18.306 Advanced Partial Differential Equations with Applications
Fall 2009

Lecture 18 2009 11 09 MON

TOPICS: Eikonal. Amplitude and curvature along rays. Behavior
near caustic. Caustic expansion.
WKBJ review. Turning points. Conneccion formulas and
Airy functions. Matching.

Equation for amplitude along rays (constant coefficients wave equation):

dA/dt + kappa*A = 0, where kappa = laplacian Phi = curvature.

Explain:

Why kappa is curvature.

How to compute kappa along rays.

kappa behaves like 1/(t0 -t), so the amplitude blows up at the caustic.

Caustic expansion:

Use coordinate system where one of the coordinates is distance from the caustic, and the other's coordinate lines are the normals to the caustic.

Strech differently in the two directions to match wave front shape (epsilon and epsilon^2/3).

Can produce description of solution near caustic, analogous to turning points in WKBJ. The two waves on one side, none in the other, given by the Airy function.

WKBJ, Turning point expansion, Airy.

WKBJ for problem y'' + (V(x)/epsilon^2) y = 0.

Oscillating and exponential solutions.

Amplitude blow up at turning points [V(x) = 0].

Turning point expansion [epsilon^2/3 layer].

Airy functions.

Behavior of the Airy function at +/- infinity.

Matching with WKBJ.

Show amplitude is epsilon^{-1/6} at turning point.

Note expansions overlap:

WKBJ valid for |x| >> epsilon^2 if turning point x = 0

--- wave vector is size \sqrt{|x|}/epsilon.

Turning point valid for |x| small.