1.053J/2.003J Dynamics and Control I
Fall 2006

Exam 2
20th November, 2006

Important Notes:

1. You are allowed to use two letter-size sheets (two-sides) of notes.
2. There are three problems on the exam. You have 80 minutes to solve them.
3. Each problem carries equal weight: 20 points.

1. Problem 1 (20 Points)

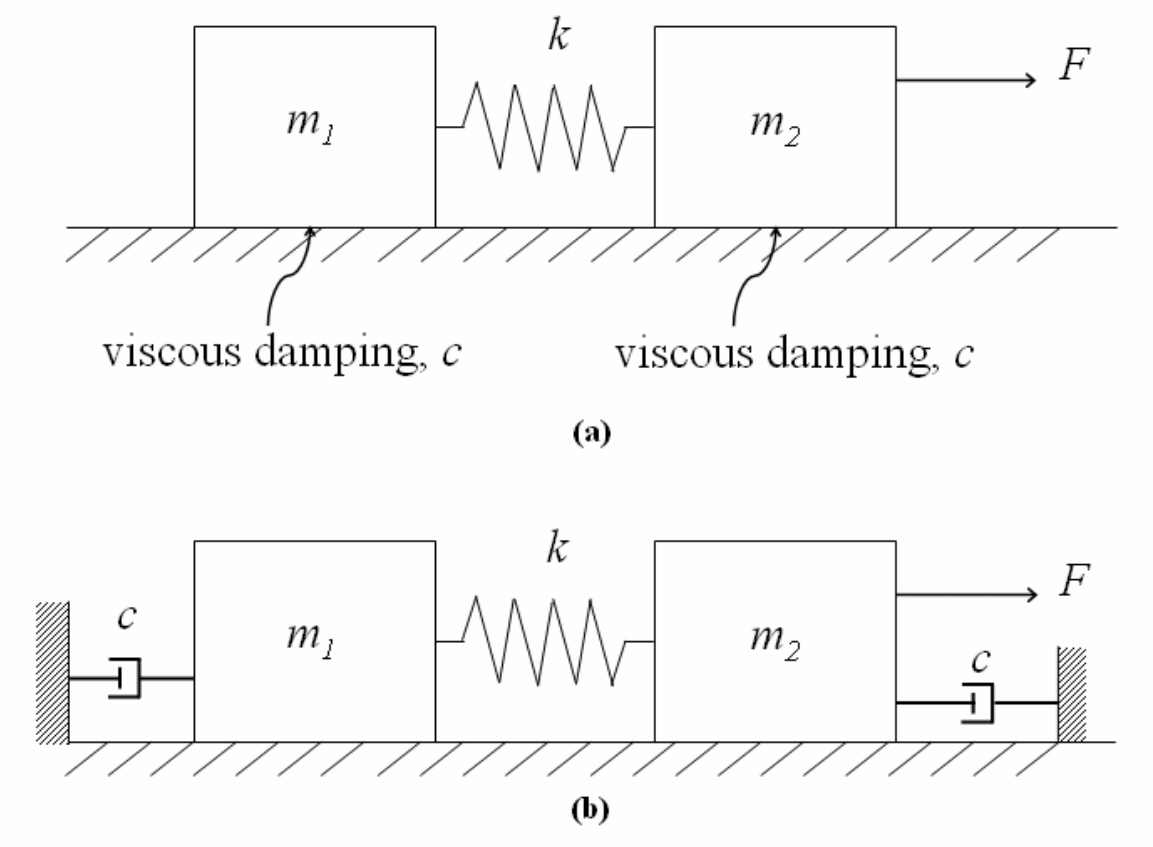

Figure 1

Two blocks of masses m_1 and m_2 are connected by a spring with a spring constant k and are free to slide on a horizontal surface as shown in Figure 1(a). The drag between each mass and the surface can be modeled as viscous damping with a dashpot constant c as shown in Figure 1(b). Initially the masses are at rest and the spring is un-stretched. At $t = 0$, a horizontal force F is applied to mass m_2. Derive the equation(s) of motion for this system.

