PROFESSOR: Last, kind of under the announcements category, is I want to talk about the muddy cards. So I've used those many times in the past. Last time was the first time I handed them out. And your comments were great.

This was really a good lecture to have handed them out. We covered lots of interesting important concepts. And so I'm going to review a couple of things that came up in the muddy cards.

A couple of the most positive comments is people really like the demos, and they really like the explanations, especially with examples. People particularly commented it was really helpful to compute angular momentum from two different points. And you get the revelation that you get two very different answers. So that's a really important point.

And somebody then asked the question, said, well, I thought that vectors were independent of the coordinate system that you select. It's true. A velocity ought to be a velocity no matter whether it's r theta or x, y, z.

But why? That seems to kind of violate that notion that vectors should be independent of coordinates. And yet we computed an angular momentum with respect to one place and respect to a different place, and we got different answers. How do you resolve that? Yeah.

AUDIENCE: It's sort of like since it's moving, the coordinates shouldn't matter. Like if it was equilibrium, it wouldn't matter where you would put the [INAUDIBLE].

PROFESSOR: OK, well, you're getting close. Here was the problem I think we did. And we chose this point, which I'll call 2, and this point, which I'll call 1. And we computed h1. I'll just call h1 with respect to point 1. We'll call this A. We computed with respect to 1. And we computed the angular momentum of A with respect to 2.

But in this case, angular momentum of a particle with respect to some location, origin of a coordinate system, is defined as r of the partial with respect to the coordinate system crossed with the linear momentum of the particle that you're-- we'll call it B here, just the name of the particle.

The definition-- these are both vectors. You arch changing the vector. It's a different vector. Because this changes in the two parts. So it's not a constant vector at all. By its definition, it is just something different when you move to a different place and this piece changes. This piece is invariant, but this piece is not. And that's the answer to that one.

Lots of people were still not clear about Coriolis. We'll work on that as time goes by. And people were interested in how to pick reference frames and so forth. Somebody made the suggestion, try using some colored chalk. It would help. I don't own any colored chalk.

My assistant just walked in with some. She found some at the last minute. So I'll try doing that. That's a good idea. And someone else says, take a break in an hour. And that's a pretty good idea, too. I'll try to remember to do that. So the muddy cards are great. Please today we'll do the same thing.

So let's start with this topic. It's a subject which we constantly use throughout the course doing dynamics. You have to be able to figure out coordinate systems, degrees of freedom, drawing free body diagrams. So I'm going to do a few quick examples-- coordinates, fbd's. I picked some examples here just to emphasize a few different points.

Here we have a slope. I've got a wheel. It's a rigid body. I pick a preliminary coordinate system. Sometimes you do that just to help you think about it. And now let's talk about degrees of freedom. What do we mean by degrees of freedom?

I'm going to define it as the number of independent coordinates necessary to describe the motion. So it's the number of independent coordinates that you need. Now, with few exceptions, I can compute that by multiplying 6 times the number of rigid bodies in the problem plus 3 times the number of particles minus the constraints, the number of constraints.

So this is the number of rigid bodies. This is the number of particles. And C, this is the constraints. So take a look at this problem. A wheel's a rigid body. The difference between a rigid body and a particle is a rigid body is big enough. It has mass at some extent. Its rotational inertia will matter.

So here we've got a wheel. So we've got certainly one rigid body but no particles. So in this case, n is 1, m is 0. And so the number of degrees of freedom that we should come up with in this problem is going to look like 6 minus C.

And so then the problem becomes-- let's identify what the constraints are in the problem. So I drew my initial little coordinate system just so that I can use language like x, y, and z direction. So z's coming out of the board. So in this problem, let's figure out how many constraints there are.

How about the y direction? The simplest kind of constraints are things that just allow no motion. Can it move in the y direction? No, OK, so y, y dot, y double dot, are 0. So that's a hard constraint. You can't move through the wall in the y direction.

You have to make assumptions when you're doing problems. You have to try to simplify things as much as you can so that you make them easy. So really I'm going to assume in this problem-- I haven't shown any constraints or wheels or guides or rollers. But I'm going to assume that it won't move in the z direction. So that's another constraint.

So this implies 1. This implies another constraint. If I don't do this, if I don't assume that, then I just end up with another equation of motion. For every degree of freedom, you end up with a problem.

