a\, e^{kz}\cos(\omega t - kx)$.

$$\delta F_x^{FK} = -2\,\rho\, g\, a\sin(\omega t)\sin\left(\frac{k}{2}(B-b)\right) \quad (37)$$

For $B >> b$ using $g = \omega^2/k$ we get a force:

$$F_x^{FK}(t) \simeq -2\,\rho\, a\,\omega^2\sin(\omega t)\left\{bT\cos(kB/2) + \delta/2\;\sin(kB/2)\right\} \quad (38)$$