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2.002 MECHANICS AND MATERIALS II

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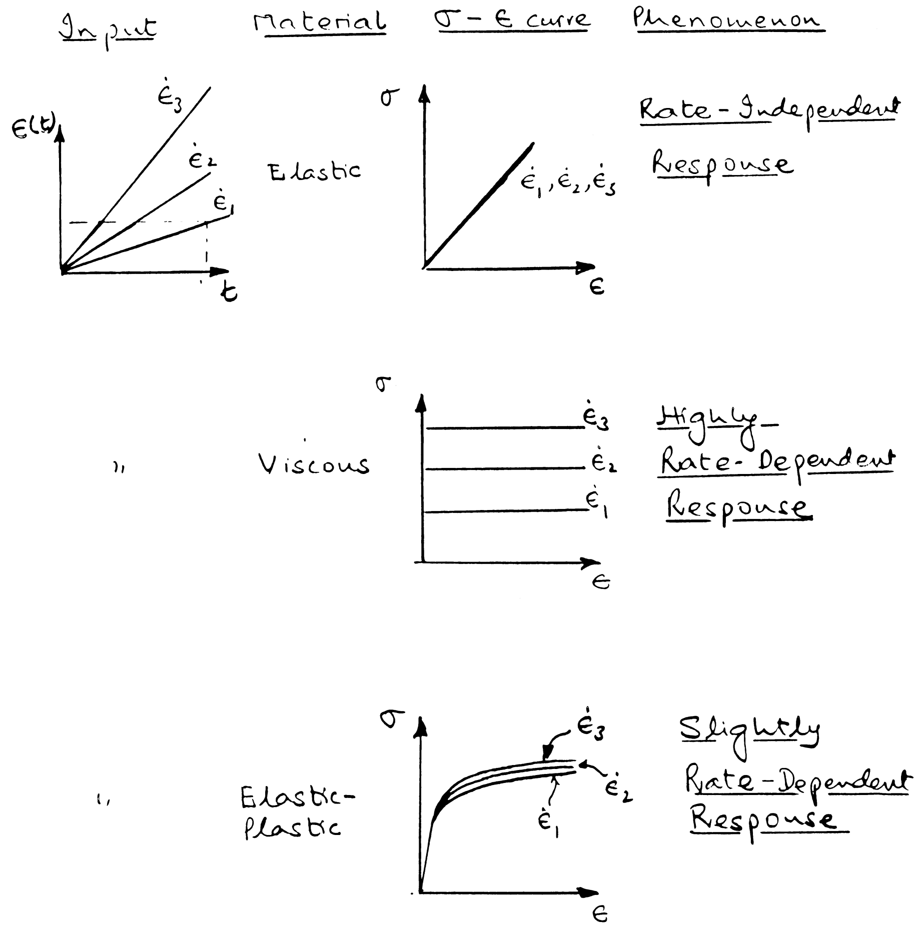
Creep and Creep Fracture: Part I