You're going to need an equation of motion. So if I did not make this assumption, I'd say that the summation of the forces in the z direction is equal to 0. And that's equal to the mass times the acceleration of the body in the z acceleration. It's a vector, vector component.

And then you just say, oh well, that's a trivial equation of motion. And so you could deal with it that way. But we'll just assume that we have no motion in the z. And that gives us another constraint.

Now we can assume again that there's constraints in the problem, or it's well behaved, in that the thing won't fall over. It won't roll over. And it won't change direction running down the hill. So we'll assume no rotation about the x or y axes. So that implies two more constraints.

And finally, the constraints can come in many flavors. Finally we know that in this problem that I need to think in terms of a rotation. So there's a positive rotation in that coordinate system. So now I have a rotational coordinate that I can think about.

But this now says that if there's no slip, I can say that the distance it rolls down the hill is minus r theta. That's a constraint. x and theta are not independent of one another.

We're looking for the number of independent coordinates required to completely describe the motion. x and r are not independent because of this no slip condition. And so that implies yet another. OK, so we've got one, two, three, four, five constraints. And we said that the number of degrees of freedom in this problem is equal to 6 minus 5, which is 1.

So you take the single coordinate. You could have told me that long ago that that's what it's going to take. But this is the sort of thinking you have to go through to come up with all these constraints. So this is going to take a single coordinate. It could be x. It could be theta.

But you don't actually need them both. You use them both for a while, because it's convenient. But in the final analysis, you'll be able to write an equation of motion just in terms of x or just in terms of theta.

OK, free body diagram of our wheel-- we said no slip. So here's your slope. You know you've got mg. You know there's going to be a normal force from the slope. There may also be some tangential force that makes it impossible for it not to slip, some f. So there's the free body diagram, in this case.

What if-- I'll do e here, or do a case ii here, slip allowed. Then how many constraints do we have? Just four. Because now you can no longer say that this is true. They're independent of one another.

They take on values not controlled by this formulation. So 6 minus 4 gives you 2. And you're going to end up having to have both x and theta probably as your chosen coordinates to do the problem. And you'll end up with two equations of motion.

So I've got a hockey puck here. I've drawn kind of a 3D perspective of this. So it has a coordinate system out here. The z-axis is going like that. So here's my z. Here's my x. Here's my y in the plane of the ice that this thing is sliding on. So this is my z-coordinate.

And I have string wrapped around it. And I've got a piece of string coming off like that. And I'm pulling on it with some tension. Because otherwise the only constraints are it's sitting on this icy surface, which I'm going to assume is-- well, I don't even have to assume it's frictionless. I could.

So let's figure out how many equations of motion we're going to need here. So the number of degrees of freedom, 6 times-- this is a rigid body. It's not a particle. And there's only one of them. So it's 6 times 1 minus C and the number of constraints.

So can it move into the table, into the surface? No, so that's a constraint in z. Are there any constraints in the x or y? It can rotate about z. But it can't rotate about the x or y-axes. All right, so constraint into z is one. Can't rotate about x or y-- two, three. Are there any others? Who thinks I may have missed one?

OK, I'd say we've got 6 minus 3. We're going to need three equations of motion to be able to actually describe the motion of this thing. And we'd probably use-- this is a pretty good coordinate system. We'd probably use an x, a y, and some theta with respect to z-axis.

All right, another quick example-- so we've got a rod leaning against a wall. It's length L. Actually, I don't want to do that. It's L long. It's got a center of mass here at the middle, uniform rod. Let this be x, y, z coming out of the board, maybe give it an angle here just to help us describe motions.

You might start off with a preliminary little coordinate system just so you could think that, OK, no motion in the x, no rotation in this. But once you get the problem, you're ready to set up the equations of motion, you might decide, OK, I know I need two coordinates. And the ones I've preliminarily chosen aren't too good. Then you change, and you pick the really good ones. But I'm just making the preliminary assessments so I can assess the problem here.

Again, the degrees of freedom-- 6 times 1 minus C. Because I only have one rigid body. Now, the constraints here are a little more subtle. Let me just discuss these two points. What can you say about the motion at point A? This is right here where it touches the wall.

This thing is sliding down the wall. It might be frictionless, might not. Whether or not friction acts doesn't really change the number of degrees of freedom unless you invoke things like no slip. What can you say kinematically about motion, about the motion at A? Does the wall restrict the motion?

AUDIENCE: Yes.

PROFESSOR: In what direction?

AUDIENCE: x.

PROFESSOR: In which direction?

