1.053J/2.003J Dynamics and Control I
Fall 2007

Exam 1
24th October, 2007

Important Notes:

1. You are allowed to use one letter-size sheet (two-sides) of notes.
2. There are three problems totaling to 100 points. You have 80 minutes to solve them.

1. A Robotic Arm (35 points)

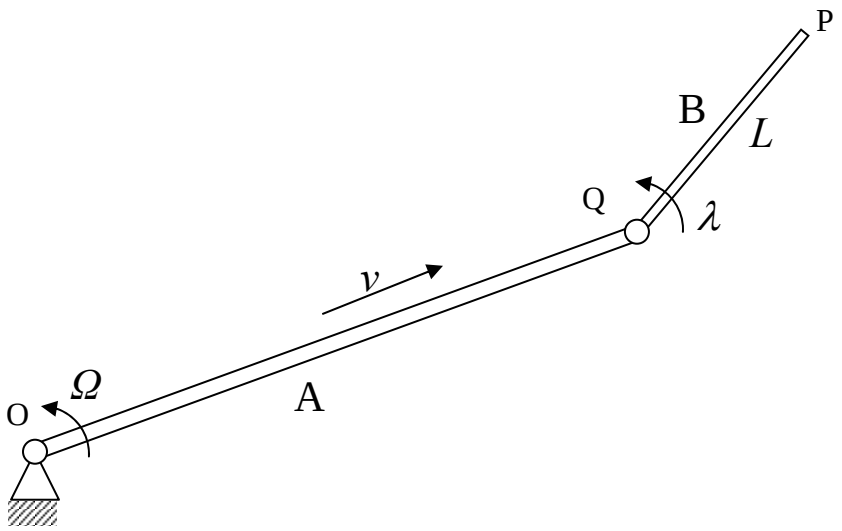

Figure 1

A robotic arm shown in the figure above consists of link A and link B. Link A is connected to a motor at point O and rotates at constant angular speed Ω. Link B is attached to the other end of link A, at point Q, and rotates at constant angular speed λ *with respect to link* A. Link A is also telescoping, *i.e.*, increasing in length at rate v from an initial length L. Link B is rigid and of constant length L.

A) Find the acceleration of point Q with respect to an observer on the ground. (20 points)
B) Find the acceleration of point P with respect to an observer on the ground. (15 points)