RATE-DEPENDENCE AND RATE-INDEPENDENCE OF PLASTIC RESPONSE



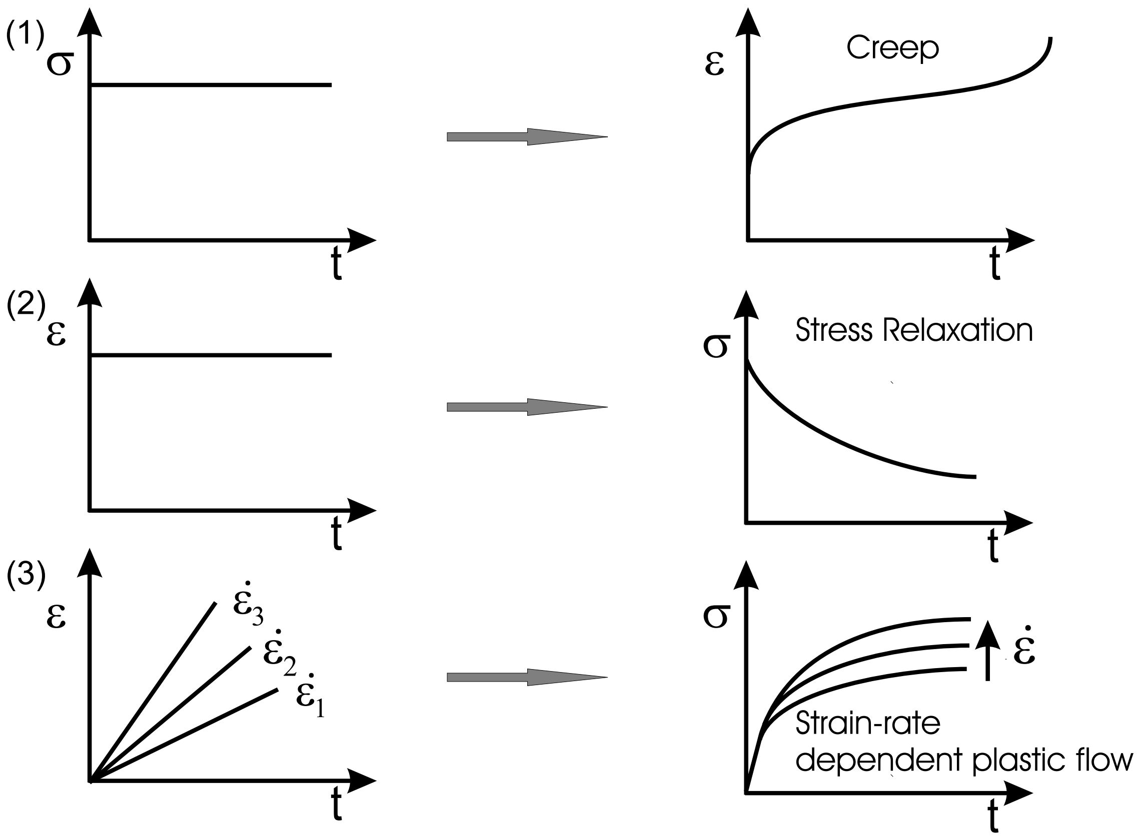
- Plastic deformation in metals is **thermally-activated** and **inherently rate-dependent**.

- However, the plastic stress-strain response of most single and polycrystalline materials at absolute temperatures $T < 0.35\,T_m$, where T_m is the melting temperature of the material in degrees absolute, is only slightly rate-sensitive, and in this temperature regime it is often be modeled as rate-independent. **We shall first consider a rate-independent theory.**

Material	Melting Temp, C	T_m, K	$0.35\,T_m$, K	$\equiv$ C
Ti	1668	1941	679	406
Fe	1536	1809	633	360
Cu	1083	1356	452	201
Al	660	933	327	54
Pb	327	660	231	-42

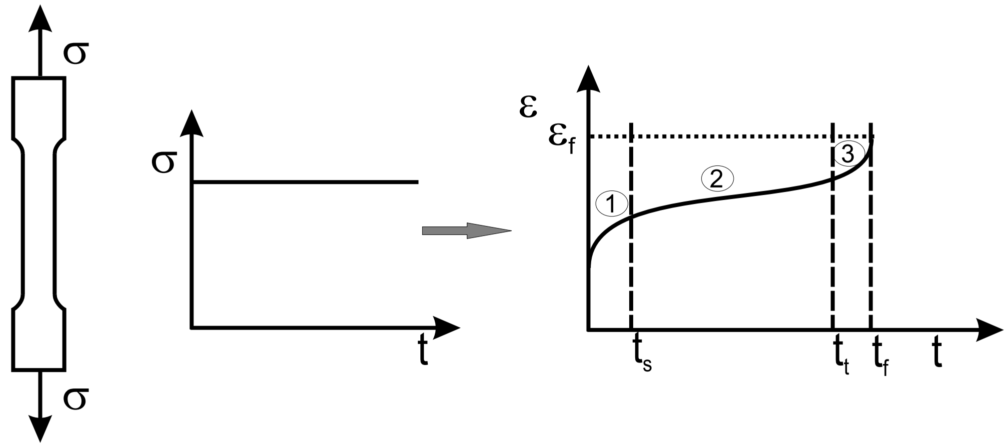
Consequences of Viscoplastic Deformation at High Homologous Temperature

