18.306 Advanced Partial Differential Equations with Applications
Fall 2009

Lecture 08 2009 10 05 MON

TOPICS: More on envelopes. Infinite slopes at envelope.
Shocks. Conservation and entropy. Irreversibility.
Examples from traffic flow.

Continue with c_t + c*c_x = 0 and c(x, 0) = C(x).
Show alternative definition of envelope of a smooth family of curves:
Curve such that each point belongs to a family member, and is tangent
to the member here.
Hence: characteristics are tangent to the boundary of multiple values.
Generic drawing of multiple valued region now justified.

Back to conservation form: \rho_t + q_x = 0.

Introduce shocks to knock out multiple-valued regions.
Now pde + discontinuities satisfying some conditions:
Rankine-Hugoniot jump conditions (conservation)
Lax entropy conditions (causality)
System is now IRREVERSIBLE (show how information is lost at shocks).

Simple examples in Traffic Flow:
Red light turns green (show how Lax Entropy crucial here).
Green light turns red.
Green-red
Red-green
Meaning and (qualitative) comparison with reality.

Generic prescription for shocks forming out of a smooth solution.