13 ELECTRIC MOTORS

Modern underwater vehicles and surface vessels are making increased use of electrical actuators, for all range of tasks including weaponry, control surfaces, and main propulsion. This section gives a very brief introduction to the most prevalent electrical actuators: The DC motor, the AC induction motor, and the AC synchronous motor. For the latter two technologies, we consider the case of three-phase power, which is generally preferred over single-phase because of much higher power density; three-phase motors also have simpler starting conditions. AC motors are generally simpler in construction and more robust than DC motors, but at the cost of increased control complexity.

This section provides working parametric models of these three motor types. As with gas turbines and diesel engines, the dynamic response of the actuator is quite fast compared to that of the system being controlled, say a submarine or surface vessel. Thus, we concentrate on portraying the *quasi-static* properties of the actuator – in particular, the torque/speed characteristics as a function of the control settings and electrical power applied.

The discussion below on these various motors is generally invertible (at least for DC and AC synchronous devices) to cover both motors (electrical power in, mechanical power out) and generators (mechanical power in, electrical power out). We will only cover motors in this section, however; a thorough treatment of generators can be found in the references listed. The book by Bradley (19??) has been drawn on heavily in the following.

13.1 Basic Relations

13.1.1 Concepts

First we need the notion of a magnetic flux, analagous to an electrical current, denoted Φ; a common unit is the Weber or Volt-second. The flux density

$$B = \Phi / A \tag{167}$$

is simply the flux per unit area, given in Teslas such that $1T = 1W/m^2$. Correponding to electrical field (Volts/m) is the magnetic field intensity H, in Amperes/meter:

$$H = \frac{B}{\mu_o \mu_r} = \frac{\Phi}{A \mu_o \mu_r}; \tag{168}$$

$\mu_o \approx 4\pi \times 10^-7 Henries/meter$ is the permeability of free space, and μ_r is a (dimensionless) relative permeability. The product $\mu_o\mu_r$ represents the real permeability of the material, and is thus the analog of electrical conductivity. A small area A or low relative permeability drives up the field intensity for a given flux Φ.

13.1.2 Faraday's Law

The voltage generated in a conductor experiencing a time rate of change in magnetic flux is given as

$$e = -\frac{d\Phi}{dt} \tag{169}$$

This voltage is commonly called the back-electromotive force or back-e.m.f., since it typically opposes the driving current; it is in fact a limiting factor in DC motors.

13.1.3 Ampere's Law

Current passing through a conductor in a closed loop generates a perpendicular magnetic field intensity given by

$$I = \oint H dl. \tag{170}$$

An important point is that N circular wraps of the same conductor carrying current I induce the field $H = \pi DNI$, where D is the diameter of the circle.

13.1.4 Force

Forces are generated from the orthogonal components of magnetic flux density B and current I:

$$F = I \times B. \tag{171}$$

The units of this force is N/m, and so represents a distributed force on the conductor.

13.2 DC Motors

The DC motor in its simplest form can be described by three relations:

$$\begin{aligned} e_a &= K\Phi\omega \\ V &= e_a + R_a i_a \\ T &= K\Phi i_a, \end{aligned}$$

where

- K is a constant of the motor
- Φ is the airgap magnetic flux per pole (Webers)
- ω is the rotational speed of the motor (rad/s)
- e_a is the back-e.m.f.
- V is the applied voltage
- R_a is the armature resistance on the rotor
- i_a is the current delivered to the armature on the rotor
- T is developed torque

The magnetic field in a typical motor is stationary (on the stator) and is created by permanent magnets or by coils, i.e., Faraday's law. Current is applied to the rotor armature through slip rings, and thus the force on each conductor in the armature is given by $\vec{F} = \vec{i_a} \times \vec{B}$. Back-e.m.f. is created because the conductors in the rotor rotate through the stationary field, causing a relative rate of change of flux. The armature voltage loop contains the back-e.m.f. plus the resistive losses in the windings. As expected, torque scales with the product of magnetic flux and current.
There are three main varieties of DC motors, all of which make use of the relations above. Speed control of the DC motor is primarily through the voltage V, either directly or through pulse-width modulation, but the stator flux could also be controlled in the shunt/independent configurations.

