2.004 Dynamics and Control II
Spring 2008

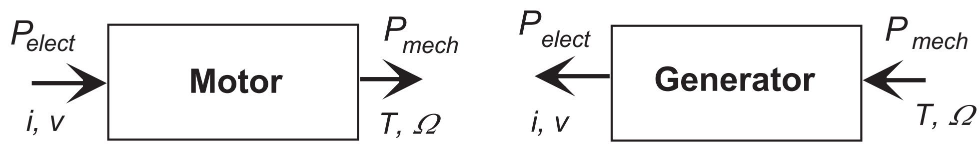
Massachusetts Institute of Technology
Department of Mechanical Engineering

2.004 *Dynamics and Control II*
Spring Term 2008

Solution of Problem Set 2

Assigned: Feb. 15, 2008 Due: Feb. 22, 2008

Problem 1:



(a) Assuming an ideal dc motor:

$$Power_{in} = Power_{elect} = v_b i_m = K_v \Omega_m i_m$$

$$Power_{out} = Power_{mech} = T_m \Omega_m = K_m i_m \Omega_m$$

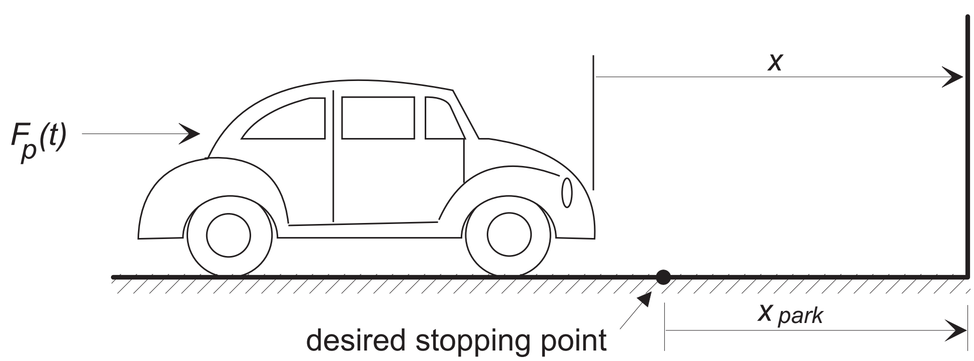
$$Power_{in} = Power_{out} \Rightarrow K_v \Omega_m i_m = K_m i_m \Omega_m \Rightarrow K_v = K_m$$

(b) Using the data sheet:

$$K_m = 30.2 \, \frac{mNm}{A} = 30.2 \times 10^{-3} \, \frac{Nm}{A}$$

$$K_v = \frac{1}{317 \, \frac{rpm}{V}} = \frac{1}{317} \, \frac{V}{rpm} = \frac{1}{317 \times \frac{2\pi}{60}} \, \frac{V}{\frac{rad}{s}} = 30.1 \times 10^{-3} \, \frac{V}{\frac{rad}{s}}$$

Problem 2:



Using an approach similar to that we used in class (with the same basic mass/friction linear model):

(a)

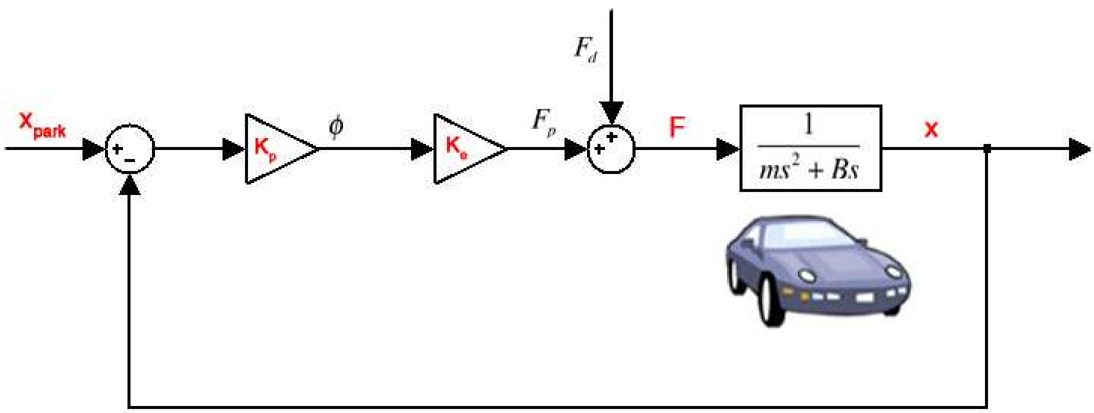
$$m\frac{dv}{dt} + Bv = F_p(t) + F_d(t)$$