For isothermal deformation (T=const)



Creep Test

- A typical creep test consists of instantaneously loading a cylindrical test specimen of a material to a constant stress, which is maintained at a constant temperature. The resulting strain is measured as a function of time.



t_s time to start of secondary creep

t_t time to start of tertiary creep

t_f time to fracture

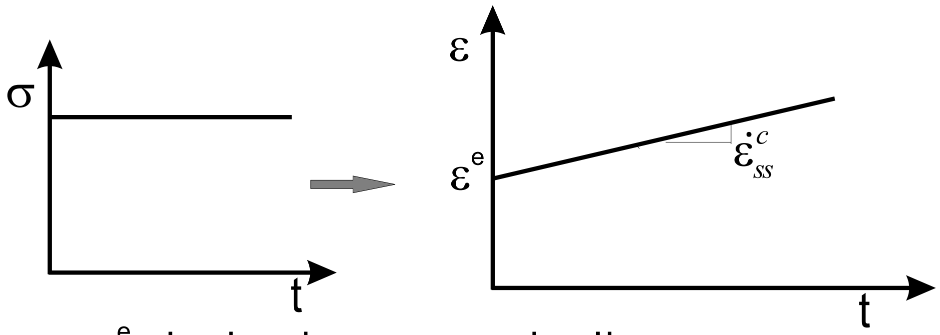
① Primary creep

② Secondary creep

③ Tertiary creep

Idealization of Creep Curve

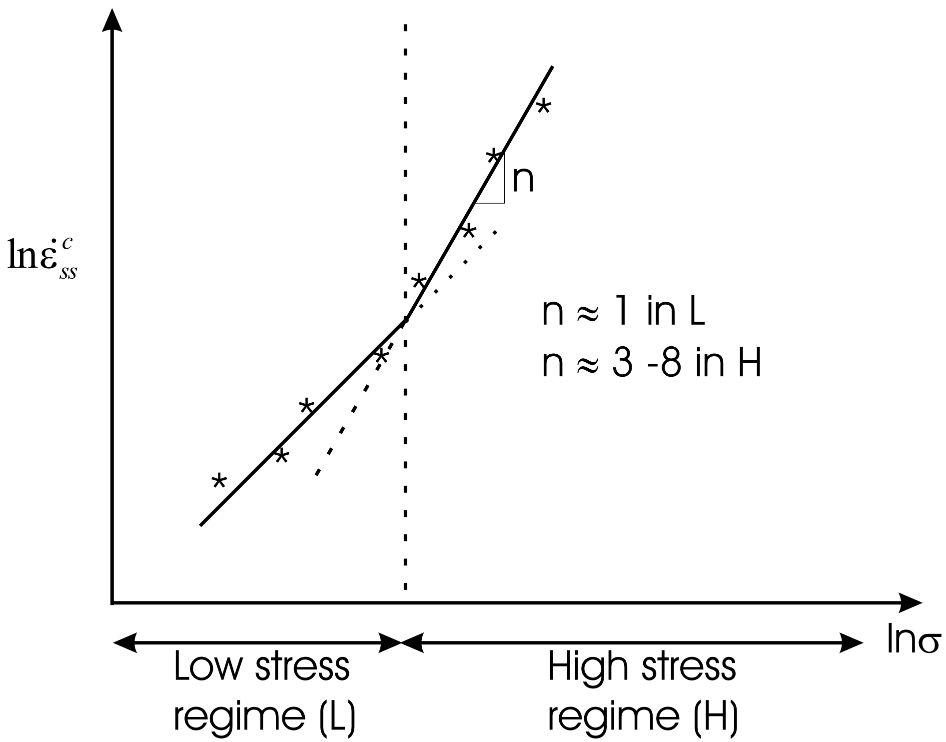
- For deformation analysis at constant temperature, the strain-time response may be idealized as



