# 2.016 Hydrodynamics

Prof. A.H. Techet

## *Why study Marine Hydrodynamics?*

The Earth's oceans are one of our least explored resources. About 70-75% of the Earth's surface is covered by water. The total area of water covering the earth's surface is 361,419,000 km$^2$, of which the oceans make up 335,258,000 km$^2$ (97%). There is the potential for many exciting discoveries in the deep: such as sources of food, medicines, energy, and water. Advanced engineering knowledge is needed to propel current ocean exploration capabilities and to assure that our ocean resources will persist for generations to come. Understanding marine hydrodynamics can help us to design better ocean vessels and to understand physical ocean processes.

Marine Hydrodynamics encompasses many topics, including how surface ships, underwater vehicles and surface platforms interact with their environment. Waves and currents significantly affect these structures and a solid understanding of the fluid dynamics and forces that arise from fluid motions are crucial to designing and building complex ocean systems. Such systems include:

- Design of offshore platforms, such as tension leg platforms and marine risers
- Offshore instrumentation buoys for oceanographic measurement
- Design of surface ships and propellers
- Design of underwater vehicles

Studying marine hydrodynamics provides a greater understanding of a wide range of phenomena of considerable complexity involving fluids. Understanding these phenomenon allow us to make predictions for practical ocean engineering applications. A fluid is a continuous medium made up of small particles. In general, fluid mechanics allows us to study groups of these particles without having to know what each individual particle is doing.

The study of fluids at rest is called hydrostatics, and in this case, hydrostatic pressure is the predominate forcing. Of course, moving fluids are more interesting. When the fluid starts moving or a body in a fluid moves relative to the fluid, things start getting interesting and often quite complicated.

 

## Hydrodynamics v. Aerodynamics

- Water is almost 1000 times denser than air!
- Hydrostatic pressure is important
- Added Mass is an important factor in dynamic systems in the ocean.

## Basic Fluid Properties

*What defines a fluid?*

Fluid vs. Solid:
- Solids at rest will deform only so far under forces.
- Fluids cannot rest – they must be in motion to sustain shear stress.

| Similarities |
|---|
| The continuum hypothesis is used for both fluids and solids. |
| The fundamental laws of mechanics apply to both fluids and solids.<br>- Conservation of Mass<br>- Newton's law of motion (conservation of momentum)<br>- First law of thermodynamics (conservation of energy) |
| Both have constitutive laws relating stress and rate of strain. |

Liquid vs. Gas:
- Liquid settles under gravity and forms a free surface.
- Gas fills the entire containing space evenly.

(Note that "fluid" is a general term encompassing both liquids and gases).

*How can we describe a fluid?*

Fluid is made up of molecules that move freely relative to each other. This motion is imperceptible in bulk fluid processes like we will consider in this class. The variation of the properties within the fluid is so smooth that we can use differential calculus to analyze the substance. Thus we will assume that a fluid is a continuous medium for the remainder of this course. This is the "continuum hypothesis".

For example a bulk of fluid has mass Δm associated with volume ΔV the average density of the fluid is Δm/ΔV. Considering a fluid to be continuous allows us to take the limit as this volume goes to zero (thus dividing our bulk of fluid into smaller and smaller parcels). Such that the mass density of the fluid at a point can be defined as:

$$\rho \equiv \lim_{V \to 0} \frac{\Delta m}{\Delta V} \tag{1.1}$$

 

Continuum fails when the volume gets so small that the volume is less that 10 x $l_m^2$ (where $l_m$ is the average path length a molecule follows before bumping into another molecule – this is like $10^{-7}$ *m* for air at sea level – so 1000 times smaller than a strand of hair).

Throughout the course we will talk about a particle of fluid – this is just a very small element of fluid – large enough however to have its properties defined as in the limit above.

*Units of Measurement:*

Dimensional units used in this class will primarily be SI units (from the French *Le Système International d'Unités*). Later in the course we will look at the use of dimensional analysis in scaling laboratory model testing or comparing flow at different speeds or length scales. It is always a good idea to check your units when doing engineering problems.

