

2.087

Engineering Math: Differential Equations and Linear Algebra
MIT, Fall 2014

Practice Quiz #1
Open Book, MATLAB[®] Allowed

7 problems in 85 minutes. A sensible pace might be 11 minutes per problem leaving time to check your work. Manage your time accordingly. Show your work as clearly as you can within the time constraints.

1) (10 points)

a) Find a particular solution $y_1(x)$ to the equation $y' + y = 3e^{-x+1}$.

b) Find a particular solution $y_2(x)$ to the equation $y' + y = 4$.

c) Find a solution $y_n(x)$ to the equation $y' + y = 0$.

d) Find a solution $y_c(x)$ to the equation $y' + y = 3e^{-x+1} + 4$. Express the solution in terms of $y_1(x)$, $y_2(x)$, and $y_n(x)$.

2) (10 points) Find a solution to the equation $y' = -5y + 3t^3 \delta(t - 2)$ that satisfies the condition $y(1) = 5$.

3) (20 points)

a. Find the equilibrium solutions to the following differential equation on the interval $0 \leq y < 2\pi$.

$$y' = \sin y$$

b. Determine which equilibrium solutions are stable and which are unstable. Justify your answer graphically.

c. Determine the concavity of the solution in the interval of interest. [Note that the concavity may change multiple times in this interval!]

d. Use the information from parts a–c, to sketch the solution. Sketch as many integral curves as you need to fully represent the behavior of the solution for any initial condition on the interval $0 \leq y < 2\pi$.

NOTE: and is released at $t=0$.

4) (15 points) A pendulum of mass M and length L is initially held at an angle θ of $\pi/8$ from vertical its angular velocity is zero radians per second. At $t=2$ sec, a torque of 3Nm is applied to the pendulum and that torque of 3Nm is held constant until $t=3$ sec. Express the function $q(t)$ in the differential equation below without recourse to inequality constraints using the Heaviside step function.

$$ML\ddot{\theta} + Mg \sin(\theta) = q(t)$$

5) (15 points) Find a solution to the equation $y' = -4ty$ that satisfies the condition $y(0) = 1$.

6) (15 points) The logistic equation is $\frac{dy}{dt} = ay - by^2$ and its solution is $y(t) = \frac{a}{de^{-at} + b}$.

a) The US Population grew from 313,873,685 in 2012 to 316,128,839 in 2014. What parameters of the logistic equation a , b , and d are consistent with these data?

NOTE: This data may underspecify the model. You are free to make assumptions including ones that make the problem much easier.

b) Based on the model you created in (a), what is the steady state US Population we expect to observe in the future?

c) Based on the model you created in (a), what is/was the inflection point?

7. (20 points)

A 1 liter pot of tea has an initial temperature of 95°C. The pot is set outside to cool and the outside air temperature, T_a , is 20°C when the pot is placed outside at $t=0$ but is dropping at 1°C per minute as a cold front is approaching. Newton's law of cooling states that rate of change of the pot's temperature $\frac{dT}{dt}$ is proportional to the difference between the ambient Temperature and that of the pot $T_a - T$

a) Write a differential equation relating temperature T and its derivative $\frac{dT}{dt}$. Is the equation linear or non-linear?

b) Solve the differential equation to find $T(t)$ that satisfies the stipulated initial condition $T(0) = 95^\circ\text{C}$.

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