MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF OCEAN ENGINEERING

DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

13.013J/1.053J Dynamics and Vibration

Fall 2002

Problem Set 5

Issued: Day 12
Due: 11am, Day 16

a) 4-59

b) 4-61

c) 5-5

d) 5-6

e) 5-7

f) Supplementary Problem 1, attached .The supplementary problem is intended to be solved by the entire class jointly.

g) Self-Evaluation in another sheet as per the instructions in the first class.

All students are supposed to work on all the problems assigned.

Supplementary Problem 1

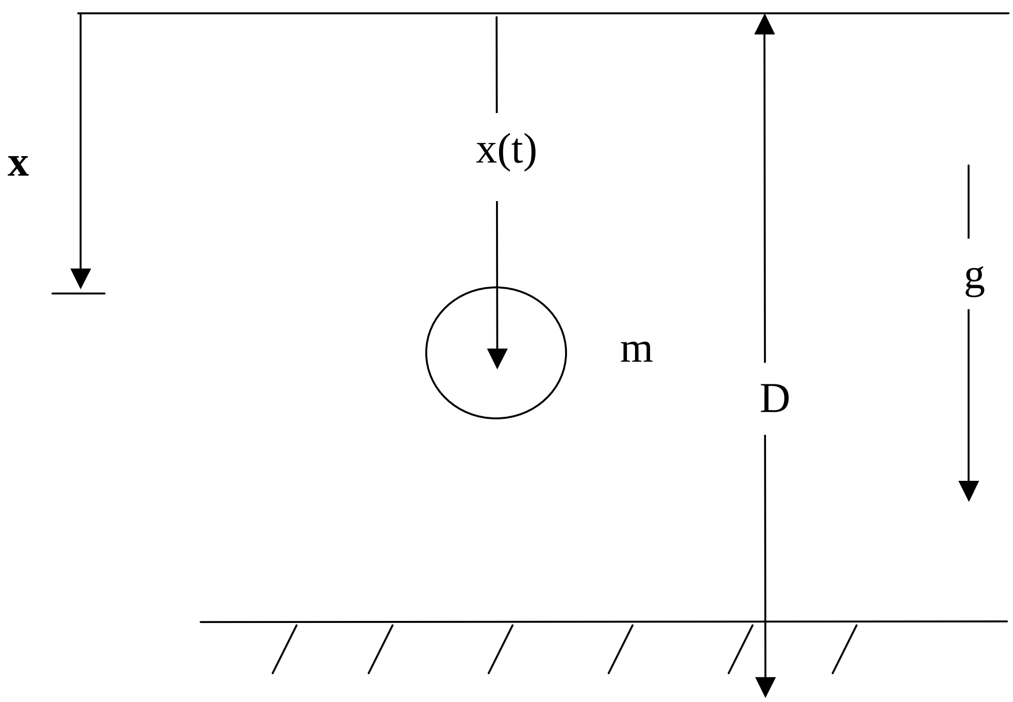

Figure 1

A rigid hollow negatively buoyant sphere of mass *m* with frontal area *A* and volume *V* is given an initial downward velocity v_i at the surface of the ocean of depth *D* and water density ρ_w. The fluid force on the sphere is given by

$$F_x(t) = -\frac{1}{2}\rho_w c_d A \dot{x}\left|\dot{x}\right| - c_m \rho_w V \ddot{x} - \rho_w g V$$

where c_d , c_m are drag and added mass coefficients which are taken as constants, and frontal area $A = \pi R^2$ and volume $V = 4\,\pi R^3 / 3$, *R* is the radius of the sphere and *x(t)* is the depth of the sphere at time *t*, and *g* is the acceleration of gravity. The drag coefficient c_d depends on the Reynolds number ($R_e = \frac{|\dot{x}|2R}{\upsilon}$) of the sphere, where υ = kinematic viscosity of the fluid. For Reynolds numbers above around the value of $1x10^6$, the drag coefficient c_d may be taken as a constant. See *Figure 2*. In this problem we assume that both c_d *and* c_m *are constants.*

a. Determine the equation of motion of the sphere and the initial conditions, needed in solving this equation. Assuming that the depth D is sufficient for the sphere to reach its terminal maximum speed, derive an expression for the terminal speed.

b. Solve the equation of motion and determine the velocity and position of the sphere as a function of time.

Hints: Set $v = \dot{x}$ and use

$$\int \frac{dv}{k^2 - v^2} = \frac{1}{2k} \ln \left| \frac{k+v}{k-v} \right|$$

or use Maple on Athena for the integrals. **Hint**: Note that k above will turn out to be the terminal speed of the sphere.

c. What will the velocity of the sphere be just prior to collision with the ocean bottom at depth *D*? **Hint** : Determine the time T of traversal of the entire water column first and use question (b)

d. Write down an expression for the work of the drag force on the sphere (ie. of first term of $F_x(t)$) during its traversal of the entire water column from $x=0$ to $x=D$.

e. Numerical example for questions (a) to (d).

$m = 1500\ kg$

$R = 0.5\ m$

Density of sea water (at about 15^o C) = $\rho_w = 1025\ kg/m^3$

Kinematic viscosity of sea water (at about 15^o C) = $\upsilon = 1.19x10^{-6}\ m^2/s$

Acceleration due to gravity = $g = 9.81\ m/s^2$

$V_i = 1\ m/s$

$D = 1000\ m$

c_d (for Re > Re initial $\cong 1x10^6$) = *0.18*

c_m (Ideal fluid assumption) = *0.5*

Using for example Matlab, Maple or Excel, plot

1. Variation of $x(t)$ against *t.*
2. Variation of $v(t)$ against *t.*
3. Variation of Inertial term = $(m+c_m\rho_w V)\ddot{x}$ in equation of motion against *t.*
4. Variation of Drag force against *t.*
5. What is the value of the effective weight $W_e = (m-\rho_w V)g$