AUDIENCE: x.

PROFESSOR: In x direction, right. So is this body constrained in the x direction?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Pardon? I hear a no. I hear some yeses. What about at B? Is it constrained at B? In the--

AUDIENCE: y.

PROFESSOR: y direction, OK. I didn't bring my big foam disk. But earlier in the term, I said that the definition of translation is that all points on a rigid body do what if you're rectilinear or curvilinear translation as opposed to rotation?

AUDIENCE: Move in parallel.

PROFESSOR: All points move in parallel, exactly right. That means that if we use real strict definitions of translation and rotation, that if I constrain the motion of any point on that object, that object is now not allowed to translate in that direction by the definition of translation.

So this point constrains it in the x direction. This constrains it in the y direction. And we're going to-- so it's constrained in translation. And that implies 2. And b, we'll assume that z motion is 0. We'll just assume there's nothing going on out of the plane.

So that gives me another one. And I'll assume no rotation. I'm not going to allow any rotation in this problem. I'm not interested in rotation about the x or about the y, about these axes.

And that implies two more. So we have two, three, four, five. And I better not have any more than that or the thing can't move. So in this case, this is 6 minus 5 equals 1.

I need a single coordinate to describe the motion. And if you look at it, you say, well, that's kind of intuitive and obvious. If I specify the x position here, I could figure everything out. If I know the length and the x, I could figure out where it is. If I know the y position and the length, I could figure out where it is. If I know theta, I could figure it out. I only need one.

But that strict definition of translation is really helpful here. This thing, this object, is in pure rotation. And if it's in pure rotation, it must rotate about some point. Where? You know how to find that?

What's the velocity here? It's got gravity acting on it, so it's probably down. But it's parallel to the wall. It has to be. What's the velocity here? It's got to be parallel to the wall.

If I draw perpendicular to that-- if I'm saying, this is rotation. All points in the body rotate at the same rate. But their speed is determined by the distance away from the center of rotation. But if it's pure rotation, there must be a center of rotation somewhere.

And it must be perpendicular to any velocity vector. So you draw the perpendicular. You draw the perpendicular. And here is the instantaneous center of rotation, the ICR. There's a little short section in the book on that. When this thing drops down to here, same kind of arguments hold. But the center of rotation has changed locations. Yeah.

AUDIENCE: So is this not translating x and y?

PROFESSOR: So now let's talk about the center of mass. So she asked-- excuse me, I should repeat the question. You guys aren't holding me to that very well. Raise your hands if I don't repeat an important question.

She says, is it not translating? We've determined that-- let me ask you, does the center of mass move? Does Newton's second law apply to the motion of the center of mass?

AUDIENCE: [INAUDIBLE]?

PROFESSOR: Yeah, it's got to. So the center of mass translates-- no doubt about that. Newton's second law applies to it. So we're not saying that there isn't motion of the system in the x and y. We're just saying that that motion is caused by rotation. It's not caused by what is strictly defined as translation.

Free body diagram, the reason-- let's think about a free body diagram for this. Here's our rod. There must be-- I'll let it be frictionless to keep the problem simple for a moment. That means there must be just a normal force in the x direction here. I'll call it Nx. And there must be a normal force in the y, call it Ny. The center of mass, there must be an Mg.

So now that I set the problem up this way, how many unknowns are there in the problem? If I want to calculate literally the motion of this thing, find an equation of motion and solve it, how many unknowns do I have? How many do you think?

AUDIENCE: One.

PROFESSOR: I hear one. I see two on the board. But is two the right answer? I hear three. Who said three? All right, what's the third one?

AUDIENCE: Acceleration?

PROFESSOR: How do you describe acceleration with a coordinate of some kind? So there's yet another unknown. It's probably the thing that you're trying to solve for. It's actually the motion itself described by theta for x or y, whatever you do. There's at least three unknowns in this problem the way you see it in this free body diagram.

So you've got to figure out ways around that. This instantaneous center of rotation gives you one possible way around that. Because about an instantaneous center, it's not moving. It's an axis. It's not moving.

We have a little formula that says the time rate of change of angular momentum-- torque is related to the time rate of change of angular momentum. And you can have a messy formula or a not so messy. And it's not so messy when the axis of rotation is stationary.

So at this instant in time, the axis is stationary. You can say that the torques about this point, the ICR, summation of the torques with respect to the ICR, is equal to d, is now a rigid body with respect to the ICR, dt. OK, in this problem, what are the torques? Where do the torques come from? Is there any torque caused by Nx?