ε^e Instantaneous elastic response
$\dot{\varepsilon}_{ss}^{c}$ Steady-state creep strain rate

$$\begin{aligned} \epsilon &= \epsilon^e + \dot{\epsilon}_{ss}^c t \\ \dot{\epsilon} &= \dot{\epsilon}^e + \dot{\epsilon}_{ss}^c \\ \dot{\epsilon}^e &= \frac{\dot{\sigma}}{E} \end{aligned}$$

Stress Dependence of $\dot{\epsilon}_{ss}^{c}$ at Constant Temperature



$$\dot{\epsilon}_{ss}^{c} = \dot{\epsilon}_0 \left(\frac{\sigma}{s}\right)^n$$

Temperature-Dependence of $\dot{\epsilon}^c_{ss}$ at Constant σ - Preliminaries

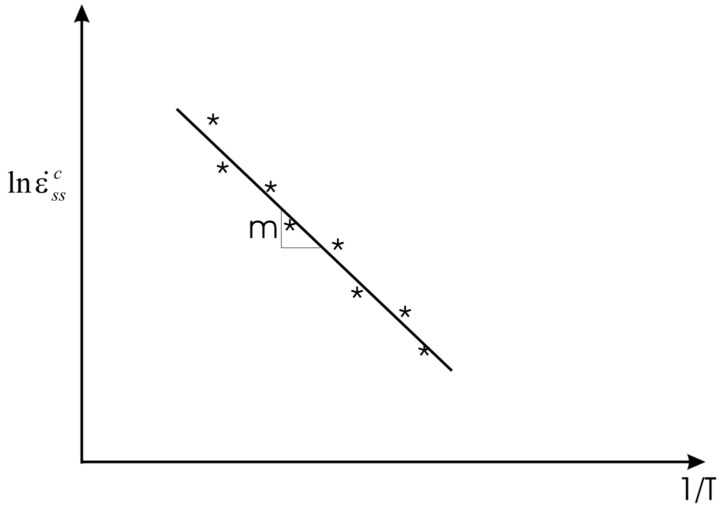
- Avogadro's number: $N_A = 6.022 \times 10^{23}$ atoms/molecules per mole

- Boltzmann's constant $k = 1.381 \times 10^{-23}\ J/K$

- Universal gas constant $R = N_A k = 8.314\ J(mol)^{-1}K^{-1}$

- Mole: 1 mole of any substance is that mass of the substance containing N_A atoms/molecules; e.g., the mass of 1 mole of C^{12} atoms is 12 grams.

Temperature Dependence of $\dot{\epsilon}_{ss}^{c}$ at Constant Stress



- The slope of the curve is $m/1 = -Q/R$, where Q is called the **activation energy for creep**, with units $J(mol)^{-1}$. Then for a constant $C(\sigma)$, the above curve can be mathematically represented as

$$ln\dot{\epsilon}_{ss}^{c} = lnC - \left(\frac{Q}{R}\right)\left(\frac{1}{T}\right) \Longleftrightarrow \dot{\epsilon}_{ss}^{c} = Cexp\left(-\frac{Q}{RT}\right)$$

An Important Observation

- Let us evaluate the increase in $\dot{\epsilon}^c_{ss}$ for a material with $Q = 270\,kJ/mol$ when the temperature is increased from $T_1 = 800^0\,C = 1073\,K$ to $T_2 = 820^0\,C = 1093\,K$.

$$\frac{\dot{\epsilon}^c_{ss1}}{\dot{\epsilon}^c_{ss2}} = exp\left\{\frac{Q}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)\right\} = 0.5746$$

- Therefore, with a temperature increase of only 20^0C, the creep rate almost doubled!!

