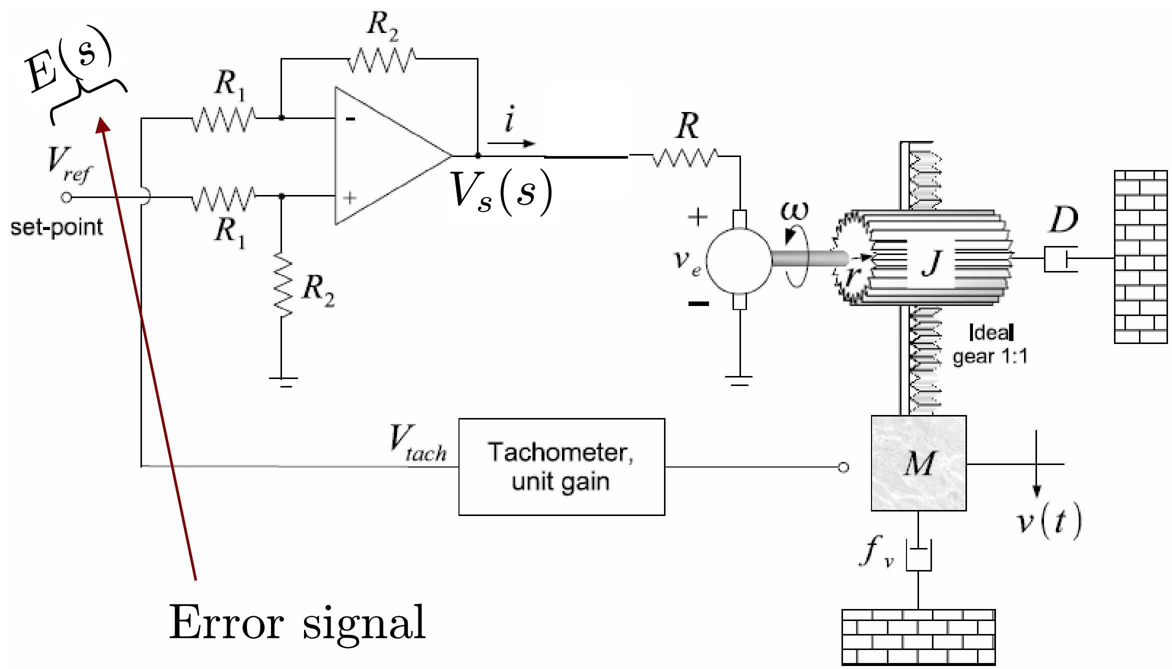
This week's goals

- **Today**
  - Physical realization of compensators
- **Wednesday**
  - Proportional-Derivative compensator
  - Lead/Lag compensators
- **Friday**
  - Introduction to state space

