2.004 MODELING DYNAMICS AND CONTROL II   Spring 2003

Problem Set No. 7

Problem 1. Pendulum mounted on elastic support. A collar of mass m slides without friction on a horizontal rigid rod and is restrained by a pair of identical springs with spring constant k. A pendulum consisting of a uniform rigid bar of length L and mass M is suspended from the collar by a frictionless pivot

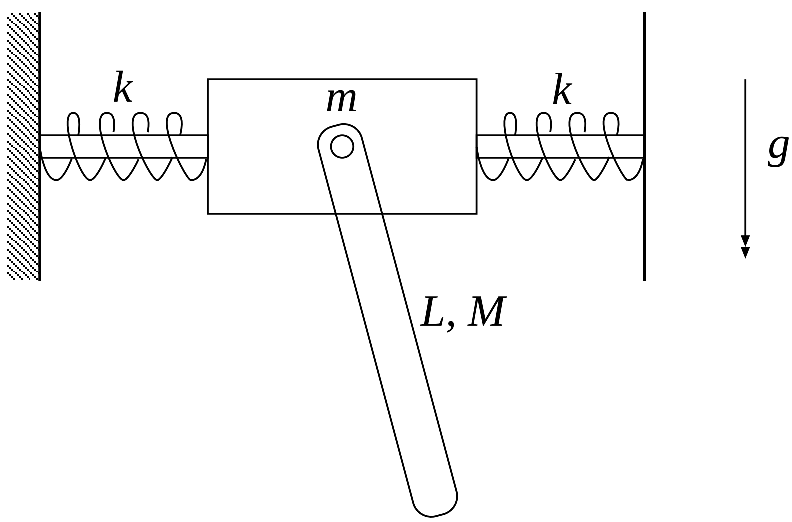

Figure 1: Pendulum supported on spring-restrained mount.

(a) Select a complete and independent set of generalized coordinates for this system.

(b) Derive differential equations of motion for these coordinates.

Problem 2. Stabilization of rocker. A rocker is machined into the shape shown from a rectangular block of metal of size $2R \times 3R \times h$, where h is the uniform height normal to the sketch. The uniform density of the material is ρ, mass per unit volume.

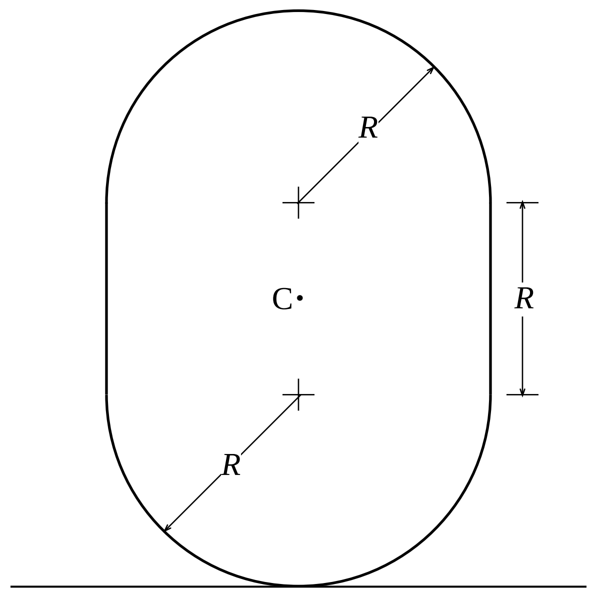

Figure 2: Dimensions of rocker.

(a) Express the mass M and the centroidal moment of inertia I_C of the rocker in terms of the parameters ρ, R, and h. Some helpful information is summarized in Fig.3. In (i) the centroidal moment of inertia of a uniform disk or cylinder is $I_C = \frac{1}{2} m_1 r^2$. In (ii) the centroidal moment of inertia of a uniform rectangular plate is $I_C = \frac{1}{12} m_2 (a^2 + b^2)$. In (iii) the centroid of a semi-circle is located a distance $\overline{y} = \frac{4}{3\pi} R$ above the base.

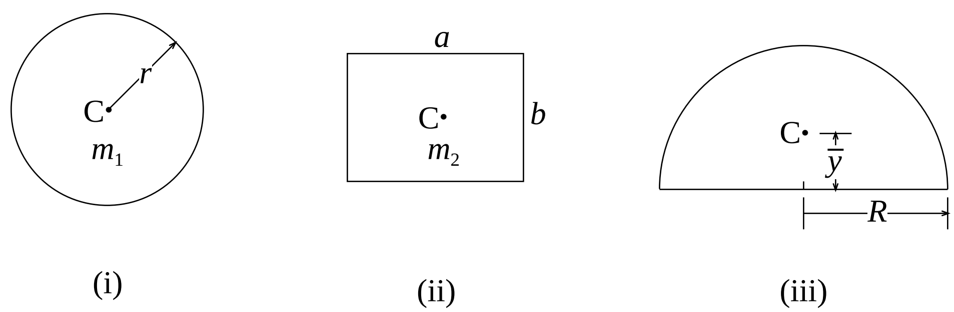

Figure 3: Useful facts about circular and rectangular shapes.