2. Problem 2 (20 Points)

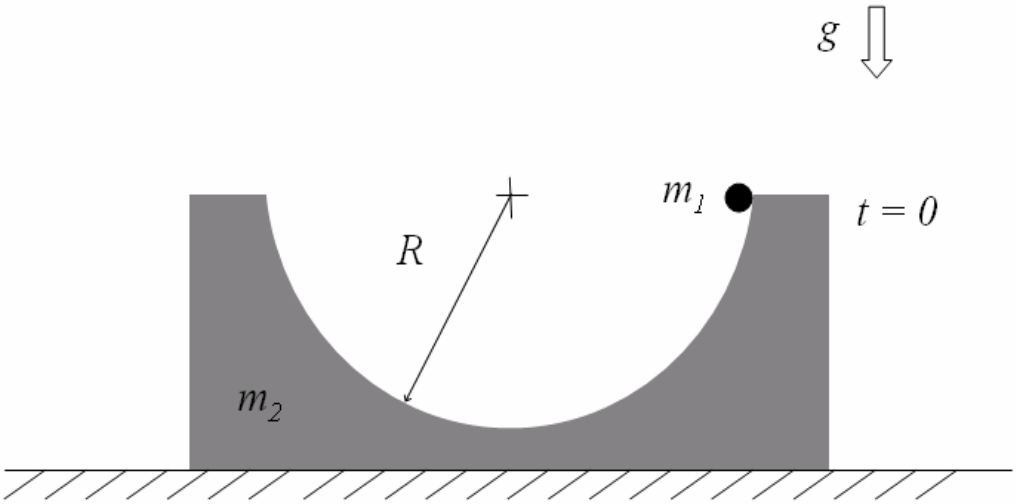

Figure 2

A block of mass m_2 with a semi-circular well of radius R is free to slide on a horizontal surface with no friction as shown in Figure 2. Before $t = 0$, the block is at rest and a point mass m_1 is held at the tip of the well. At $t = 0$, the point mass m_1 is released and both the masses are free to move thereafter. Derive the equation(s) of motion for this system.

Hint: Start by picking the generalized coordinates. List your steps.

3. Problem 3 (20 Points)

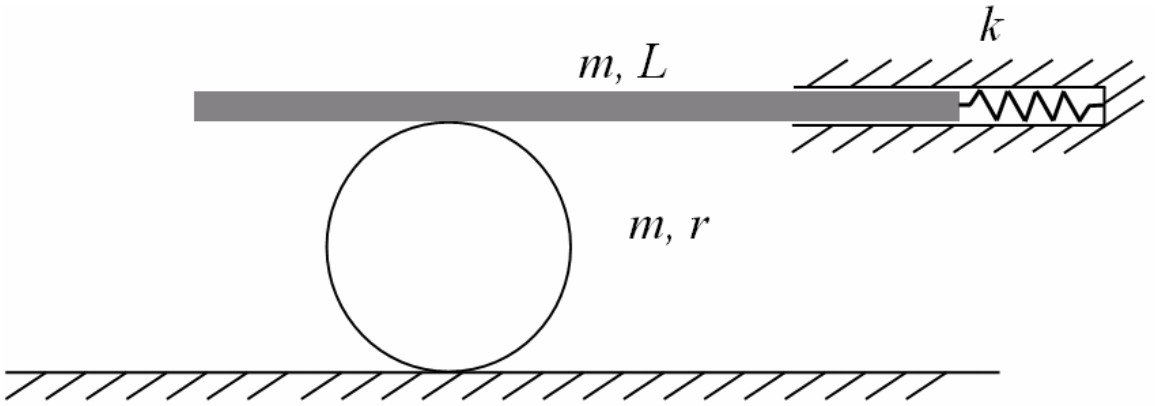

Figure 3

A disc of mass *m* and radius *r* is sandwiched between a horizontal surface and a slender rod of mass *m* which can slide in and out of a horizontal groove without friction as shown in Figure 3. The disc *rolls without slippage* with respect to both the horizontal bottom as well as the rod. The rod is connected to a spring of a spring constant *k* as shown in the figure, and the other end of the spring is attached to the inertial frame. Assume some length L_0 of the un-stretched spring – although that will turn out to be irrelevant to the problem. The rod is pulled out the groove such that the spring is stretched by some length *and* then released.

a. Show that at any instant the velocity of the rod with respect to a frame fixed to the ground is twice the velocity of the center of the disc with respect to the same frame.
b. Derive the equation(s) of motion for this system.