13.2.1 Permanent Field Magnets

Here, the magnetic field is created by permanent magnets arranged in the stator, imposing a steady Φ. The product $K\Phi$ is generally written as k_t, the torque constant of a DC motor, and has units of Nm/A. When SI units are used, k_t also describes back-e.m.f.. The three basic relations are thus rewritten

$$\begin{aligned} e_a &= k_t \omega \\ V &= e_a + R_a i_a \\ T &= k_t i_a, \end{aligned}$$

which leads via substitution to

$$\begin{aligned} \omega &= \frac{1}{k_t}\left[V - \frac{R_a T}{k_t}\right], \text{ or} \\ T &= \frac{k_t}{R_a}\left[V - k_t \omega\right]. \end{aligned}$$

This result indicates that the torque developed scales linearly with the applied voltage, but that it also scales negatively with the motor speed. Hence, at the speed $\omega = V/K_t$, no torque is created. Additionally, the maximum torque is created at zero speed.
Control of these motors is through the voltage V, or, more commonly, directly through current i_a, which gives torque directly.

13.2.2 Shunt or Independent Field Windings

The field created by the stator can be strengthened by replacing the permanent magnets with electromagnets. The field windings are commonly placed in series with the rotor circuit, in parallel with it (shunt connection), or, they may be powered from a completely separate circuit. The latter two cases are effectively equivalent, in the sense that current and hence the field strength can be modulated easily, through a variable resistance in the shunt case. We have

$$\omega = \frac{1}{K\Phi}\left[V - \frac{R_a T}{K\Phi}\right],$$

with the important property that the second term in brackets is small due to the increased field strength, compared with the permanent magnet case above. Thus, the motor speed is effectively independent of torque, which makes these motor types ideal for self-regulation applications. At very high torques and currents, however, the total available flux will be reduced because of field armature reactance; the speed starts to degrade as shown in the figure.

13.2.3 Series Windings

When the field windings are arranged in series with the rotor circuit, the flux is

$$\Phi = K_s i_a,$$

where K_s is a constant of the field winding. This additional connection requires

$$V = e_a + (R_a + R_s)i_a;$$

the field winding brings a new resistance R_s into the voltage loop. It follows through the substitutions that

$$\begin{aligned} T &= K K_s i_a^2 \rightarrow \\ I_a &= \sqrt{\frac{T}{KK_s}} \\ \omega &= \frac{V}{\sqrt{KK_sT}} - \frac{R_a + R_s}{KK_s}. \end{aligned}$$

The effects of resistance are usually quite small, so that the first term dominates, leading to a nonlinear torque/speed characteristic. The starting torque from this kind of motor is exceptionally high, and the series field winding finds wide application in railway locomotives. At the same time, it should be observed that under light loading, the series motor may well self-destruct since there is no intrinsic upper limit to speed!

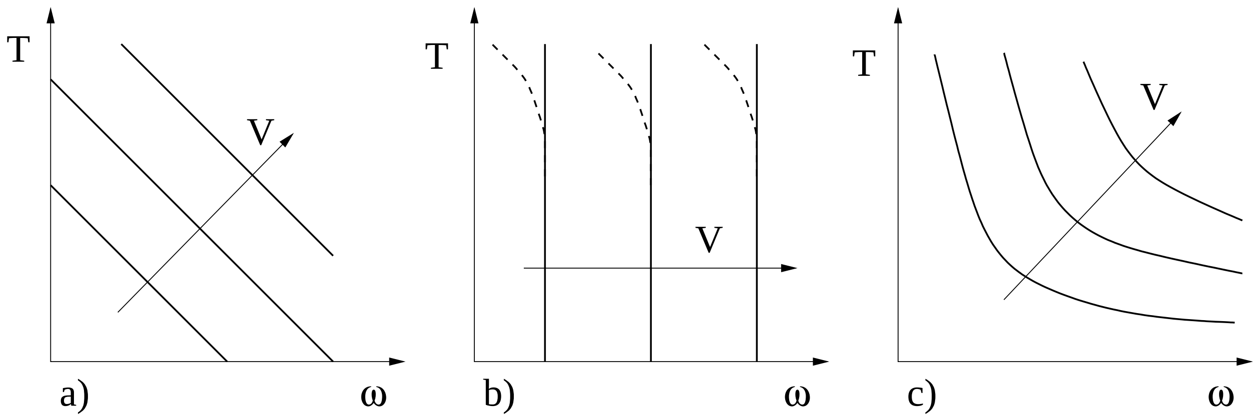

Figure 6: Torque-speed characteristics of three types of DC motors: a) permanent field magnets, b) shunt or independent field winding, c) series field winding.