2. Wheels of a railway engine (35 points)

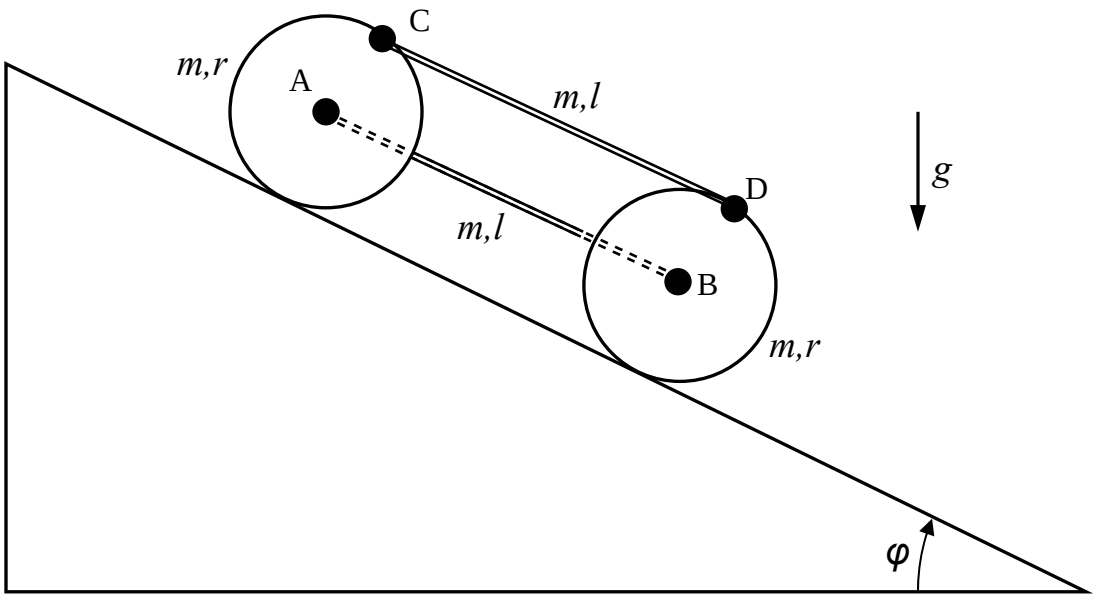

Figure 3

The wheels of a railway engine ride on axles attached to the chassis. Typically a second pinned strut, which links the wheels and keeps them rotating together, is used to distribute power. We will study the dynamics of such a system.

Axles A and B through centers of two identical rigid wheels, each of mass m and radius r, are connected by a thin rigid rod of mass m and length l. This models the chassis. In addition, as shown in the figure above, the wheels are also connected by another thin rigid rod of mass m and length l with pin joints at points C and D at the periphery of the wheels. The wheels are then allowed to roll without slipping on an inclined surface as shown. Note that gravity acts. Determine the equation(s) of motion of the system.

Hint #1: Use energy.

Hint #2: Does the link CD rotate at all?

3. Wheels with a spring (30 points).

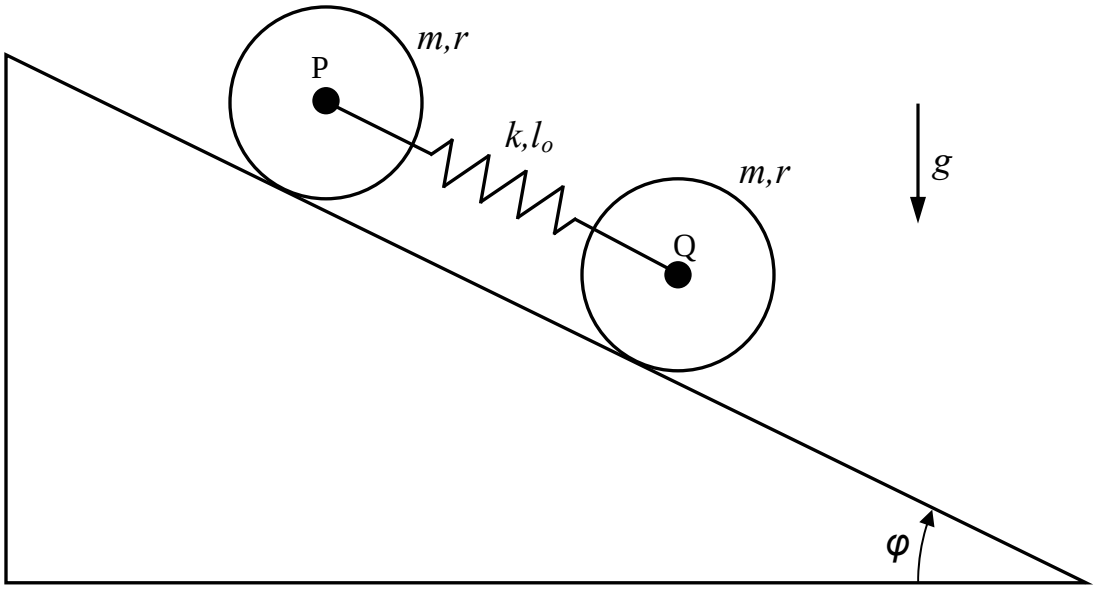

Figure 2

The axles of two identical rigid wheels are connected to each other through a mass-less spring of spring constant k as shown in the figure above. Each wheel has mass m and radius r. When not stretched, the length of the spring is l_o. Initially the system is placed on an inclined surface with the spring un-stretched. The system is set in motion and the wheels roll without slipping. Note that gravity acts.

A) How many degrees of freedom does the system have? Parameterize the system – in other words, what non-standard coordinates would you use? (10 points)
B) Determine the equations(s) of motion of the system. (20 points)