No, because there's no moment on it, right? It's pointing right at the center. Same thing-- no torque caused by this. You want to find equations that get rid of unknowns. So neither unknown appear in this equation.

Where does the torque come from? Gravity, right? And it's going to be some Mg times a moment arm. And the moment arm is going to be like that. So it's an L over 2 sine theta. And the sine you'll have to figure out from an r cross and f.

So you have an i cross j, gives you a k. But it's in the minus direction. So I think it'll come out minus. But I could be wrong. I did that on the fly. So we're not going to go further with this. But the instantaneous centers of rotation could be really handy.

OK, a final example in this stuff-- how are we doing on time? Couple of carts, so the floor constrains the motion. And I've got a spring and a dashpot and an M1 and an M2. And I want to figure out how many-- yeah?

AUDIENCE: I was just curious. Why is torque negative in your earlier solution?

PROFESSOR: Let's just figure out r cross f. So my r-- you're saying, why is the torque negative? The r is L/2 in the i hat. And the gravity crossed with the force, which is Mg in the-- but it's minus Mg in the j hat. So i cross j is positive k. But the minus sign comes from there.

AUDIENCE: What is that over to the left [INAUDIBLE]?

PROFESSOR: Well, I'm working on the center of mass here. That's my equation. And this thing is L long. So half of the length must be L/2. And I'm interested in this side of the right triangle.

AUDIENCE: Why are you interested in that side rather than the side that connects it to the--

PROFESSOR: Because this side crossed with that gives me 0. There's no moment. So this is a moment equation. I'm trying to compute moments. Yeah?

AUDIENCE: Should the moment come from the ICR load?

PROFESSOR: Oh, you're right. I don't know what I'm thinking. I could have messed this up. It's got to be about the ICR. So the force is down. Ooh, it's got to be this one.

I messed up-- good catch. So that's a cosine, still L/2. You've got a theta. You have a-- this is also theta. And we're looking now for this side. Eh, it's still sine theta, right? Does the sign still work out the right way?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Good, OK, I've got to keep rolling. I've got something else really fun I want to talk about. So let's do this example quickly. This is mostly to get you to think about free body diagrams. This-- two rigid bodies. The degrees of freedom quickly here-- 6 times 2. There's no particles-- minus c, so 12 minus c.

Now how many? What does your intuition tell you? How many independent coordinates is it going to take to solve this problem? Hold up your fingers. I see two, one, two, one. OK, one or two.

Well, I think you can find 10 constraints in this problem. If you assume it doesn't roll over and you assume it doesn't move, you can find 10. You're going to need two coordinates. Because just because you've got a spring and a dashpot, they don't fix. They don't say there's any particular relation between the motion of this and the motion of that. You're going to need an independent coordinate to describe the-- whoops, 2 times 6. This is 12 minus 10.

You don't need an independent coordinate to describe the motion of each of these masses. And I'd probably choose a coordinate that, let's say, goes from the center of mass of this one. I'll call it x1, center of mass of this one call it x2. And because I've been doing problems, vibration problems and stuff like this, for a long time, I'll tell you it's smart to start your coordinates at 0 when you have zero spring forces. Or from the static equilibrium position-- that's the good place to start.

So then your answer, if you're at the static equilibrium position, then any non-zero answers that come out of it are movement around that point. That's what mother nature does. A spring hanging on a mass hanging on a spring, it has a static equilibrium position. The vibration occurs around it. Your car is sitting on the ground. The vibration is around its static equilibrium position. So you usually choose these at the static equilibrium.

And you'll find that that makes for the simplest equations of motion. They tend to not have constants on the right hand side that's caused by gravity or offsets or things like that. So this thing, actually a problem very similar to this is on your homework. The homework says, go find what these 10 are. So you can name them pretty fast. That's why I'm not doing it for you.

What I do want to do quickly here is just talk about how you assign free body diagrams for this problem. Because people, one of the most easiest thing to get confused about is figuring out the directions of the forces that come from the springs in the dashpots, which direction acts on each.

So I've made my x's so that x1 is 0 when this thing is at its static equilibrium position. And I'm going to start by assuming that x1 and x1 dot are positive. You just establish positive motions. And you deduce the direction of the spring and dashpot forces.