Differential amplifier as proportional controller



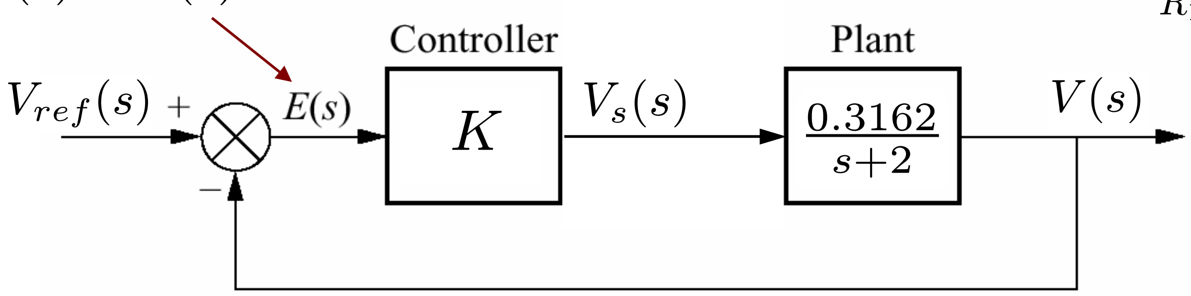
Controller gain

$$K = \frac{R_2}{R_1}.$$

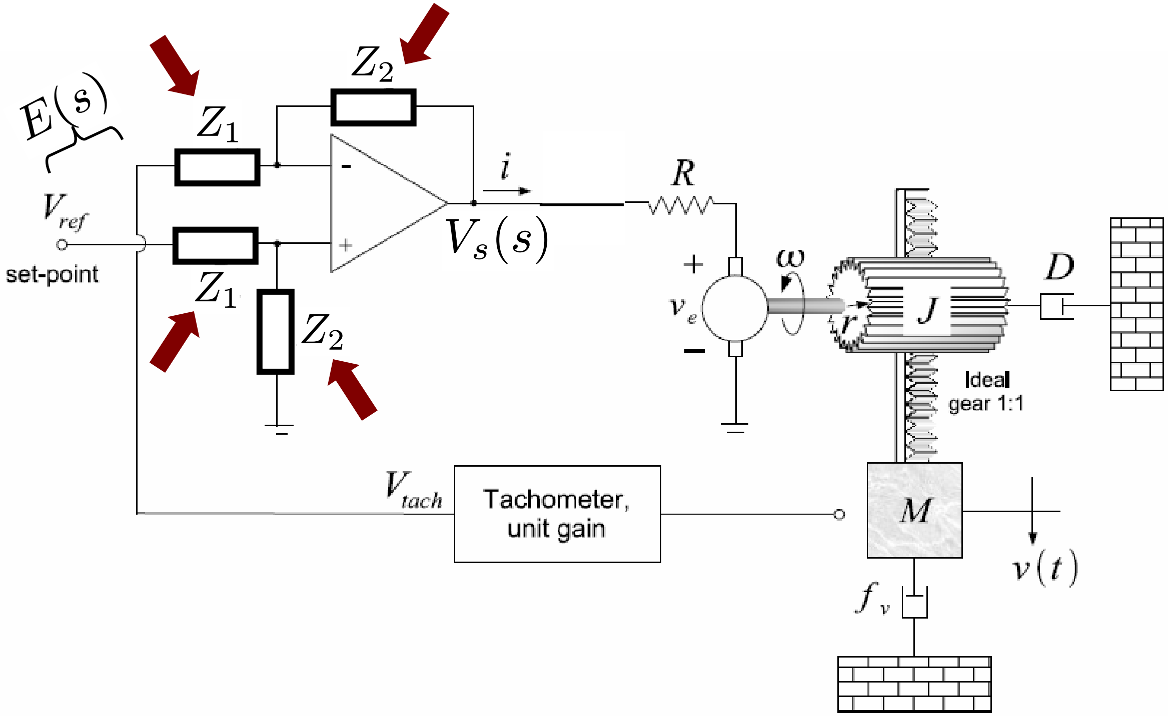
Recall the differential amplifier input–output relation

$$\begin{aligned} V_s(s) &= \frac{R_2}{R_1}\left(V_{ref}(s) - V(s)\right) \\ &= \frac{R_2}{R_1} E(s). \end{aligned}$$

$$E(s) = V_{ref}(s) - V(s)$$



Differential amplifier as *generic* controller



Input–output relation
of differential amplifier
with general impedances

$$
\begin{aligned}
V_s(s) &= \frac{Z_2(s)}{Z_1(s)}\left(V_{ref}(s) - V(s)\right) \\
&= \frac{Z_2(s)}{Z_1(s)} E(s).
\end{aligned}
$$