Some variations on the series and shunt connections are common, and referred to as *compound* DC motors. These achieve other torque/speed curves, including increasing torque with increasing speed, which can offset the speed droop due to field armature reaction effects in the shunt motor.

13.3 Three-Phase Synchronous Motor

The rotor is either fitted with permanent magnets or supplied with DC current to create a static field on the rotor. The stator field windings are driven with three (balanced) phases of an AC supply, such that a moving field is created which rotates around the stator. The torque exerted on the rotor tries to align the two fields, and so the rotor follows the rotating stator field at the same speed. Note that if the rotor speed lags that of the stator field, there is no net torque; hence the name synchronous motor.
A simple model of the synchronous motor is straightforward. As with the DC motors, the voltage loop equation for a single phase on the stator gives

$$V = e_a + j i_a X,$$

where V, e_a, and i_a are now phasors (magnitude of V and e_a measured with respect to ground), $j = \sqrt{-1}$, and X is the reactance (armature and stator leakage) of the machine. Then, let ϕ denote the angle between i_a and V. Equating the electrical (all three phases) and mechanical power gives

$$3Vi_a \cos\phi = T\omega.$$

Next, let δ denote the angle between the phasors e_a and V. It follows from the voltage loop equation that

$$\begin{aligned} i_a \cos\phi X &= e_a \sin\delta \rightarrow \\ T &= \frac{p}{2\omega}\frac{3Ve_a \sin\delta}{X}, \text{ or} \\ T &= \frac{p}{2\omega}\frac{V_{ab}e_{a,ab}\sin\delta}{X} \end{aligned}$$

where

- p is the number of poles on the rotor; two poles means one north pole and one south pole, etc.
- ω is both the rotational speed of the rotor, and the rotational speed of the stator field
- V_{ab} is the line-to-line voltage, equal to $\sqrt{3}V$
- $e_{a,ab}$ is the line-to-line back-e.m.f., equal to $\sqrt{3}e_a$
- *delta* is the angle by which the rotor field lags the stator field (rad)
- X is the synchronous reactance

The torque scales with $\sin\delta$, and thus the rotor lags the stator field when the motor is powering; in a generator, the stator field lags the rotor. If the load torque exceeds the available torque, the synchronous motor can slip one or more poles, causing a large transient disturbance.

Speed control of the three-phase synchronous machine is generally through the frequency of the three-phase power supply, ω, with the assumption that adequate voltage and current are available.

13.4 Three-Phase Induction Motor

Like the synchronous machine, the induction machine has windings on the stator to create a rotating magnetic field at frequency ω. Letting the rotor speed be ω_r, we see immediately that if $\omega \neq \omega_r$, a potential field will be induced on any conductor on the rotor. In the case of a squirrel-cage rotor design, the rotor is made of conductor bars which are shorted out through rings at the ends, and hence the potential field will cause a real current flow. Torque is then generated through the familiar $F = I \times B$ relation. The fact of unequal field and rotor speeds in the induction motor is related to several unique effects, leading to torque-speed characteristic which differs significantly from both the DC the AC synchronous machines.

First we define the slip ratio

$$s = \frac{\omega - \omega_r}{\omega}; \tag{172}$$

a slip ratio of zero means $\omega = \omega_r$ (and hence zero torque because the magnetic field seen by the rotor is constant) while a slip ratio of one implies the rotor is stopped. Most induction motors are designed to operate at a small positive slip ratio, say 0.1-0.2, for reasons described below.