There are only four primary dimensions in fluid mechanics from which all other dimensions can be derived. These are mass [M], length [L], time [T] and temperature [Θ]. (In the case of electromagnetic effects, electric current [I] may also be useful.)

Secondary dimensions are those derived as combinations of the primary dimensions, for example: velocity, acceleration, and force.

Take as an example, Newton's Second Law: **F** = m **a**

$$1 \text{ Newton force} = 1 \text{ N} \equiv 1 \text{ kg} \cdot \text{m} / \text{s}^2$$

so dimensionally we can see that force has the units of mass times length divided by time squared:

$$[F] = [M\ L\ T^{-2}]$$

 

| Secondary dimension | Primary components | SI unit |
|---|---|---|
| Area | $[L^2]$ | $m^2$ |
| Volume | $[L^3]$ | $m^3$ |
| Velocity | $[L\ T^{-1}]$ | m/s |
| Angular Velocity | $[T^{-1}]$ | 1/s |
| Acceleration | $[L\ T^{-2}]$ | $m/s^2$ |
| Pressure or stress | $[M\ L^{-1} T^{-2}]$ | $Pa = kg / (m\ s^2)$ |
| Energy, heat, work | $[M\ L^2\ T^{-2}]$ | J = N m |
| Power | $[M\ L^2\ T^{-3}]$ | W = J/s |
| Density | $[M\ L^{-3}]$ | $kg / m^3$ |
| Dynamic Viscosity | $[M\ L^{-1}\ T^{-1}]$ | kg/(m s) |
| Kinematic Viscosity | $[L^2\ T^{-1}]$ | $m^2/s$ |

We will refer back to this notation when we discuss dimensional analysis further.

*Physical Properties of a Fluid:*

Density, $\rho$, is defined as mass/unit volume.

- Can be effected by pressure and temperature (i.e. as you heat air, it becomes less dense and rises).
- Density of freshwater at 4º C is 1000 kg/m$^3$; at 20º C is 998.2 kg/m$^3$.
- Density of seawater is 1025 kg/m$^3$ at 20ºC and 1 atmosphere pressure.
- Density of air is 1.204 kg/m$^3$; at 20ºC at atmospheric pressure. NOTE: Water is 1000 times more dense than air!
- If density does not change under pressure the fluid is deemed incompressible.

Compressibility is affected by the speed of sound in the medium. Denser fluids have a higher speed of sound. For example, in air the speed of sound is 300 m/s and in water it is about 1000 m/s. Fluid will compress only if its velocity exceeds the speed of sound. In air this is not so difficult (think concord and sonic boom). Also the density of air is much less than that of water. In aerodynamics this opens up the study of subsonic (below speed of sound), transonic (near the speed of sound) and supersonic (above the speed of sound) flows.

For this course and most ocean/water flows compressibility will not be an issue and it *will be assumed that the fluid is incompressible unless otherwise noted.*

Specific Gravity, *SG* = density of substance/density of water

- If the SG is less than 1, the substance will float in water.
- Ice floats because it specific gravity is about 0.917 (at 20ºC)

Pressure, *p*:

- Pressure is a scalar quantity and always acts at right angles to a surface.
- Pressure is isotropic: it is equal in all directions and acts regardless of orientation.
- Pressure is a stress (force per area) and produces a force normal to the body surface that it acts upon.

$$\mathbf{F}\text{orce} = -p\, dA\, \mathbf{n} \tag{1.2}$$

Even a small pressure can create a huge force if the area it acts on is large.

Pressure inside or outside of a vessel causes stress within the containers walls.

Take a balloon for example: Pressure of the air inside forces the balloon walls out uniformly – if too much pressure builds and the walls are stretched too thin they will break.

In a pipe, pressure acts radially outwards and stress is built up in the pipe walls requiring a circumferential force within the pipe walls. This outward pressure causes a tensile stress in the material.

