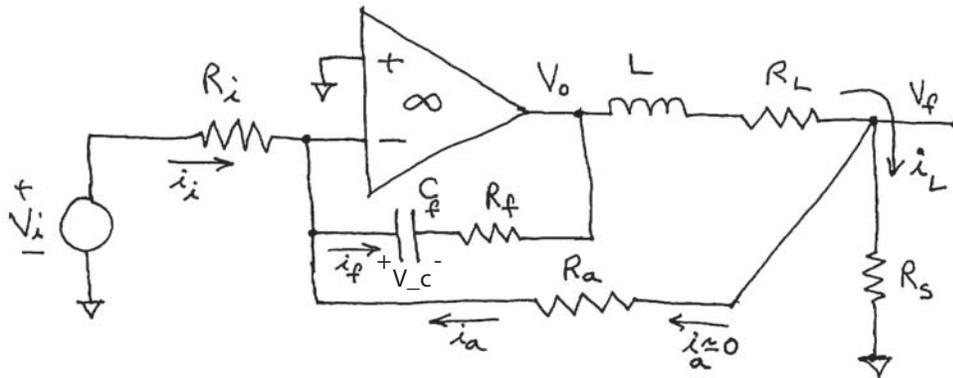**Assigned:** Wed. Apr. 9, 2014
Due: Wed. April 16, 2014, in class
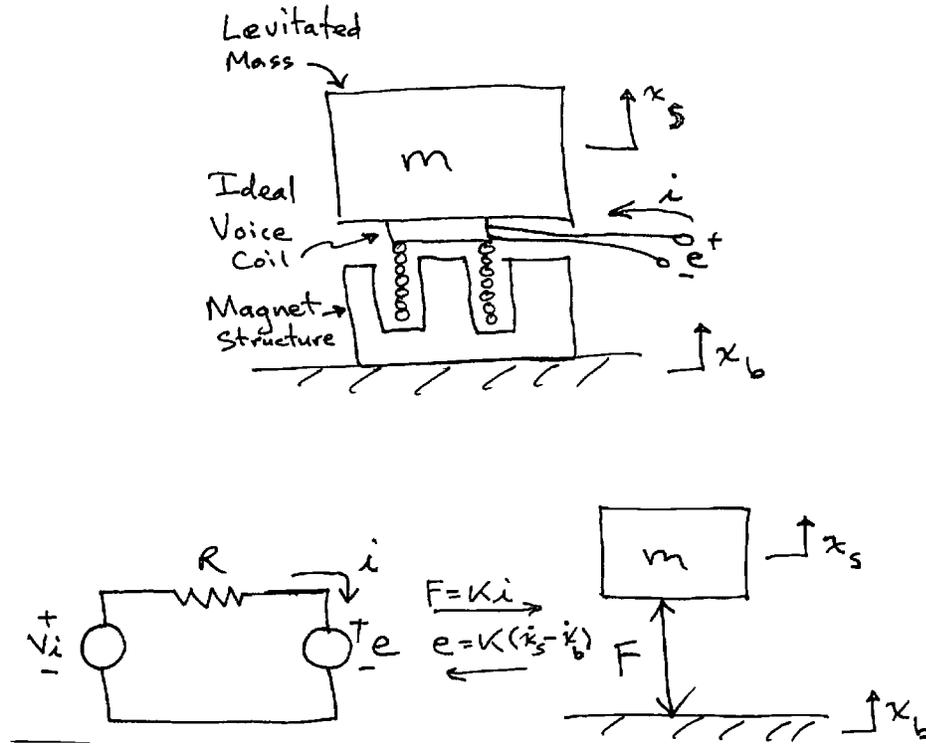**Reading:** FPE Sections 7.1–7.4

The following problems are assigned to both 2.14 and 2.140 students.

Problem 1 This problem revisits the current control circuit from Problem 1 of Problem Set 8. As
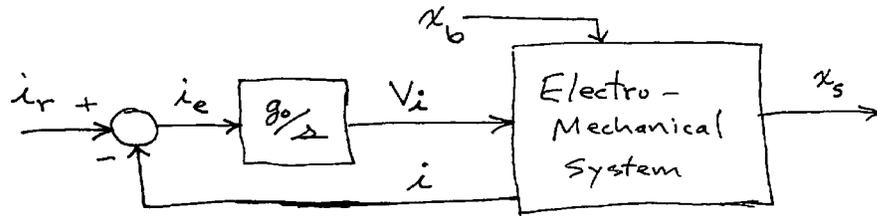before we assume that $R_s \ll R_a$ and thus that $i_a \ll i_L$.

a) Write a state-space model for the circuit in terms of state vector $\mathbf{x} = [V_c \ i_L]$, with input V_i and
outputs V_o and V_f. Note that V_c is defined as the capacitor voltage as shown in the figure.
This state-space model should be expressed in terms of the system variables, without any
numerical values.

b) Now use the numerical values as determined in Problem 1 of Problem Set 8, and enter the state-
variable model into Matlab. Compute the eigenvalues of the system, and thus the closed-loop
poles. Are these consistent with the crossover frequency and phase margin that was designed
in Problem Set 8?

c) Use Matlab commands to compute the step responses from input $V_i(t) = u_s(t)$ to outputs $V_o(t)$
and $V_f(t)$, assuming zero initial conditions.

d) Use Matlab commands to compute the transfer functions $V_o(s)/V_i(s)$ and $V_f(s)/V_i(s)$, and to
plot their respective Bode plots.

Problem 2 This problem revisits the one degree of freedom magnetic levitation system from Problem 2 of Problem Set 8.

For all of this problem, we assume that we close a current control loop on the coil by driving V_i on the basis of measurement of coil current i. In this context, we assume that i is directly measured, and that V_i is a dependent voltage source whose value is set by the current controller, *i.e.*, a power amplifier. In this simple model, we will assume that the coil has no inductance, and thus that the current can be controlled by a pure integral controller $G_c(s) = g_0/s$.

a) Write a state-space model for this system in terms of state variable $\mathbf{x} = [x_s \ \dot{x}_s \ V_i]$. Note that we have selected V_i as a state since it is the output of the integral controller. This state-space model will have inputs i_r and x_b. For the floor vibration, you may assume that you have the signals x_b, $\dot{x}_b$, and $\ddot{x}_b$ available as inputs. Which of these do you need to use in your state model? Also, set up your state model to have outputs i and x_s.

b) Now assume that the levitation system parameters take the numerical values $m = 2$ kg, $K = 5$ N/A, and $R = 8 \ \Omega$. For these values, and again assuming that the electrical current loop

2

bandwidth is much larger than the mechanical dynamics, choose a value of g_0 such that the current loop crosses over at $\omega_c = 10^4$ rad/sec. What is the resulting current loop phase margin ϕ_m?

c) For your value of g_0 chosen above, enter the state-space model into Matlab, and use Matlab commands to generate the step responses from the two inputs to the two outputs. That is, you will generate four step responses, two for the input i_r taking a step while $x_b = 0$, and two for the input x_b taking a step while $i_r = 0$.

Problem 3 FPE 7.2

Problem 4 FPE 7.6

Problem 5 FPE 7.18

The following problems are assigned to only 2.140 students. Students in 2.14 are welcome to work these, but no extra credit will be given.

Problem G1 FPE 7.19

Problem G2 FPE 7.20

2.14 / 2.140 Analysis and Design of Feedback Control Systems
Spring 2014