Next, since the magnetic flux lines pass through the rotor, the number of ampere-turns on stator and rotor is equivalent, that is, they form an ideal inductor:

$$N_r I_r = N_s I_s. \tag{173}$$

We consider per-unit quantities from here on, for which we set $N = N_r = N_s$ and hence $I_r = I_s$. If the stator flux at a particular location is $\Phi_s = \Phi_o \sin \omega t$, the associated voltage is $e_s = N d\Phi/dt = N\Phi_s^o \omega \cos \omega t$. On the rotor, the same flux applies, but it rotates more slowly: $\Phi_r = \Phi_o \sin \omega_r t = \Phi_o \sin s\omega t$. Hence the rotor voltage is $e_r = N d\Phi_r/dt = N\Phi_o s\omega \cos s\omega t$. Then the RMS voltage of the stator and rotor sides of the inductive coupling are related by

$$\frac{E_s}{E_r} = \frac{1}{s}. \tag{174}$$

The voltage seen at the stator scales inversely with the slip ratio, for a constant voltage at the rotor. In per-unit terms, the current in the rotor and stator are equivalent, and this then indicates that the rotor impedance, seen from the stator, also scales inversely with the slip ratio:

$$\begin{aligned} Z_{rs} &= \frac{1}{s}\left[R_r + jsX_r\right] \\ &= \frac{1}{s}R_r + jX_r. \end{aligned}$$

The factor of s in the rotor inductance occurs because the field seen by the rotor is actually rotating at $s\omega$.

Next, we construct the (one phase) Thevenin equivalent circuit of the stator: it has a voltage source V_t, and equivalent resistance R and inductance X. This is to be paired with the rotor resistance and inductance, *reflected to the stator*, giving the following current

$$I = \frac{V_t}{\sqrt{(R_r/s + R)^2 + (X_r + X)^2}}.$$

Finally, we need to express the torque/speed characteristic of the machine. The mechanical power is $P_m = (1 - s)\omega T$, while the power delivered across the airgap is $P_{gap} = I^2 R_r/s$. The actual power dissipated in the copper is related to the real rotor resistance: $P_{loss} = I^2 R_r$, and hence the mechanical power is $P_m = 3(P_{gap} - P_{loss}) = 3P_{gap}(1 - s)$. It follows that the efficiency of the motor is simply $\eta = 1 - s$. Combining the mechanical power with the torque equation gives

$$\begin{aligned} T &= \frac{P_m}{(1-s)\omega} = \frac{3P_{gap}}{\omega} \\ &= \frac{3V_t^2 R_r}{s\omega\left[(R_r/s + R)^2 + (X_r + X)^2\right]}. \end{aligned}$$

Maximum torque is developed at a slight slippage, with decreased values at lower speeds.

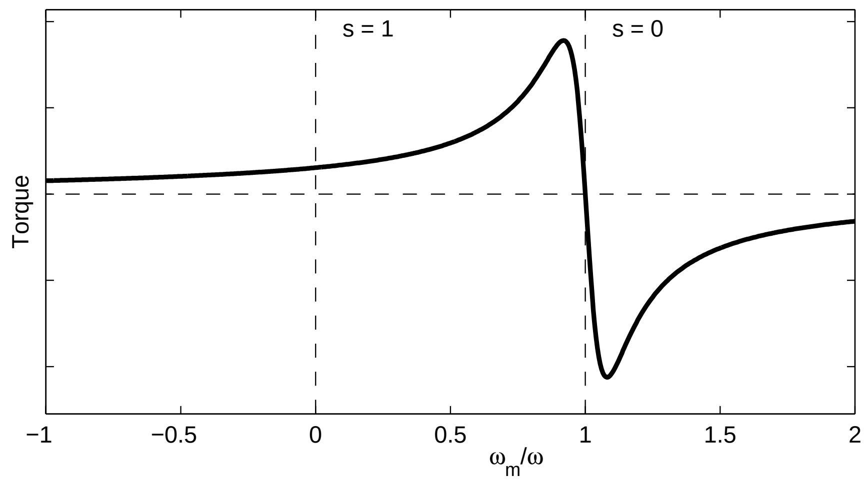

Figure 7: Torque-speed characteristics of a typical three-phase induction motor.