Ignore F_d, assume $F_p(t) = K_e\phi(t)$ and employ $v = \frac{dx}{dt}$:

$$m\frac{d^2x}{dt^2} + B\frac{dx}{dt} = K_e\phi(t)$$

$$\frac{X(s)}{\Phi(s)} = \frac{K_e}{ms^2 + Bs}$$

(b-c)



(d)

$$m\frac{d^2x}{dt^2} + B\frac{dx}{dt} = K_eK_p(x_{park} - x)$$

In the steady state, $\frac{dx}{dt} = 0$ hence:

$$x_{s.s.} = x_{park}$$

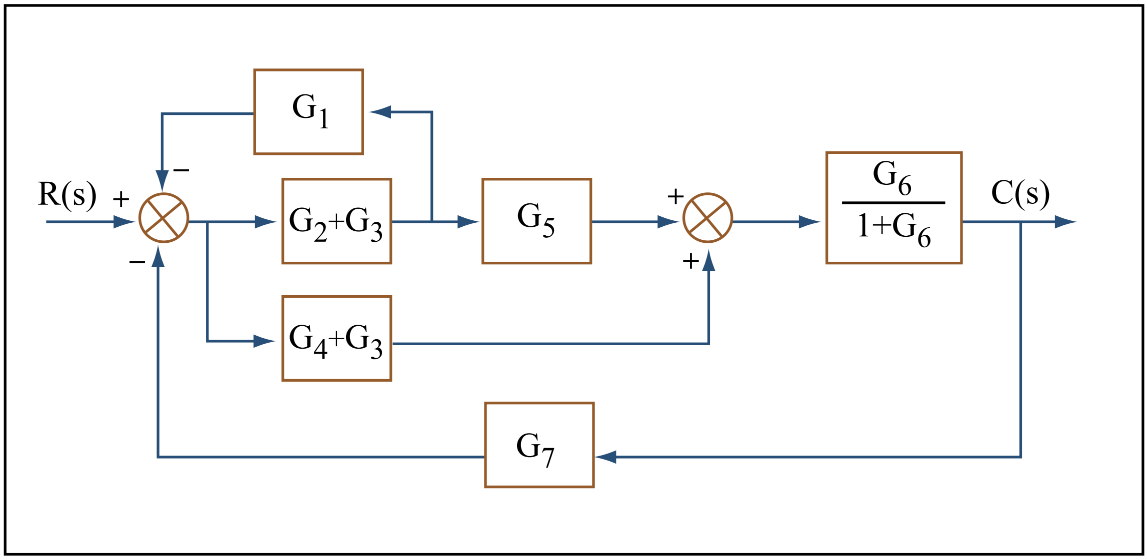
(e)

$$\begin{aligned}\text{Position Control: } & m\tfrac{d^2x}{dt^2} + B\tfrac{dx}{dt} = K_eK_p(x_{park} - x)\\ \text{Velocity Control: } & m\tfrac{dv}{dt} \ \ + Bv \ \ \ = K_eK_p(v_{desired} - v)\end{aligned}$$