(b) If the rocker is constrained to roll without slipping on the floor, the upright position shown in Fig.2 is an equilibrium position. Is this a stable equilibrium?

(c) To stabilize the rocker, it is proposed to apply a horizontal force $f(t)$ to the centroid of the rocker, as shown in Fig.4. Derive a differential equation which describes how the rocking angle θ responds to the excitation $f(t)$.

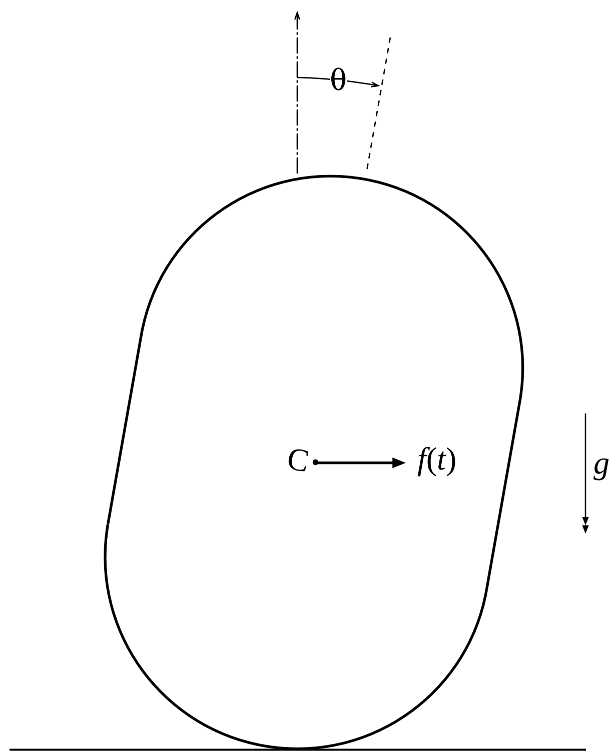

Figure 4: Force $f(t)$ is applied to rocker.

(d) Linearize the result of (c) by making the approximations $\sin\theta \approx \theta$ and $cos\theta \approx 1.0$. Transform the differential equation in the time domain into a transfer function from $F(s)$ to $\Theta(s)$ in the Laplace s-domain.

(e) It is proposed to construct the force $f(t)$ by observing the angle θ, comparing it with a desired angle θ_d and using the difference to generate (by means of a linkage driven by a motor) the force

$$f(t) = K(\theta_d - \theta)$$

where K is the effective gain, with the dimensions of force per radian. Transform this relation to the s-domain and couple it to the result of (d). Obtain the poles of the closed-loop transfer function from $\Theta_d(s)$ to $\Theta(s)$. For what range of values of the gain K is the closed-loop system stable?

Problem 3. Eigenvalue problem. The two masses slide without friction on the horizontal rigid rod, and are held in place by two springs with spring constant k.

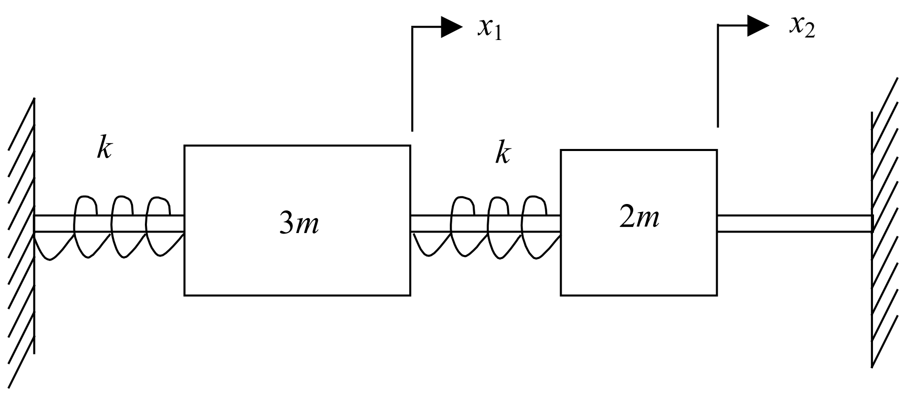

Figure 5: Mass-spring vibratory system

(a) Formulate equations of motion for $x_1(t)$ and $x_2(t)$ in the form of a matrix differential equation.

(b) Derive an eigenvalue problem of the form

$$[K]\{a\} = \omega^2[M]\{a\}$$

for natural modes of the form

$$\left\{ \begin{array}{c} x_1 \\ x_2 \end{array} \right\} = \left\{ \begin{array}{c} a_1 \sin(\omega t + \phi) \\ a_2 \sin(\omega t + \phi) \end{array} \right\}$$

(c) Solve analytically for the mode shapes $\{a_1\ a_2\}^T$ and the eigenvalues ω_1^2 and ω_2^2. Construct the *modal matrix* $[\Phi]$ whose columns are the mode shape vectors.

(d) Open MATLAB and type the command: *help eig* to learn about MATLAB's eigenvalue capabilities. Apply the command [V, D] = EIG (K, M) and compare MATLAB's solution to your solution in (c) above.