OK, and you do them one at a time. The problems we're doing here are linear. Spring force is equal to kx. Dashpot forces are Bx dot. So they're linear problems. So the superposition holds. You can just do these conceptually one at a time and figure them out. And then you add them all together to get the complete answer.

So if x1 moves in the positive direction, what is the direction that that spring puts on this mass as a result? Which direction is it? And my coordinate system is positive. Here's x1.

We'll do the same thing here. Here's x2. So if it moves in the positive direction out to here, what is the spring going to do? No other motion allowed in the system, just that one. It moves a little bit. What does the spring do on that mass?

AUDIENCE: Push back.

PROFESSOR: Push back kx. And I let the sign be indicated by the direction of the arrow. And I'll use that when I write out my equation of motion. OK, now I'm going to assume a positive bx1, so its velocity in that direction. What does the dashpot do?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Resist or [INAUDIBLE]?

AUDIENCE: Resist.

PROFESSOR: Pushes back, right? OK, so you have another force here, bx1 dot. Now, this is if you have two bodies. You have to have two free body diagrams, one for each. But I need to know what's the effect of x2 and x2 dot on this body. Well, let's just go do the same thing. Let's let x2 be positive.

Now it's the only motion. I have a positive movement of x2 that stretches the spring. What is that spring force applied to this? What direction is it in? This body is moving in that way.

AUDIENCE: Positive x direction.

PROFESSOR: Which way is the spring going to pull on this thing? It's going to pull on it, right? So k, by amount kx2, and of positive velocity, to the right. It makes the dashpot open up. What direction is the force that dashpot puts on this?

AUDIENCE: Positive x.

PROFESSOR: bx2 dot. And now in this problem, except for gravity, I've got a normal force f and an Mg down. But all the action, all the motion, is in the horizontal direction. I can write out an equation of motion for the first rigid body here. And that's a sum of the forces in the x direction on body one-- I'll give this a 1 here-- in the x is equal to M1 x1 double dot. And now I can just write it out.

It is k x2, because that one's positive, minus x1 plus b x2 dot minus x1 dot. And that has the signs right. And the whole key is just one at a time assume positive motions and deduce what happens, and then use the arrows, the direction of the arrows, to set the signs. And now there's your equation of motion.

Now we can do the same thing, sum of the forces on 2 in the x 2 direction, M2 x2 double dot. And now we would do exactly the same thing. So positive motion of x1, what does it do over here? It gives me a force through the spring in which direction, positive or negative?

AUDIENCE: Positive.

PROFESSOR: Right, so now you just end up with a kx1, bx1 dot. And you'll find that kx2 bx2 dot. And you sum it up. And you'll end up-- this should switch around, k x1 minus x2 plus b x1 dot minus x2 dot.

I've got two equations of motion. And they're mixed. So each one has both coordinates in it. So this problem has two questions of motion, and they're coupled. They're not independent. You have to solve them together.

OK, now we're going to move on to a subject which has come up in conversation. People have asked about this lots of time. And they say, what about the centrifugal force? And you sometimes use the term "fictitious force."

How many of you use or heard the word used "fictitious force"? And how many of you heard us say that centripetal acceleration is not a force, it's an acceleration? And yet we love to talk about this concept of centrifugal force, which doesn't exist. But it trips us all up. Because it's handy to think about it. And so we're going to talk about fictitious forces now.

They're handy. But they are dangerous. You really have to understand your fundamentals if you're going to use the concept of fictitious forces without getting yourself in trouble. OK, what is a fictitious force?

Well, Newton's law, let's start there, Newton's second. Sum of the forces external on the body equals a mass times acceleration. It's a vector equation. So we can break it down into its components.

So a fictitious force, you take the true acceleration times the mass. So you have a fictitious force. It's going to multiply the true acceleration times the mass and put it on the summation of forces side of the equation.

That's really all it is. And you move it over to this side of the equation. And you think of it as a force. So what you've done is you've said that the summation of the forces minus Ma-- because to move it over here, you've got to subtract it from this side-- equals 0. And you're saying, I'm just going to think of this as a force. And it makes this whole equation be conveniently equal to 0.

But now, that's kind of abstract. Let's see if we can figure out an example or two to illustrate this. I'm sure you've done this problem in physics. I'm going to pick a really elementary problem so you don't get hung up on the physics to start with.

This is the elevator problem. You've got cables pulling it up. You've got some scales in here, and you're standing on it. Here's your center of mass. We'll call that A. And I need a coordinate system, my inertial system. Newton's law only applies in inertial systems. So here's my x. I called this z here. I'll call this y.