Pressure is transmitted evenly through a fluid:

If you push your finger into a full balloon – this applied pressure will distribute through the gas (air) and cause the internal pressure to rise – If you push too hard the balloon will pop.

If you increase the pressure by 10 psi at one point in a long pipe, the pressure will eventually increase throughout the entire pipe by 10psi. This may not be instantaneously though. Pressure disturbances travels at the speed of sound! After all sound is simply a pressure disturbance propagating through the air or water.

Increases in velocity can affect pressure: assuming a small parcel of fluid is accelerated from $V_1$ to $V_2$, at the same height, the pressure change would be

$$\Delta p = -{}^{1}/_{2}\, \rho\, (V_2^{\,2} - V_1^{\,2}). \tag{1.3}$$

Pressure due to a change in velocity is called dynamic pressure.

Viscosity, ($\nu$ or $\mu$), is related to the ability of a fluid to flow freely. Different fluids flow more freely than others: Glycerin, water, vs. motor oil. Viscosity indicates how a fluid will react to stresses and strains.

 

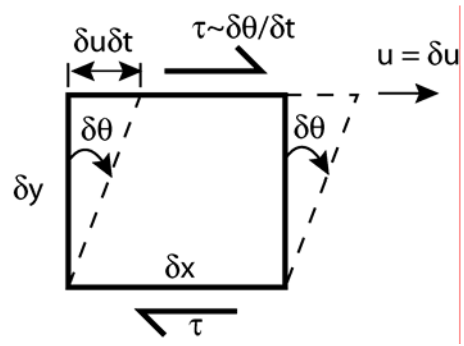


Figure 1: Strain Rate and shear stress in a fluid element.

Strain rate in a fluid is given by ${}^{\delta\theta}/_{\delta t}$. Newtonian fluids such as water, oil, and air have a linear relation between applied shear and resultant strain rate:

$$\tau \propto {}^{\delta\theta}/_{\delta t} \qquad (1.4)$$

Sir Isaac Newton first postulated this resistance law in 1687. Those that do not follow this relationship are referred to as non-Newtonian fluids (such as silly putty).

By geometry, $\tan \delta\theta = (\delta u\ \delta t) / \delta y$. If we limit the motion to small changes, $\tan \delta\theta \approx \delta\theta$, and we get that

$$d\theta/dt \quad = \quad du/dy \qquad (1.5)$$

shear strain rate = velocity gradient

Since shear stress is linearly proportional to shear strain rate, it is so related to velocity gradient as well. In hydrodynamics, the idea of a strain rate is not as important as the velocity gradient in the fluid, so we write the proportional relation for shear stress in terms of the velocity gradient.

$$\tau = \mu\, d\theta/dt = \mu\, du/dy. \qquad (1.6)$$

The constant of proportionality, $\mu$, is called the dynamic viscosity, since it relates to the *forces* on the fluid. The dynamic viscosity has units of stress-time.

$$[\mu] = [kg/(m\ s)]=[N\ s\ /\ m] \qquad (1.7)$$

 

Kinematic Viscosity, ν, so called because it relates to the *motion* of the fluid, is the ratio of the dynamic viscosity to density:

$$\nu = \mu / \rho . \tag{1.8}$$

Units of kinematic viscosity are [length$^2$/time] or [m$^2$/s].

Both dynamic viscosity and kinematic viscosity can change with temperature.

| Temp.@20° C | Air | Water |
|---|---|---|
| Dynamic Viscosity (kg/m s) | $1.82 \times 10^{-5}$ N s/m$^2$ | $1.0 \times 10^{-3}$ N s/m$^2$ |
| Kinematic Viscosity (m$^2$/s) | $1.51 \times 10^{-5}$ m$^2$/s | $1 \times 10^{-6}$ m$^2$/s |

Viscosity is most important near a body boundary. You can also think of viscosity as a measure of the "stickiness" of the fluid. Any amount of viscosity causes the fluid to "stick" to a surface without slip. This "no-slip" condition requires that for a stationary body the fluid velocity must approach zero as it approaches the wall. The no-slip condition results in a thin shear-layer, called a boundary layer, to develop when fluid moves over a plate.

