HOMEWORK #9, DUE THURSDAY MAY 9TH

1. Herstein, Chapter 4, §6, 2.
2. Herstein, Chapter 4, §6, 3.
3. Herstein, Chapter 4, §6, 4.
4. Herstein, Chapter 4, §6, 5.
5. Let F be a field and ϕ an automorphism of $F[x]$ such that $\phi(a) = a$ for every $a \in F$.
(i) If $f(x) \in F[x]$ prove that $f(x)$ is irreducible in $F[x]$ if and only if $g(x) = \phi(f(x))$ is irreducible.
(ii) Prove that if $f \in F[x]$ then $\deg \phi(f) = \deg f$.
6. Let F be a field, $b \neq 0$, c elements of F. Define a function

$$\phi\colon F[x] \longrightarrow F[x] \qquad \text{by} \qquad \phi(f(x)) = f(bx + c).$$

for every $f(x) \in F[x]$. Prove that ϕ is automorphism of $F[x]$ such that $\phi(a) = a$ for every $a \in F$.
7. Let ϕ be an automorphism of $F[x]$ such that $\phi(a) = a$ for every $a \in F$. Prove that there exists $b \neq 0$, c, such that $\phi(f(x)) = f(bx + c)$ for every $f(x) \in F[x]$.
8. (i) Find an automorphism of $\mathbb{Q}[x]$, not equal to the identity, such that ϕ^2 is equal to the identity.
(ii) Given any integer $n > 0$, exhibit an automorphism ϕ of $\mathbb{C}[x]$ of order n.
9. (i) If F is a field of characteristic $p \neq 0$, show that

$$(a + b)^p = a^p + b^p,$$

for all a, $b \in F$.
(ii) If F is a field of characteristic $p \neq 0$, show that the map

$$\phi\colon F \longrightarrow F \qquad \text{given by} \qquad \phi(a) = a^q$$

is a ring homomorphism, where $q = p^n$ is a power of p.
(iii) Show that ϕ is injective.
(iv) If F is a finite field show that ϕ is an automorphism.
Challenge Problem: 10. Give an example of a field F of characteristic p such that $\phi(a) = a^p$ is not surjective.

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