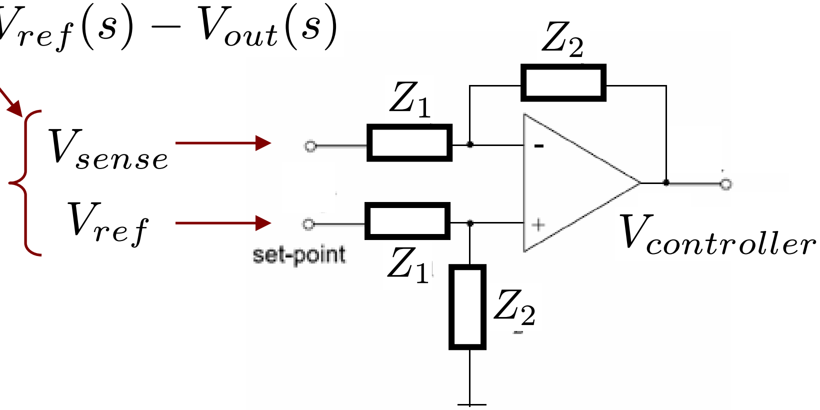
Controller transfer function

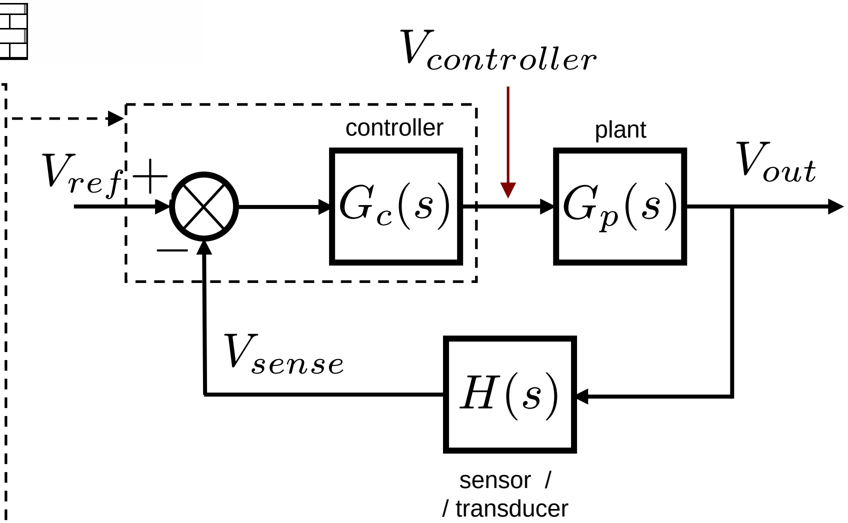
$$G_c(s) = \frac{Z_2(s)}{Z_1(s)}$$

$$E(s) = V_{ref}(s) - V_{out}(s)$$

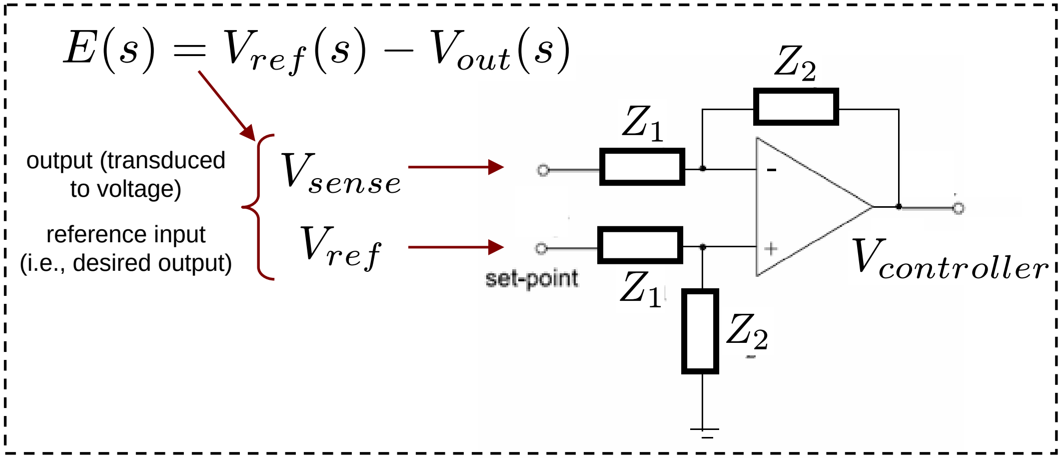
output (transduced
to voltage)

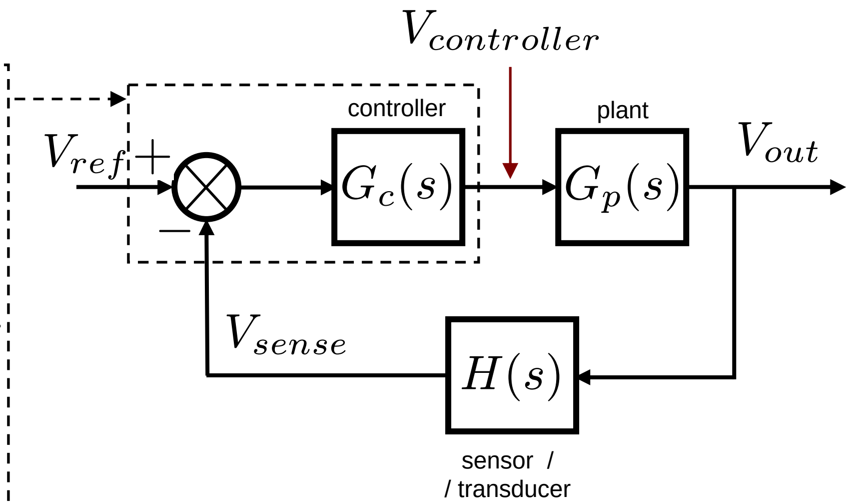
reference input
(i.e., desired output)

