

### 1.3.2 An open problem about spike models

**Open Problem 1.3 (Spike Model for cut–SDP [MS15]. As since been solved [MS15])** Let  $W$  denote a symmetric Wigner matrix with i.i.d. entries  $W_{ij} \sim \mathcal{N}(0, 1)$ . Also, given  $B \in \mathbb{R}^{n \times n}$  symmetric, define:

$$Q(B) = \max \{ \text{Tr}(BX) : X \succeq 0, X_{ii} = 1 \}.$$

Define  $q(\xi)$  as

$$q(\xi) = \lim_{n \rightarrow \infty} \frac{1}{n} \mathbb{E} Q \left( \frac{\xi}{n} \mathbf{1}\mathbf{1}^T + \frac{1}{\sqrt{n}} W \right).$$

What is the value of  $\xi_*$ , defined as

$$\xi_* = \inf \{ \xi \geq 0 : q(\xi) > 2 \}.$$

It is known that, if  $0 \leq \xi \leq 1$ ,  $q(\xi) = 2$  [MS15].

One can show that  $\frac{1}{n} Q(B) \leq \lambda_{\max}(B)$ . In fact,

$$\max \{ \text{Tr}(BX) : X \succeq 0, X_{ii} = 1 \} \leq \max \{ \text{Tr}(BX) : X \succeq 0, \text{Tr} X = n \}.$$

It is also not difficult to show (hint: take the spectral decomposition of  $X$ ) that

$$\max \left\{ \text{Tr}(BX) : X \succeq 0, \sum_{i=1}^n X_{ii} = n \right\} = \lambda_{\max}(B).$$

This means that for  $\xi > 1$ ,  $q(\xi) \leq \xi + \frac{1}{\xi}$ .

## Tghgt gpeg

[MS15] A. Montanari and S. Sen. Semidefinite programs on sparse random graphs. *Available online at arXiv:1504.05910 [cs.DM]*, 2015.

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