Problem Set 7, revised

0. Hand in your hour test with all the problems corrected, including more careful or efficient presentation of problems that you already did correctly. (You can improve your score up to half way to 100 from your present score.)

1. Let $f \in L^1(\mathbb{R}/2\pi\mathbb{Z})$ and denote its Fourier coefficients by

$$\hat{f}(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-inx}\, dx$$

Prove that

a) f is even if and only if $\hat{f}(n) = \hat{f}(-n)$ for all n

b) f is odd if and only if $\hat{f}(n) = -\hat{f}(-n)$ for all n

c) f is real-valued if and only if $\hat{f}(n) = \overline{\hat{f}(-n)}$ for all n.

2. Compute the Fourier coefficients of the following functions. Note which symmetries of Problem 1 hold and express the series both in terms of complex exponentials and in terms of sine or cosine functions where appropriate. What does Parseval's formula tell us in each case?

a) $f(x) = x$, $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$

b) $g(x) = |x|$, $-\pi < x < \pi$ and $g(x + 2\pi) = g(x)$

c) $h(x) = f(x + \pi)$

3. A series $\displaystyle\sum_{n=0}^{\infty} a_n$ is called Cesaro summable if the Cesaro means

$$\sigma_N = (s_0 + \cdots + s_{N-1})/N$$

of the partial sums $s_N = \displaystyle\sum_{0}^{N} a_n$ converge. Show that if s_N converges then σ_N converges to the same limit. Give an example showing that the converse is false.

18.103 Fourier Analysis
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