18.112 Functions of a Complex Variable

Fall 2008

Lecture 23: The Extension of $\zeta(s)$ to the whole plane and the Functional Equation

(Text 214-217)

See Riemann's Collected Works p. 146.
In Theorem 10 we consider the function $z \to (-z)^{s-1}$ with z outside the positive real axis R^+. Angles are measured from the positive real axis from $-\pi$ to $+\pi$. Consider contour C.

If z is on the upper part of the cut R^+, $-z$ is below the negative real axis so

$$\arg(-z) = -\pi \text{ so } (-z)^{s-1} = x^{s-1}e^{-(s-1)\pi i}.$$

If z is on the lower par of the cut R^+ then $-z$ is above the negative real axis so $\arg(-z) = +\pi$ so $(-z)^{s-1} = x^{s-1}e^{(s-1)i\pi}$.