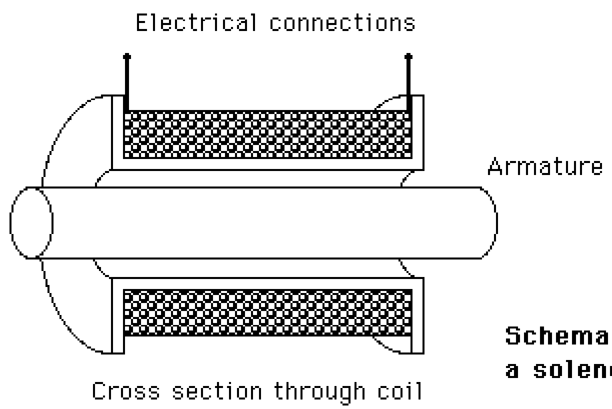
EXAMPLE: ELECTROMAGNETIC SOLENOID

A common electromechanical actuator for linear (translational) motion is a solenoid.



Schematic of a solenoid

Current in the coil sets up a magnetic field that tends to center the movable armature.

On the electrical side the device behaves like an inductor

— but the inductance depends on the position of the movable armature.

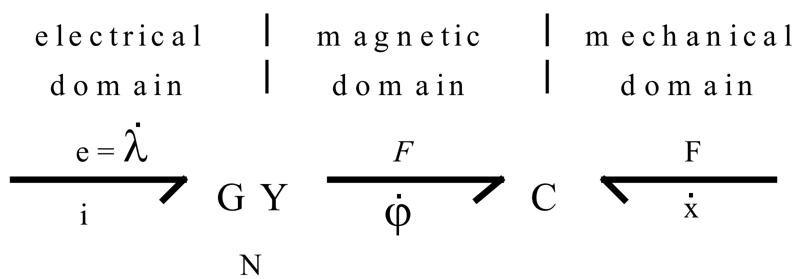
This "position-modulated inductor" is properly represented by a two-port energy-storage element with an electrical port and a mechanical port.

On the mechanical side, a force is required to displace the armature from its center position

— the device looks like a spring.

An inductor may be represented by a gyrator (coupling the electrical and magnetic domains) and a capacitor representing magnetic energy storage.

A bond graph for this model is as follows.



EQUIVALENT BEHAVIOR:

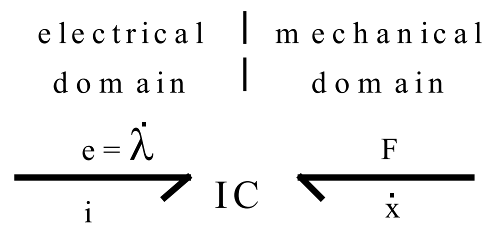
To find an equivalent model without the magnetic domain bring the magnetic behavior into the electrical domain.

a capacitor through a gyrator behaves like an inductor

the mechanical side still behaves like a spring.

This model can be represented by a (new) multiport element with "mixed" behavior

- **like an inertia (inductor) on one port**
- **like a capacitor (spring) on the other**



This element is often simply called a "multiport IC".

CONSTITUTIVE EQUATIONS

—two needed

$$i = i(\lambda, x)$$

$$F = F(\lambda, x)$$

Electrical constitutive equation—assume electrical linearity

$$i = \frac{\lambda}{L(x)}$$

where L(x) is a position-dependent inductance.

Mechanical constitutive equation—find the total stored energy.

$$E = \frac{\lambda 2}{2\, L(x)}$$

Force is the gradient of energy with respect to displacement.

$$F = \frac{\partial E}{\partial x} = -\frac{\lambda^2}{2} \frac{\partial L(x)/\partial x}{L(x)^2}$$

We need to know the function L(x) relating inductance to armature position.

With the armature centered, idealized coil inductance (neglecting fringing effects) is

$L = N^2 \mu_r \mu_o A/l$

where N is number of turns, μ_r is relative permeability, μ_o is the permeability of air or vacuum, A is coil cross sectional area and l is coil length.

