Coloring knots

A *knot* is a smooth embedding of a circle into $\mathbb{R}^3$. Two knots are *equivalent* if one can be deformed to the other smoothly. The *knot class* of a given knot is the set of knots equivalent to it.

There is a combinatorial way of studying knot equivalence, by taking a knot and projecting to the plane $\mathbb{R}^2$, in such a way that the resulting *knot projection* has only simple undercrossings. Knots in the same class can have many different projections, but any two are related by a sequence of the following *Reidemeister moves*:

I II III

You should convince yourself that these moves do indeed allow you to move between any two projections of equivalent knots.

The maximal paths not containing any undercrossings are the *arcs* of the knot projection. A *coloring* of the projection is a function from the set of arcs to the set {red, green, blue}, with the property that, at each crossing, the three arcs meeting there have the same color, or else have all different colors. A coloring is *nontrivial* if it involves more than just one color. Show that admitting a nontrivial coloring is a knot invariant.

Give some examples: prove that some knots are not equivalent to each other by this method. A main part of the project should be to explore the following question: how does the notion of coloring generalize?