PDE Examples Sheet

Problem 1. Prove that a Harmonic function with an interior maximum is constant.

Problem 2. Write out the laplacian in plane-polar coordinates.

Problem 3. A Green's function on $\mathbb{R}^n$ is a harmonic function on $\mathbb{R}^n \setminus \{0\}$ which depends only on the radius (for example $\log r$ on $\mathbb{R}^2$). Find non-trivial Green's functions for all dimensions.

Problem 4. The heat equation for a function $u : \mathbb{R} \times [0, \infty)$ is $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$. Find all solutions of the form $u = f(t)g(x)$.

Problem 5. Find all solutions u of the heat equation on $[0, 1] \times [0, \infty)$ with the $u = 0$ on $(\{0\} \cup \{1\}) \times [0, \infty)$.