So I need to write an equation of motion about this person riding up in the elevator. We take a look at it. How many degrees of freedom? Intuitively obvious-- it's probably how many?

AUDIENCE: One.

PROFESSOR: One-- all sorts of constraints. It's only going up. So we're going to say we need one degree of freedom. Free body diagrams-- well, this object has some force pushing up on it. This is the person.

This is your mass. It has a force pulling down on it, mg. And that's it. Now this N is going to work out to be what the scales read, right? Because it's coming through the scales. So that's really what we're looking for.

And the problem here is, define the weight on the scales. So we say that the summation of the forces in the y direction-- and in this problem, we would say it's the mass times the acceleration of point A with respect to O. And the sum of those forces-- you've got an N minus mg. And that's a pretty simple equation.

Now I'm going to specify. It's given that acceleration of A with respect to O is 1/4 of g. So that's what the cables in the elevator are doing. It's making this thing move, and it's going up at 1/4 of g, the N acceleration. So it's getting faster and faster.

OK, so if I solve this for N, I will get-- let me stop there for a second. It's normally what I would just do. But since we're talking about fictitious forces, I need to go through that step for a second.

So now I say that, well, the summation of the forces in the y direction here minus M acceleration of A with respect to O, that total is equal to 0. And that then is N minus Mg minus M times g over 4.

And now I have taken this upward acceleration. And I've treated it like a force. I've just moved it to this other side, set the whole thing equal to 0. This is the fictitious force. I'm going to say, OK, that's all the forces in the system, solve for N. And of course you get N is M times g plus g/4. And that's 5/4 mg. And so you read 25% heavier.

It's a really trivial example. But the notion is that you think of this as a force that's been applied. It's the mass times acceleration with a minus sign in front of it. It'll always turn out like that. It's minus the mass times the acceleration.

Now, the acceleration can come from Coriolis acceleration. It can come from centripetal acceleration. So if we do this in a rotating thing, this fictitious force might be the centrifugal force, which is minus M times the centripetal acceleration. And we'll do an example like that. All right, stop-- there we go.

OK, trivial example-- let's see if we can find something a little harder. How are we doing on time? We're doing pretty good. So let's do an example where the notion is really quite powerful. Now, I showed you this last time. And we talked a lot about--

we did the time derivative is the angular momentum. And we computed the torques that this thing exerts around different axes with respect to the point of attachment to this.

Well, this is an example which, if you're comfortable with fictitious forces, you can figure out those torques really rather quickly. So I'm going to-- these two problems are identical. This problem, or with a shaft running like that, are really exactly identical. But I'm going to pretend that my thing is made this way. Because it makes it easier to see where these torques are coming from.

So here's my z. And I've got my rotation about the z-axis. Here's this mass, my coordinates. Here's my r hat. Here's my z k hat, is this distance here. And I'm going to set conditions in the problem, my rotation rate. It's also theta dot about the k z-axis. And omega dot, theta double dot-- also in the k. It's not restricted. So I'm allowing this thing to accelerate.

But r dot equals r double dot is 0. So it's not changing a position. It's just fixed. And I need to figure out the forces on this thing and talk about how we might consider some of them as fictitious forces.

So let's think about in the r direction-- summation of the forces here in this r hat direction. So it's one vector component. I don't have to carry along all of the other baggage. It's equal to-- and I'll call this B.

This is going to be the point about which I care about things. There's a fixed coordinate system here, Oxyz. But then a rotating coordinate system-- we'll call this A. And it's going to have my coordinates. I'll use polar coordinates, r theta z. But this one rotates. But it also has its origin right there coincident with O.

So the sum in the r direction, then, we have the mass times the acceleration of B with respect to O. And the acceleration of B with respect to O is the acceleration of-- we'll just write out the whole formula using cylindrical coordinates.

r double dot minus r theta dot squared, this is in the r hat, plus r theta double dot plus 2r dot theta dot in the theta hat direction. That's my full expression for--

whoops, not quite full. Forgot a term, right? There we go.

That's my full expression for acceleration using cylindrical coordinates, but with a moving, possibly translating and rotating reference frame. This piece here counts for what?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Acceleration of what? In general, why do we have that term?

AUDIENCE: [INAUDIBLE].