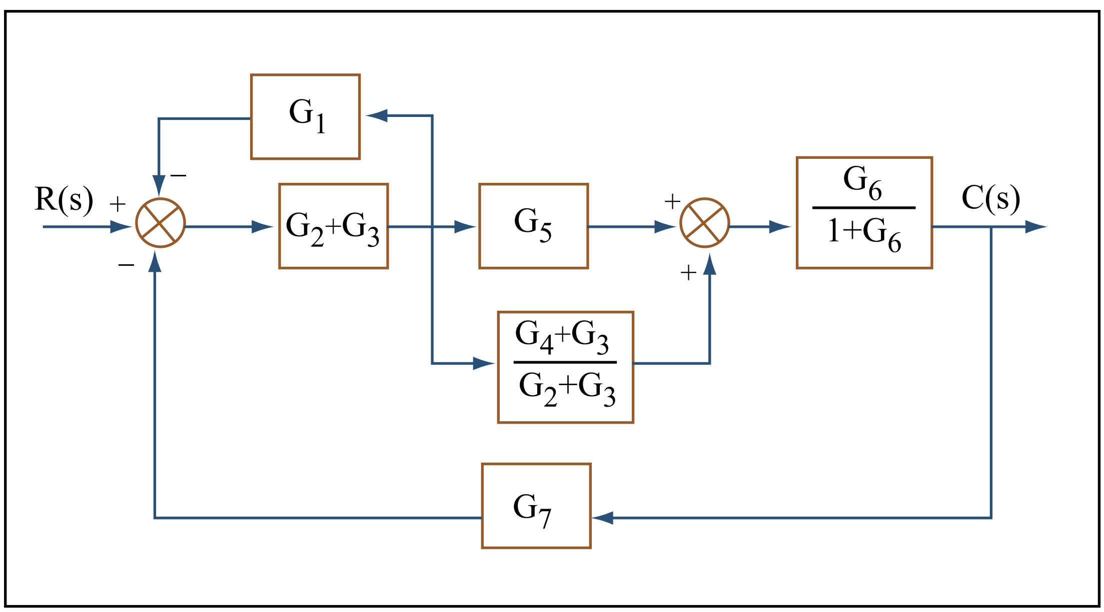
In the position control $v_{s.s.} = 0$, all the left terms of the equation are zero and a zero error can be held. On the other hand for the velocity control, $\frac{dv}{dt}|_{s.s.} = a_{s.s.} = 0$ but $v_{s.s.} \neq 0$, the drag force is not zero, and then $v_{s.s.} \neq v_{desired}$.

<u>**Problem 3:**</u> Nise, Chapter 5, Problem 4.

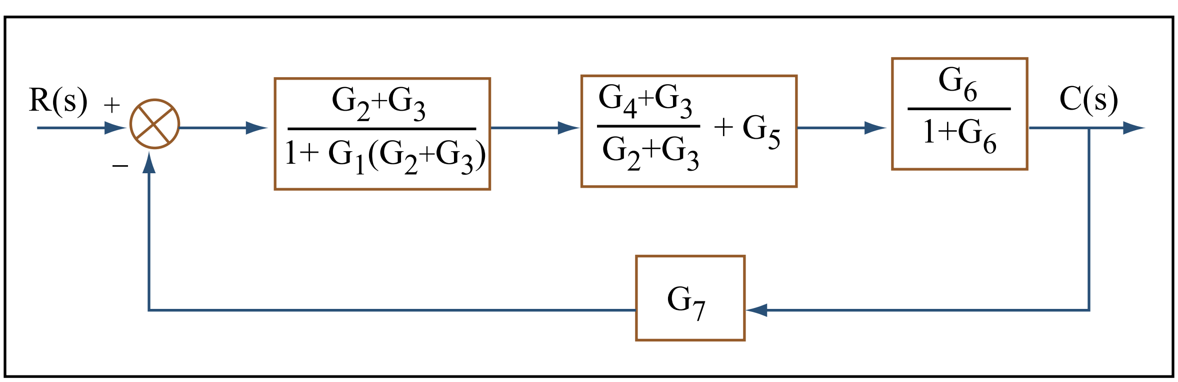
(1) Split G_3 and combine it with G_2 and G_4. Also use feedback formula on G_6 loop:



(2) Push $G_2 + G_3$ to the left past the pickoff point:



(3) Using the feedback formula and combining parallel blocks:

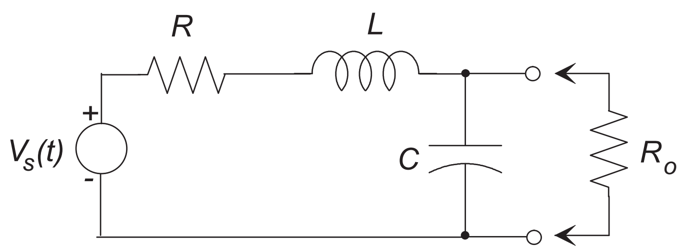


Figures by MIT OpenCourseWare.

(4) Multiplying the blocks of the forward path and applying the feedback formula:

$$T(s) = \frac{G_6G_4+G_6G_3+G_6G_5G_3+G_6G_5G_2}{1+G_6+G_3G_1+G_2G_1+G_7G_6G_4+G_7G_6G_3+G_6G_3G_1+G_6G_2G_1+G_7G_6G_5G_3+G_7G_6G_5G_2}$$

Problem 4:

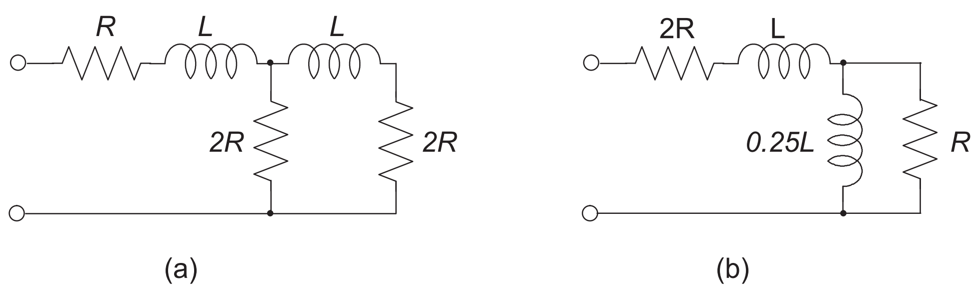


$$\begin{aligned} Z_o &= R_o \,||\, \frac{1}{Cs} = \frac{R_o \frac{1}{Cs}}{R_o + \frac{1}{Cs}} = \frac{R_o}{R_oCs+1} \\ Z_s &= R + Ls + Z_o = R + Ls + \frac{R_o}{R_oCs+1} \end{aligned}$$

Voltage Division:

$$\begin{aligned} G(s) &= \frac{V_o}{V_s} = \frac{Z_o}{Z_s} = \frac{\frac{R_o}{R_oCs+1}}{R + Ls + \frac{R_o}{R_oCs+1}} \\ &= \frac{R_o}{LR_oCs^2 + (L + RR_oC)s + (R + R_o)} \end{aligned}$$

Problem 5: Input impedance does not characterize a system uniquely. In other words, systems with different characteristics, like below examples, can have the same input impedance.



(a)

$$\begin{aligned} Z_a &= R + Ls + 2R \,||\, (Ls + 2R) \\ &= R + Ls + \frac{2R(Ls+2R)}{2R + (Ls+2R)} \\ &= R + Ls + \frac{2RLs + 4R^2}{4R + Ls} \\ &= R + Ls + \frac{0.5RLs + R^2}{R + 0.25Ls} \end{aligned}$$

(b)

$$
\begin{aligned}
Z_b &= 2R + Ls + R \mid\mid (0.25Ls) \\
&= 2R + Ls + \frac{R(0.25Ls)}{R + 0.25Ls} \\
&= R + Ls + R + \frac{R(0.25Ls)}{R + 0.25Ls} \\
&= R + Ls + \frac{0.5RLs + R^2}{R + 0.25Ls} \\
&= Z_a
\end{aligned}
$$