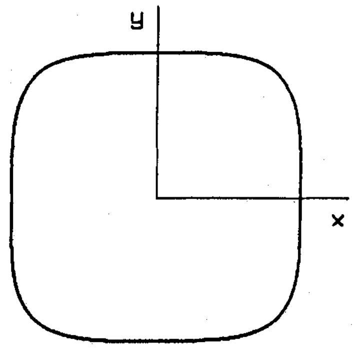
18.330 Problem Set 3 Due in class: Fri 27 Feb 04

(7) For the **pseudosquare** $x^4 + y^4 = 1$ pictured on the right, calculate

(a) the area A, and

(b) the circumference S

to our usual exquisite accuracy.



(8) (a) To a commendable final accuracy, practice **Romberg** extrapolations on the trapezoidal-rule estimates $T_1, T_2, T_4, T_8 \ldots$ for

$$\int_0^\pi \frac{\sin x}{x}\, dx \quad .$$

(b) Likewise exercise the 3, 5, 7 and 9-point closed **Newton-Cotes** formulas whose coefficients are given overleaf. Compare their errors with those of Romberg at the same orders of accuracy.

(c) Finally, polish off this integral also via **Taylor** series.

(9) For the more challenging integral $\int_0^\pi \sqrt{\sin x}\, dx$:

(a) Examine carefully the manner in which similar trapezoidal estimates $T_1, T_2, T_4, T_8 \ldots$ seem to be converging toward their eventual limit, and then speed them along to at least 6-decimal accuracy via some variants of **Aitken or Richardson**.

(b) Startle yourself by examining how fast even the humble old trapezoidal rule manages to evaluate this integral after we rephrase things slightly via $x = (\pi/2)(1 - \cos\theta)$.

Comparison of Romberg and Closed NC Weights and Errors

Given f(x) at		x = 0	1	2	3	4	5	6	7	8 h:		Too large by:
2nd order:	2pt NC											
	T_1	4								4	$\times\ h$	64
	T_2	2				4				2		16
	T_4	1		2		2		2		1		4
Trapezoidal	T_8	0.5	1	1	1	1	1	1	1	0.5		$1 \times \frac{2}{3} h^3 f''(\xi)$
4th order:	3pt NC											
	T_{12}	4				16				4	$\times\ \frac{h}{3}$	256
	T_{24}	2		8		4		8		2		16
Simpson	T_{48}	1	4	2	4	2	4	2	4	1		$1 \times \frac{2}{45} h^5 f^{iv}$
6th order:	5pt NC											
	T_{124}	7		32		12		32		7	$\times\ \frac{4h}{45}$	64
2× Extrap.	T_{248}	3.5	16	6	16	7	16	6	16	3.5		$1 \times \frac{16}{945} h^7 f^{vi}$
8th order:	7pt NC (with altered x's)											
		41	216		27	272	27		216	41	$\times\ \frac{h}{105}$	$\left(\frac{4}{3}\right)^9 \cdot \frac{9}{1400} h^9 f^{viii}$
3× Extrap.	T_{1248}	217	1024	352	1024	436	1024	352	1024	217	$\times\ \frac{4h}{2835}$	$1 \times \frac{128}{315} h^9 f^{viii}$
10th order:	9pt NC											
		989	5888	-928	10496	-4540	10496	-928	5888	989	$\times\ \frac{4h}{14175}$	$\frac{2368}{467775} h^{11} f^{x}$