2.016 HW # 5

1) a) $u_i = [u_1, u_2, u_3] = \vec{u}$

b) $u_i v_i = u_1 v_1 + u_2 v_2 + u_3 v_3 = \vec{u} \cdot \vec{v}$

c) $\dfrac{\partial u_i}{\partial x_i} = \dfrac{\partial u_1}{\partial x_1} + \dfrac{\partial u_2}{\partial x_2} + \dfrac{\partial u_3}{\partial x_3} = \vec{\nabla} \cdot \vec{u}$

d)
$$u_i \frac{\partial}{\partial x_i} v_k = \left[u_1 \frac{\partial v_1}{\partial x_1} + u_2 \frac{\partial v_1}{\partial x_2} + u_3 \frac{\partial v_1}{\partial x_3},\; u_1 \frac{\partial v_2}{\partial x_1} + u_2 \frac{\partial v_2}{\partial x_2} + u_3 \frac{\partial v_2}{\partial x_3},\; u_1 \frac{\partial v_3}{\partial x_1} + u_2 \frac{\partial v_3}{\partial x_2} + u_3 \frac{\partial v_3}{\partial x_3} \right] = (\vec{u} \cdot \vec{\nabla}) \vec{v}$$

e)
$$\varepsilon_{ijk} \frac{\partial u_k}{\partial x_j} = \left[\varepsilon_{123} \frac{\partial u_3}{\partial x_2} + \varepsilon_{132} \frac{\partial u_2}{\partial x_3},\; \varepsilon_{231} \frac{\partial u_1}{\partial x_3} + \varepsilon_{213} \frac{\partial u_3}{\partial x_1},\; \varepsilon_{312} \frac{\partial u_2}{\partial x_1} + \varepsilon_{321} \frac{\partial u_1}{\partial x_2} \right]$$
$$= \left[\frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3},\; \frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1},\; \frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2} \right]$$
$$= \vec{\nabla} \times \vec{u}$$

Note: There are many more ε_{ijk} terms here that I did not write down because the ε_{ijk}'s are all 0.

$$\varepsilon_{ijk} = \begin{cases} 0, & \text{if any } i, j, k \text{ are equal} \\ 1, & \text{if} \\ -1, & \text{if} \end{cases}$$

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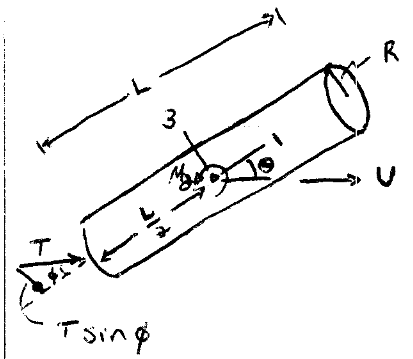
2.



The Monk moment balances the moment due to the off-axis thrust. In steady state, the sum of the moments is zero.

$$\Sigma M = (T \sin\phi)\left(\frac{L}{2}\right) + M_2 = 0$$

where $M_2 = -U^2 \sin\theta \cos\theta \, (m_{33} - m_{11}) = -\frac{1}{2} U^2 \sin(2\theta)\, (m_{33} - m_{11})$

and $m_{33} = \rho \pi R^2 L$

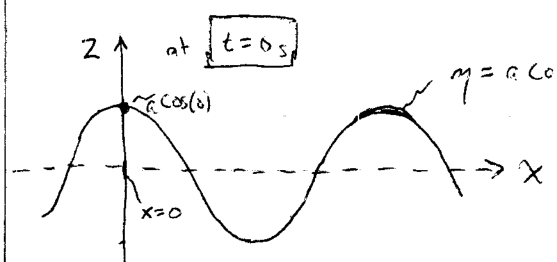
$m_{11} = 0$ for slender body $R \ll L$

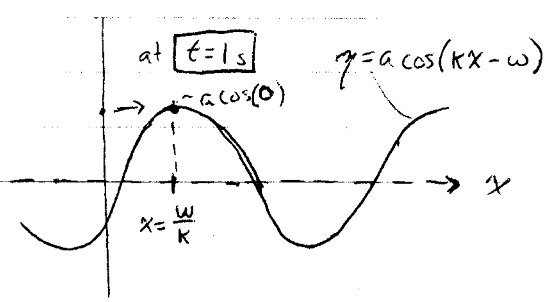
$$T \sin\phi \frac{L}{\cancel{2}} - \frac{1}{\cancel{2}} U^2 \sin(2\theta)\, \rho\pi R^2 L = 0$$

$$\sin(2\theta) = \frac{T \sin\phi}{U^2 \rho \pi R^2}$$

$$\boxed{\theta = \frac{1}{2} \arcsin\left(\frac{T \sin\phi}{U^2 \rho \pi R^2}\right)}$$

3. $\eta = a \cos(kx - \omega t)$





The wave moves to the right. In one second, the wave moved $\frac{\omega}{k}$ metess to the right.

$\hookrightarrow$ phase speed $= \frac{\omega}{k}$ ✓

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4.

$$\omega^2 = gk\tanh(kH) \rightarrow \begin{cases} \omega^2 = gk^2H & (\text{shallow},\ kH \ll 1) \\ \omega^2 = gk & (\text{deep},\ kH \gg 1) \end{cases}$$

$H = 2m$

		λ	$k=\frac{2\pi}{\lambda}$	kH	$\tanh(kH)$	$\omega = [gk\tanh(kH)]^{1/2}$	ω_{approx}	error
a	shallow	125 m	0.05 $\frac{1}{m}$	0.1	0.0997	0.221 $\frac{1}{s}$	0.221 $\frac{1}{s}$	≈0%
b	shallow	21 m	0.3 $\frac{1}{m}$	0.6	0.4621	1.26 $\frac{1}{s}$	1.33 $\frac{1}{s}$	5%
c		12.6 m	0.5 $\frac{1}{m}$	1	0.7616	1.93 $\frac{1}{s}$	2.21 $\frac{1}{s}$ / 2.21 $\frac{1}{s}$	15% / 15%
d	deep	6.28 m	1 $\frac{1}{m}$	2	0.9951	3.07 $\frac{1}{s}$	3.13 $\frac{1}{s}$	2%
e	deep	2.1 m	3 $\frac{1}{m}$	6	1.0000	5.42 $\frac{1}{s}$	5.42 $\frac{1}{s}$	≈ 0%

5.

	$V_p = \frac{\omega}{k}$	$V_{p_{approx}} = \frac{\omega_{approx}}{k}$	% error
a	4.42 m/s	4.42 m/s	≈ 0
b	4.20 m/s	4.43 m/s	5%
c	3.86 m/s	4.42 m/s	15%
d	3.07 m/s	3.13 m/s	2%
e	1.81 m/s	1.81 m/s	≈ 0

6.

	$C_g = \frac{1}{2}V_p\left\{1 + \frac{kH}{\sinh(kH)\cosh(kH)}\right\}$	$C_{g_{approx}} = \begin{cases} V_{p_{approx}} \\ \frac{1}{2}V_{p_{approx}} \end{cases}$	% error
a	4.41 m/s	4.42 m/s	0.2 %
b	3.76 m/s	4.43 m/s	18%
c	2.99 m/s	4.42 m/s / 2.21 m/s	48% / 26%
d	1.76 m/s	1.57 m/s	12%
e	0.91 m/s	0.91 m/s	≈ 0%

7. $\frac{H}{\lambda} = \frac{2m}{21m} = \frac{1}{10} > \frac{1}{20}$ ⇝ intermediate depth wave