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2.002 MECHANICS AND MATERIALS II
PRACTICE QUIZ I

Distributed: Wednesday, March 10, 2004

This quiz consists of four (4) questions. A brief summary of each question's content and associated points is given below:

1. (10 points) This is the credit for your (up to) two (2) pages of self-prepared notes. **Please be sure to put your name on each sheet**, and hand it in with the test booklet. You are already done with this one!

2. (20 points) A multi-part, lab-based question.

3. (40 points) A question about thermal loading of a laminated plate.

4. (30 points) Three short-answer questions.

Note: you are encouraged to write out your understanding of the problem, and of "what to do" in order to solve the problem, even if you find yourself having "algebraic/numerical difficulties" in actually doing so: let me see what you are <u>thinking</u>, instead of just what you happen to write down....

The last page of the quiz contains "useful" information. Please refer to this page for equations, etc., as needed.

If you have any questions about the quiz, please ask for clarification.

Good luck!

Problem 1 (10 points)

Attach your self-prepared 2-sheet (4-page) notes/outline to the quiz booklet.

Be sure your name is on each sheet.

Problem 2 (20 points)

(Lab-Based Problem)

In your own words, describe the following terms used in the description of linear viscoelastic behavior, and briefly illustrate (with schematic figures, simple equations, etc.) how these features are used, measured, or are otherwise identified:

- (4 points) **stress relaxation function**

- (4 points) **creep function**

- (4 points) **correspondence principle**

- (4 points) **mechanical damping**

(4 points) Broadly describe the essential molecular structure and major features of the mechanical behavior of an **amorphous polymer** (1 page, maximum).

Problem 3 (40 points)

A large 3-layer laminated plate is constructed by bonding identical layers of aluminum (AL) on the top and bottom of a plate of polycarbonate (PC), as shown in Figure 1. The outer layers of AL have thickness $t_{(AL)} = 0.25\,mm$, while the central layer has thickness $t_{(PC)} = 4\,mm$, for a total plate thickness $t = t_{(PC)} + 2t_{(AL)} = 4.5\,mm$.

Material properties for each material are as follows: for AL, $E^{(AL)} = 72\,GPa$; $\nu^{(AL)} = 0.3$, and $\alpha^{(AL)} = 23 \times 10^{-6}/^\circ K$. For the PC, $E^{(PC)} = 2.5\,GPa$; $\nu^{(PC)} = 0.4$, and $\alpha^{(PC)} = 65 \times 10^{-6}/^\circ K$.

The cartesian coordinate directions 1 and 2 lie in the plane of the plate, with the origin of the x_3 coordinate axis located at the mid-thickness of the plate. The in-plane dimensions of the plate are large compared to thickness t. After bonding, at room temperature, all stress and strain components are initially zero throughout the plate.

Then, the temperature in the laminated plate is <u>uniformly</u> increased by an amount $\Delta T = 50^\circ C$, without application of external loading.
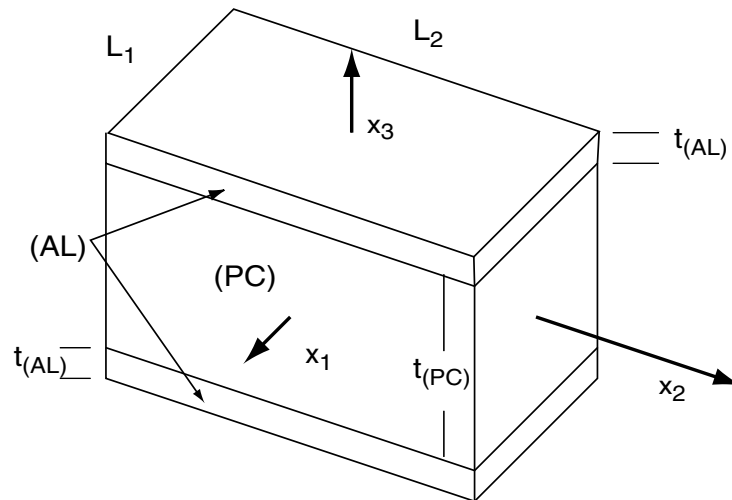

Figure 1: Schematic of a rectangular block cut from the central portion of the 3-layer laminate.

- (**5 points**) Briefly explain why you should expect each material layer in the central portion of the laminated plate to be in a state of **plane stress**, with due regard for the **traction boundary conditions** on the top and bottom surfaces of the plate (at $x_3 = \pm t/2$).

- (**5 points**) Assume that the total strain within each layer is piecewise constant; that is, $\epsilon_{ij} = \epsilon_{ij}^{(AL)}$ in both of the outer layers, but $\epsilon_{ij} = \epsilon_{ij}^{(PC)}$ in the central layer, where

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$\epsilon_{ij}^{(AL)}$ and $\epsilon_{ij}^{(PC)}$ are respective sets of constants (to be determined). Explain why it is necessary for $\epsilon_{11}^{(AL)} = \epsilon_{11}^{(PC)}$, $\epsilon_{22}^{(AL)} = \epsilon_{22}^{(PC)}$, and $\epsilon_{12}^{(AL)} = \epsilon_{12}^{(PC)}$.