- **Caution: the temperature T must be expressed in kelvins**

Combined Stress and Temperature Dependence of $\dot{\epsilon}^c_{ss}$

$$\dot{\epsilon}^c = \left\{ Aexp\left(-\frac{Q}{RT}\right)\right\} \left(\frac{\sigma}{s}\right)^n$$

A	pre-exponential factor (s^{-1})
Q	creep activation energy (J/mol)
n	creep exponent
s	reference stress which produces a strain rate $\dot{\epsilon}_0$

$\dot{\epsilon}_0 = Aexp\left(-\frac{Q}{RT}\right)$

Summary of One-Dimensional Creep Equation

$$\dot{\epsilon} = \dot{\epsilon}^e + \dot{\epsilon}^c \quad (1)$$

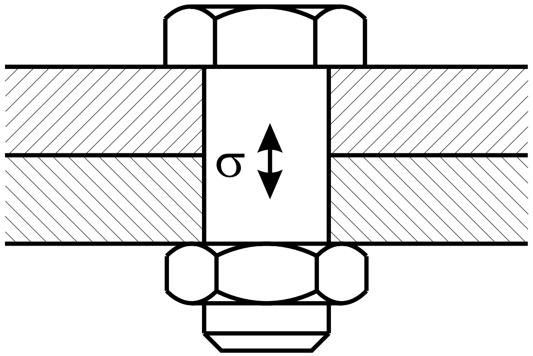
$$\dot{\epsilon}^e = \frac{\dot{\sigma}}{E} \quad ; \ E = E(T) \quad (2)$$

$$\dot{\epsilon}^c = \left\{ Aexp\left(-\frac{Q}{RT}\right)\right\}\left(\frac{\sigma}{s}\right)^n \quad (3)$$

- Note that equation (3) states that the rate of creep (or ‘viscoplastic’) strain increases exponentially with temperature, so that the time required for a given amount of creep strain decreases exponentially with temperature.

Example Problem: Stress Relaxation

- Consider a bolt with pre-tension $\sigma = \sigma_i$ at time $t = 0$. Given that the bolt is maintained at constant temperature, determine the pre-tension at some time t. The isothermal constitutive equation for steady state creep is given by $\dot{\epsilon}^c = B\sigma^n$ with $n \neq 1$



Example Problem: Stress Relaxation (cont.)

$$
\begin{aligned}
\dot{\epsilon} &= \dot{\epsilon}^e + \dot{\epsilon}^c \\
\Rightarrow 0 &= \dot{\epsilon}^e + \dot{\epsilon}^c \qquad \text{since } \epsilon = const \text{ in the bolt} \\
\Rightarrow 0 &= \frac{\dot{\sigma}}{E} + B\sigma^n \Rightarrow \frac{1}{E}\frac{d\sigma}{dt} = -B\sigma^n \\
\Rightarrow \sigma^{-n} d\sigma &= -EB\, dt \Rightarrow \int_{\sigma_i=\sigma(0)}^{\sigma(t)} \sigma^{-n} d\sigma = -EB \int_0^t dt \\
\Rightarrow \sigma(t)^{-(n-1)} - \sigma_i^{-(n-1)} &= (n-1)EBt \\
\Rightarrow \sigma(t) &= \frac{\sigma_i}{\left[1 + (n-1)\, t\, B\sigma_i^n\, (E/\sigma_i)\right]^{1/(n-1)}}
\end{aligned}
$$

- Defining the characteristic relaxation time t_r such that $\sigma(t_r) = \sigma_i/2$, we get

$$t_r = \frac{2^{(n-1)} - 1}{(n-1)EB\sigma_i^{(n-1)}} = \frac{2^{(n-1)} - 1}{(n-1)\dot{\epsilon}_i^c/(\sigma_i/E)}$$