With the armature removed—displaced an infinite distance—idealized coil inductance is

$L_\infty = N^2 \mu_o A/l$

In practice $\mu_r \gg \mu_o$ so $L \gg L_\infty$

We expect the inductance to be large with the armature centered and to decline smoothly to a small value as the armature is withdrawn to either side.

The precise form of L(x) may be determined in several ways

—by experiment

—using Finite-Element codes to compute the magnetic field for different armature positions.

An approximation:

For pedagogic simplicity we will use the following function.

$$L(x) = L\, e^{-(x/x_c)^2}$$

where x_c is a characteristic length of the armature

and it has been assumed that $L_\infty \approx 0$

CAUTION! This is *not* accurate!

It has no better justification than that

- **—it is analytically simple**
- **—it has approximately the right shape.**

Mechanical constitutive equation:

$$\frac{\partial L(x)}{\partial x} = -\frac{2x}{x_c^2} L e^{-(x/x_c)^2}$$

$$F = \frac{\lambda^2}{2} \frac{2x}{x_c^2} \frac{L e^{-(x/x_c)^2}}{\left(L e^{-(x/x_c)^2}\right)^2}$$

$$F = \frac{\lambda^2 x e^{(x/x_c)^2}}{L x_c^2}$$

This equation implies that force grows without bound as armature displacement increases.

DOES THIS MAKE SENSE PHYSICALLY?

Shouldn't the force should decline as the armature is removed?

CHECK FOR ERRORS:

Multiport stores energy, therefore should obey Maxwell's reciprocity.

Partial derivatives:

$$\frac{\partial i}{\partial x} = \frac{2\lambda x e^{(x/x_c)^2}}{L x_c^2}$$

$$\frac{\partial F}{\partial \lambda} = \frac{2\lambda x e^{(x/x_c)^2}}{L x_c^2}$$

— identical, as required.

The answer to this puzzle lies in our implicit assumptions

- **—that displacement and flux linkage are independent input variables.**

If the flux density could be held constant, the force *would* grow with separation

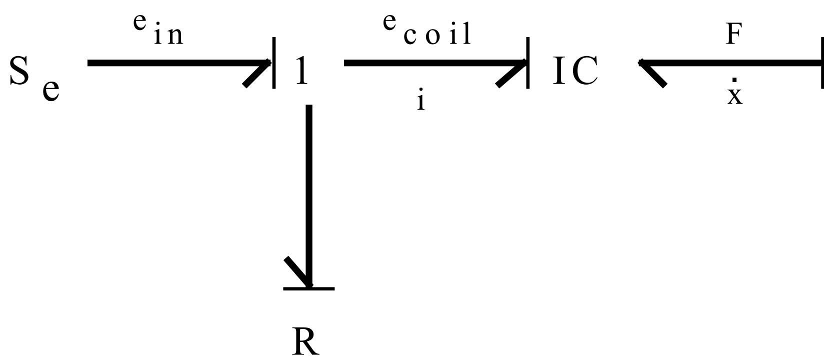
- **—but this is unlikely.**

It requires current to grow without bound as armature displacement increases.

For example:

include the inevitable resistance of the coil

assume a constant voltage input



at steady state for fixed x, the *current* is constant, not voltage.

$$\dot{\lambda}_{\text{steady-state}} = e_{\text{coil,steady-state}} = e_{\text{in}} - i_{\text{coil,steady-state}} R = 0$$

$$i_{\text{coil,steady-state}} = \frac{e_{\text{in}}}{R}$$

To express force as a function of current,

$F = F(i, x)$

we may use the electrical constitutive equation to eliminate flux linkage.

$$F = \frac{i^2 L x e^{-(x/x_c)^2}}{x_c^2}$$

In this case,

if $x < x_c$

—force increases as x is increased

if $x > x_c$

—force declines rapidly to zero

consistent with common experience.

NOTES:

Behavior (e.g., force-displacement relation) depends on boundary conditions.

Force as a function of current and displacement corresponds to differential causality on the inertia side of the multiport.