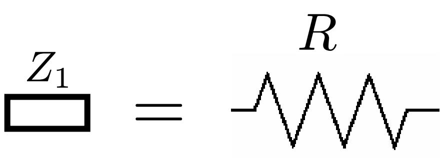


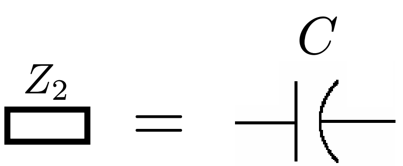


Example 1: Ideal integral controller





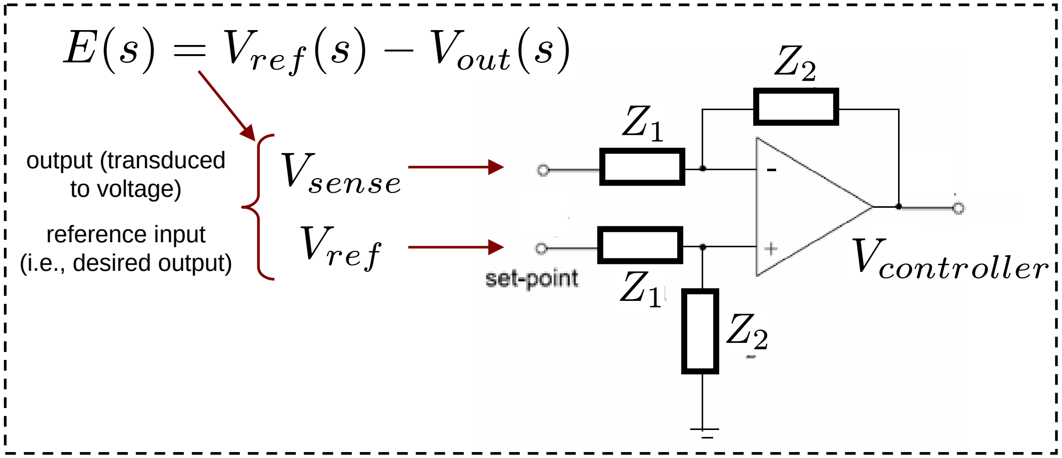


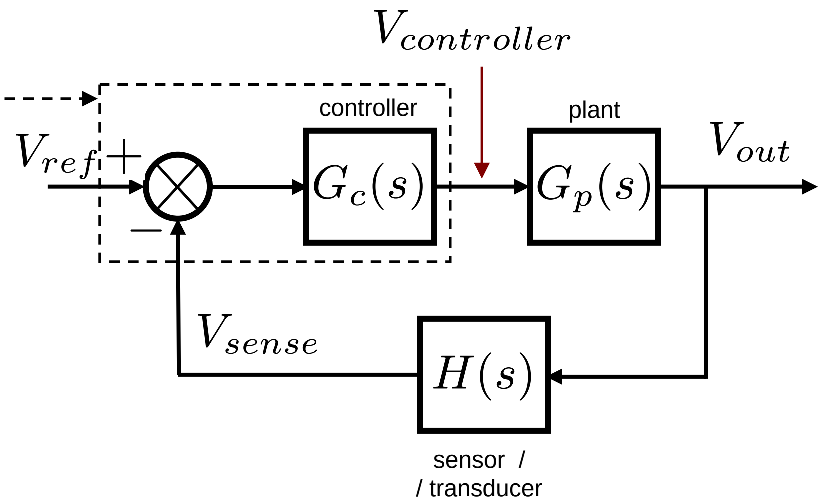


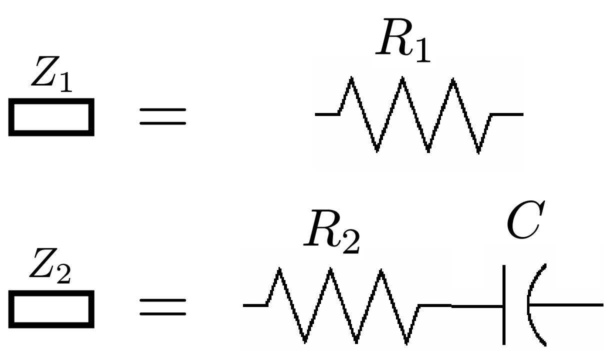
Controller transfer function

$$G_c(s) = \frac{Z_2(s)}{Z_1(s)} = \frac{1/(Cs)}{R} = \frac{1/(RC)}{s}.$$

Example 2: Proportional-integral controller







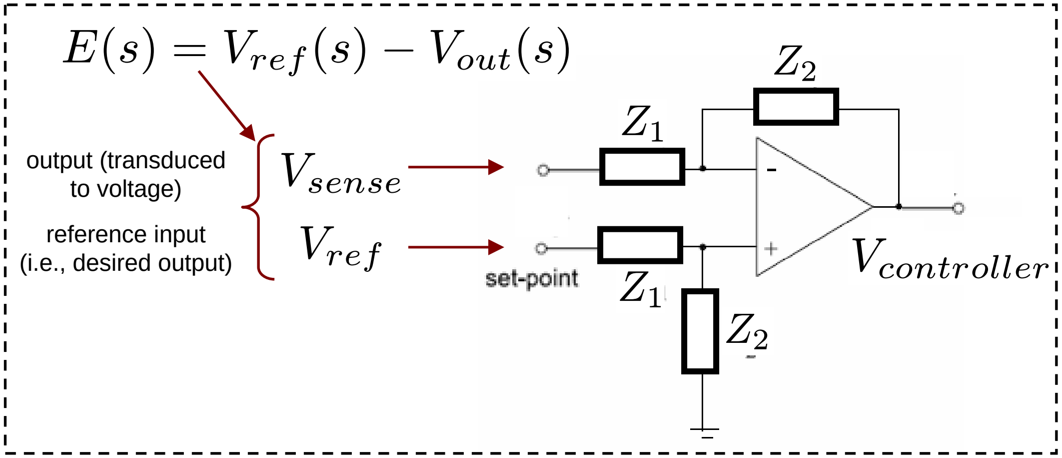
Controller transfer function

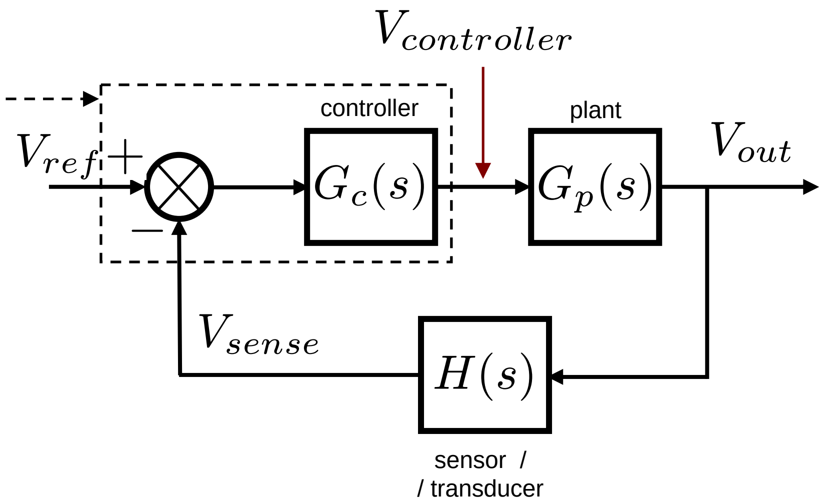