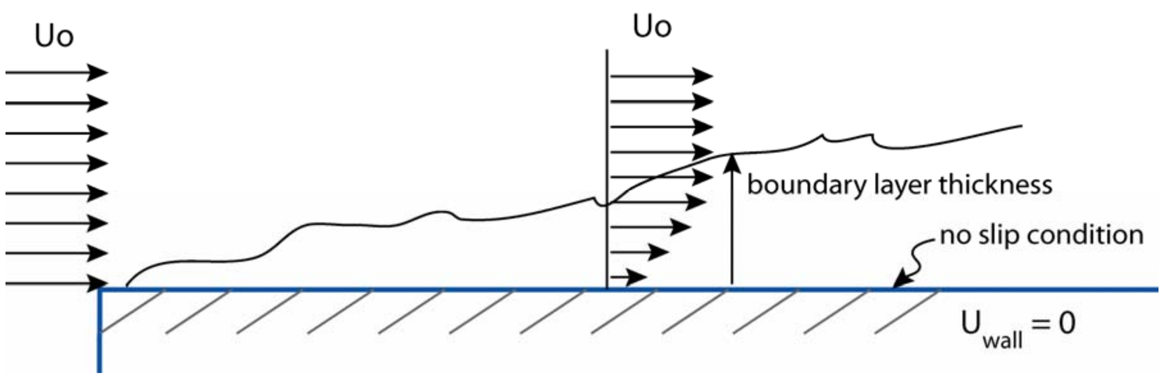


Figure 2: Boundary Layer Schematic

For water moving at $U_0$ = 1 m/s, 1 meter down the plate the boundary layer has only grown to be 0.005 m thick. That's really thin!

Boundary layers play a crucial role in marine hydrodynamics from ocean floor boundary layers, to boundary layers on ship or submarine hulls to those on offshore structures, hydrofoils and propellers.

CONCEPT QUESTION: How would you go about determining the viscosity of a fluid? Think about different ways to design an apparatus that allows us to measure the fluid viscosity.

Non-Dimensional Parameters are very useful to describe engineering problems, especially in fluids where experiments, models and reality do not always occur on the same scale, velocity or size! In order to compare two similar cases, say an experiment with a model ship and a full size Navy Destroyer (100 times larger than the model), we use non-dimensional parameters to scale the experiments to match reality. Often we cannot quite match up exactly, but that where experimental experience and numerical models come in handy!

Reynolds Number is an important number in fluid mechanics. We'll introduce it here in the context of viscosity and discuss it further in later lectures. Reynolds number is defined as a ratio of the inertial forces to the viscous forces.

$$R_e = {}^{U\,L}/_{\nu} \tag{1.9}$$

where U the fluid velocity, L the useful length scale, and ν is the kinematic viscosity. The Reynolds number indicates, among other things, how thick a boundary layer will be.

Surface Tension, σ

At the interface between a liquid and a gas (in other words where there is a density discontinuity), surface tension plays an important role. Surface tension arises from the attractive forces of that exist between molecules at the interface. Liquid surfaces tend to contract to the smallest possible surface area – thus the reason water droplets tend to form spherically. Molecules fully submerged within a fluid tend to repel due to their close packing. However those on the surface are missing half of their neighbors and thus a tension arises across the surface layer. This mechanical effect is that of surface tension.

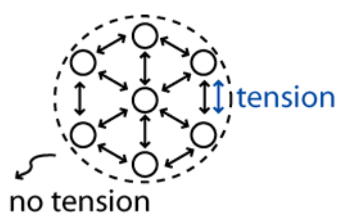


Figure 3: Surface tension in a blob of water

Surface tension has the units work per unit area:

$$[\sigma] = [\text{work}]/[\text{area}] = \text{N/m}$$

 