PROFESSOR: It's the translational acceleration of the A frame. So we can account for it. In this case, what is that? OK, so this term is 0. But this formula you really need to know. Because it accounts for all of the accelerations in the problem.

You know that, you just go in and start assigning, picking out things. What's this? 0. Is this 0? No, how about this one? Not 0-- I'm allowing it to accelerate. This one-- definitely 0. And this one? 0.

So I don't have many terms left in this. So this then is the mass times minus r theta dot squared. And this is r hat plus r theta double dot in the theta hat direction. Now I said I was summing the forces in the r. I put all of them in at the moment. It may be smarter to do that. Let me reverse course here.

This is the vector summation of all the forces for a moment. And then we'll take the r component next. So just the r component, the summation of the forces in the r, we look at this, and we say, oh, it's just that minus Mr theta dot squared r hat. So that's the r direction. But now we need a free body diagram. Yeah?

AUDIENCE: Why did you take the 2r theta [INAUDIBLE]?

PROFESSOR: Oh, because that's r dot 0. So in my r direction, this is my acceleration. That's the only acceleration in the r direction. It's what we know to be centripetal. And the minus tells us it's inward. Free body diagram-- need that.

So looking-- a side view. Here's your bead. I'm going to draw it as an unknown here. There is an unknown force in the r direction that comes from this bar holding it. It's applying-- there's got to be a force that makes this go in a circle. And that bar is what provides it.

It's the only thing [INAUDIBLE]. I'll just call this unknown and r. And I'm just drawing it in the positive direction. The way you can do this, if you're not sure the direction, draw it positive. And the sign that falls out tells you what the right answer is.

There'll be some force in the z provided by the rod. There'll be Mg. And that should be it. If we did a top view, then there'd be an unknown in the theta direction. You'd also see the N in the r direction. And anything else? No, the Mg is down where you can't see it. So these are your two free body diagrams.

So now, in this direction, this is the acceleration. And the external forces are just that. So the sum of the forces is Nr. Now I want to treat it, bring in this concept of a fictitious force. I'm going to move the acceleration term to the other side.

Nr-- unknown positive. Now I'm going to move that acceleration term over here minus-- ah, but now it's minus the acceleration. So this term looks like that, has a minus sign. If you move it over here, that actually becomes plus, equals 0.

Now this is your fictitious force. Sometimes people call them inertial forces. And it acts like it's pulling out on the object. So the force that the mass appears to apply to the rod is this centrifugal force pulling out.

And of course now we can solve for Nr. And we find it's minus Mr theta dot squared, which we knew all along. It's the force applied to the mass by the arm as it spins around. It has to pull in on it to make it go in that circle.

Now, we could do the same thing in the theta direction. The theta direction will have an Mr theta double dot. And when we look at the theta free body diagram, it's plus N theta. You could solve that, and you immediately come up with a solution for the force in the theta direction. I'm just going to write that one down. When you solve for this one, you get a minus sign. This one ends up plus, Mr theta double dot.

I'm not going to go through the gyrations of getting to this. You can do that one. Now, I want to do one quick thing with this. Once you develop confidence in knowing when you can use a fictitious force and not get in trouble, this is the sort of thing you might do.

Here's my system. It's rotating. And I want a quick estimate of, what's the torque? What's the bending moment about this point caused by the fact that it is the centripetal acceleration? Well, centripetal acceleration is equivalent to having this fictitious force outward on this of an amount Mr theta dot squared. And this is the moment arm z. what's the torque that that causes about this point? This is just levers now, forces and lever arms.

AUDIENCE: 0.

PROFESSOR: No, not 0. About this point here-- this is O. And I've got that centrifugal force pulling out on this fictitious force. Yeah?

AUDIENCE: zMR of theta dot squared?

PROFESSOR: Yeah, in the-- this direction, which is theta, right? So that's the moment the torque about this point caused by that-- torque at O is minus Mr theta dot squared times z, the moment arm. And it's not minus. It's in that direction. So it's plus. It's in the theta hat direction, the torque, that that force is applying about this point.

We could have solved this problem the way we did in the last lecture, very carefully going through the dh dt's and following it all out. And we would have gotten that answer for the torque except for a minus sign.

Now, why the difference? This is the torque that the centripetal force causes down here. When you do dh dt, you get the torque required to make what's happening happen. It's just the opposite.

This is the torque. This is putting about that. There must be an equal and opposite torque that this system puts on this arm out here to keep it from flopping out.