$$G_c(s) = \frac{Z_2(s)}{Z_1(s)} = \frac{R_2 + 1/\left(Cs\right)}{R_1} = \frac{R_2}{R_1}\frac{s + 1/\left(R_2 C\right)}{s}$$

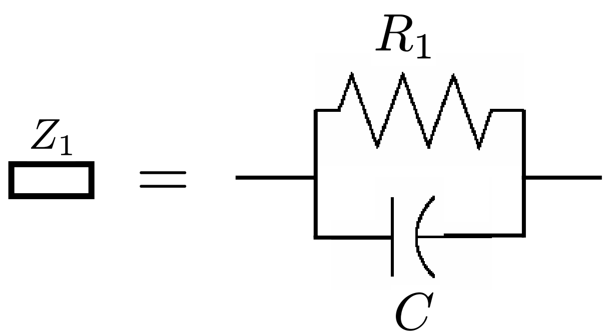
$$= K_p \frac{s+z}{s}, \quad \text{where } K_p = \frac{R_2}{R_1}, \; z = \frac{1}{R_2 C}$$

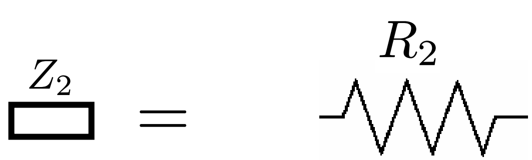
$$= K_p + \frac{K_i}{s}, \quad \text{where } K_i = \frac{1}{R_1 C}.$$

Example 3: Proportional-derivative controller









Controller transfer function

$$G_c(s) = \frac{Z_2(s)}{Z_1(s)} = \frac{R_2}{R_1/\left(1 + R_1 C s\right)} = \frac{R_2}{R_1} + R_2 C s$$

$$= K_p + K_d s, \quad \text{where } K_p = \frac{R_2}{R_1}, \; K_d = R_2 C.$$