- (**5 points**) With constant temperature change ΔT in all layers, and constant total strain within each layer, explain why the stress components must be constant within respective layers; i.e., $\sigma_{ij} = \sigma_{ij}^{(AL)}$ in the outer layers, and $\sigma_{ij} = \sigma_{ij}^{(PC)}$ in the central layer, again for to-be-determined sets of constants $\sigma_{ij}^{(AL)}$ and $\sigma_{ij}^{(PC)}$.

- (**5 points**) Construct a free body diagram of small rectangular region [mathematically!] cut from the central portion of the large plate, of full thickness t, and exposing a rectangular interior surface of length L_2 perpendicular to the x_1 axis, and a similar surface of length L_1 perpendicular to the x_2 axis, as shown in Figure 1. Assuming that $\epsilon_{12}^{(AL)} = \epsilon_{12}^{(PC)} = 0$, show that **force equilibrium** in the x_1 direction requires that

$$0 = \int_{-t/2}^{t/2} \sigma_{11}(x_3)\, dx_3 = 2t_{(AL)}\, \sigma_{11}^{(AL)} + t_{(PC)}\, \sigma_{11}^{(PC)},$$

with a similar relation holding for the x_2 component of force balance.

- (**15 points**) Assuming that the total strain components $\epsilon_{11} = \epsilon_{22} \equiv \epsilon_0$ throughout the plate, derive symbolic equations that let you solve for values of ϵ_0, $\epsilon_{33}^{(AL)}$, $\epsilon_{33}^{(PC)}$, and $\sigma_{11}^{(AL)}$ and $\sigma_{22}^{(AL)}$, as well as $\sigma_{11}^{(PC)}$ and $\sigma_{22}^{(PC)}$. Provide numerical values (including units) for all quantities.

- (**5 points**) By how much does the total thickness of the plate change (in mm), in response to the temperature change?

Problem 4 (30 points)

- **(10 points)** A large shaft of diameter D passes through two bearings, within which the shaft rotates. At one end of the shaft, beyond the nearest bearing, power is transmitted to the shaft through a large, belt-driven pulley. In this region, the shaft diameter is to be reduced to a value "d" $(d < D)$ in order to fit the end of the shaft onto the central hole in the pulley (hole diameter: $\sim d$). Briefly discuss design features and details that will help keep stress suitably small in the shaft.

- **(10 points)** In a certain two-dimensional region, the following polynomial expression has been proposed to describe the spatial variation of the in-plane components of the strain field:

$$\epsilon_{11}(x_1, x_2) = \epsilon_{11}^0 + \frac{x_1^2 x_2^2}{\ell^4};$$

$$\epsilon_{22}(x_1, x_2) = \epsilon_{22}^0 + \frac{x_1^2 x_2^2}{\ell^4};$$

$$\epsilon_{12}(x_1, x_2) = \epsilon_{12}^0 + \frac{x_1^3 x_2 + x_1 x_2^3}{3\ell^4},$$

where ϵ_{ij}^0 are dimensionless constants, and ℓ is a length-dimensioned constant. Is it possible for this polynomial expression to be exact? Explain why or why not.

- **(10 points)** The mechanical behavior of by far the majority of engineering structures is analyzed using linear elasticity. List three possible types of phenomena that might limit the applicability of a linear elastic analysis. (Hint: consider the "three pillars" of solid mechanics, and how each is used within linear elasticity.)

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Isotropic Linear Thermal-Elasticity
(Cartesian Coordinates)

Stress/Strain/Temperature-Change Relations:

$$\epsilon_{ij} = \alpha \Delta T \, \delta_{ij} + \frac{1}{E} \left[(1 + \nu) \, \sigma_{ij} - \nu \left(\sum_{k=1}^{3} \sigma_{kk} \right) \delta_{ij} \right].$$

$$\sigma_{ij} = \frac{E}{(1 + \nu)} \left[\epsilon_{ij} + \frac{\nu}{(1 - 2\nu)} \left(\sum_{m=1}^{3} \epsilon_{mm} \right) \delta_{ij} - \frac{(1 + \nu)}{(1 - 2\nu)} \alpha \Delta T \, \delta_{ij} \right].$$

Strain-displacement Relations:

$$\epsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right).$$

Equilibrium equations (with body force and acceleration):

$$\sum_{j=1}^{3} \frac{\partial \sigma_{ij}}{\partial x_j} + \rho b_i = \rho \frac{\partial^2 u_i}{\partial t^2}$$

Mises Stress Measure:

$$\bar{\sigma} \equiv \sqrt{\frac{1}{2} \left[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 \right] + 3 \left[\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2 \right]}.$$

Traction on elemental area with unit normal "n":

$$t_i(\mathbf{n}) = \sum_{j=1}^{3} \sigma_{ij} \, n_j.$$