So this is applying a torque here. It must resist with a torque. And so when you do dh dt, you're going to end up with a minus Mr theta dot squared theta hat. So you've got to be careful what you mean. But from an engineering point of view, if I were just trying to calculate the bending moment down here and deciding whether or not this shaft sticking out here is going to break off or not, I could make a very quick estimate of the bending moment by knowing this centrifugal force times the moment arm. Yeah?

AUDIENCE: Is that supposed to be a z in the dh dt expression? Does there have to be a z in that expression?

PROFESSOR: A v, a velocity?

AUDIENCE: A z.

PROFESSOR: A z, yeah, right. I'm just getting a little speedy in writing the equation. Good catch. All right, so now, we also have a theta direction thing here, right? We can think of a fictitious force in the theta direction. And that was the Mr theta double dot term.

Does it generate torques that you could quickly compute? So as this thing is trying to accelerate in the positive acceleration, omega dot, theta double dot, it's trying to accelerate into the board. The bar is having to push that thing into the board.

There's a force in the positive theta hat direction. But the fictitious inertial force is the mass pushing back on the rod, pushing this way, Mr theta double dot pushing this way. What moments does that cause about this point?

AUDIENCE: [INAUDIBLE].

PROFESSOR: So there's actually two moment arms. If you have a force this way now, the force in, I'll call it, the theta direction is Mr theta double dot. Now, this is my fictitious force. This is this fictitious force. It's in the minus theta hat direction. It's pushing back. It's in the minus theta hat direction, Mr theta double dot.

And a torque is some R cross with the force. And this R is that, RBA. And it's composed of r R hat and z k hat here. So we have to do a little r r hat plus z k hat crossed with minus Mr theta double dot theta hat. So you get an r hat cross theta gives you a k.

A k cross theta hat gives you an r. You're going to get two terms out of this. One is going to look like Mr squared theta double dot. And that's a torque. And that is the one that it takes to-- that's the torque about this axis. There's a force times this moment arm. That's a torque about this axis.

That's the actual torque it takes to speed this thing up. And then there's a torque about this axis, which is this length times this force. And that's the other term. That'll give you the term in the R direction. It'll be a twist. It'll be trying to twist this bar like that. Because there's a force in this direction. Yeah.

AUDIENCE: It seems like these forces are coming from the systems. They aren't even fictitious.

PROFESSOR: So she's asking, it's like the forces are real. But the force-- I mean, this is why fictitious forces, this concept of them, is so dangerous. It's because it's really tempting to start thinking of them like real forces.

So any time you get stuck, you go back to the basics. And you say, Newton's law, F equals ma, you see torques is dh dt plus that v cross p term if you need it. And you work it out carefully using all the vector math.

And then you are dealing-- and then Coriolis, centripetal, Euler terms are only accelerations. You treat them as pure accelerations. That's all they are. Now, to cause accelerations, you need to apply forces. And we give these forces names. Because it's helpful conceptually to think about these things as forces sometimes. But they are not real forces.

But it's quick and easy. If you get too comfortable with them, they are great assets to your intuition. So I know immediately just looking at this thing, as soon as I see a machine that has a part that rotates like this, I say, rotational motion. That's central, circular motion, must be the equivalent of a centrifugal force.

This mass is going to pull in that direction. Because it's going around and around.

And it's going to cause a moment about this thing. I know for sure that's got to be there. And if I'm going to do it rigorously, I call it centripetal acceleration. And I compute. It's really the force that this bar puts on that mass to make it go in a circle.

But it's handy to think of it as a force if what I'm interested in is the force. The other even better-- let's go back to the old really simple demonstration. This thing going around constant speed, there's definitely a tension in the string. And I am pulling. The tension is inward, right? That tension causes an acceleration of r theta dot squared. And that acceleration is inward.

But it's easy. If you ask me, come on, Kim, quick, tell me, what's the tension in the string, I just say, that's easy-- Mr theta dot squared, centrifugal force. They're handy.

But any time you get stuck, go back and be really strict and say, what's acceleration? And what's real external forces? Gravity is an external force. The force that the bar puts on the mass, that's a real force. But these other things are just accelerations.

All right, OK, hey, perfect. I didn't get to the thing I really wanted to do. So I'll do it next time. And I'm going to show you this thing. And I want you to fill out your muddy cards. I'm giving you a couple minutes. So this is a mechanical shaker. You see it moving already. And we're going to talk about that next time.