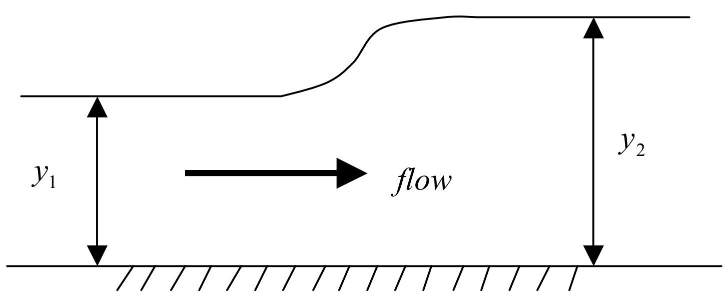
2.20 Problem Set 4B **Name: ____________________**

1. A hydraulic jump occurs in an open channel as shown. Consider the flow to be 2D (no flow in the *z*-direction). We can see that there are two relevant length scales for the flow, y_1 and y_2. The other physical variables are the two-dimensional flow rate q (has dimensions L^2/T), the density ρ, the dynamic viscosity μ, and the acceleration due to gravity *g*. Using dimensional analysis, find the non-dimensional functional relationship between these variables in the form

$$y_2 / y_1 = f(\Pi_a, \Pi_b)$$

where Π_a and Π_b are two dimensionless parameters you must find.



2. Consider a body of water that has a free surface (like a lake). Define the surface of the still water to be $z = 0$, with *z* positive upward. Recall that in the Navier-Stokes equations we can combine the hydrostatic pressure term $p_s = -\rho g z$ with the total pressure *p* by defining the dynamic pressure $p_d = p - p_s$:

(1) $$\frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \vec{\nabla} \vec{V} = -\frac{1}{\rho} \vec{\nabla} p_d + \nu \nabla^2 \vec{V}$$

(a) Suppose you are studying a submarine moving at a steady speed on the surface of the lake, and that the submarine has length *L* and speed *U*. Choose the characteristic pressure to be ρU^2. Write each dimensionless variable x^*, y^*, z^*, $\vec{V}^*$, and p^* in terms of the corresponding dimensional variable and the characteristic length, velocity, and pressure.

(b) Re-write the Navier-Stokes equations (1) for the steady flow case in dimensionless form using the dimensionless variables, noting that $\vec{\nabla}^* = L\vec{\nabla}$. Put your answer in the form $(\vec{V}^* \cdot \vec{\nabla}^*)\vec{V}^* = \ldots$.

From this form of the N-S equations, we see that one similarity parameter is _________. This is the inverse of what is commonly called the __________________ number.

(c) Write the dimensionless form p_d^* of the dynamic pressure $p_d = p - p_s = p + \rho g z$ in terms of p^* and z^*.

(d) Returning to the submarine cruising on the surface of the lake, we know that to solve for the flow around the submarine we have a dynamic boundary condition on the free surface, which in dimensional terms is: $p = p_{atmosphere}$ *on the free surface*. Using the expression for p_d^* that you obtained in (c), write this boundary condition in dimensionless terms, i.e., $p^* = p^*_{atmosphere} = p_d^* + p_s^* = ?$ *on the free surface*. Your answer should be in terms of p^*, $p^*_{atmosphere}$, p_d^*, and z^*.

From this form of the dynamic boundary condition, we see that another similarity parameter for the flow is _________. This is a form of what is commonly called the ________________ number.

IMPORTANT POINT: *To have similitude between a prototype and a model, it is necessary not only for any similarity parameters in the governing equations to be the same, but also for any similarity parameters in the dimensionless boundary conditions to be the same!*

3. A model of a submarine 1/16th the size of the prototype (real) submarine is built. We want to determine various characteristics of the prototype submarine when it is traveling at 20 knots on the surface (where we decide to neglect viscous effects) and at 20 knots far below the surface. Both the model and the prototype operate in water.

(a) At what speed would we tow the model on the surface?

(b) At what speed would we tow the model below the surface?

(c) If the towing speed in part (b) is not practical for our tow tank, what two parameters could we possibly change to make it more practical while maintaining similitude, assuming that the size and speed of the prototype does not change and that we do not have use water for the model test? In which direction would each of the parameters have to change (increase or decrease)?

(d) Suppose that for the 1/16th scale model we decide that we cannot ignore viscous effects after all when the submarine is on the surface. If we want similarity for both gravity (wave) effects *and* viscous effects, what kinematic viscosity (m^2 / s) of the model fluid would be required?