WRITING ASSIGNMENT 2, 18.100C

Goal. Most mathematics writing is done for other mathematicians. In this assignment, you will present your work on an interesting analysis problem to your peers.

Assignment. Choose one of problems 1, 2 or 3. Write a 2–4 page solution to the problem in which you not only present a proof but also introduce and motivate the problem. If space allows, include interesting examples and extensions of your work.

Your audience is a fellow 18.100 student, though you should imagine that they have no familiarity with the problem. Feel free to assume facility with all the material covered thus far in class. Carefully explain the logic of your solution so that your work is both comprehensible and interesting. Use guiding text to help your audience understand your argument.

You are encouraged to collaborate on these problems, but please cite any peers, instructors, or resources you use when working on your problem.

Technical details. Write your paper in LaTeX in the amsart document class, 11-point font. Do not manually alter any of the formatting, but do use theorem and proof environments to help with the readability of your document.

Submit both .tex and .pdf files . Your grade will be based on clarity of exposition, mathematical correctness, and readability of LaTeX in the final version of your paper. The assignment is worth 35 points.

Note. This writing assignment is due two days after Exam 1, but the problems are review problems for the exam. I recommend preparing a draft of your paper before the exam as it should be good preparation for the test.

Questions.

(1) (modified from question 8 in the recitation worksheet) Suppose we have a subset $E \subset [0, 1]$ which has countably many limit points. Then is it true that E is countable?

(2) (Rudin p. 45 ex. 23, also question 11 in the worksheet) A collection $\{V_\alpha\}$ of open subsets of X is said to be a base for X if the following is true: For every $x \in X$ and every open set $G \subset X$ such that $x \in G$, we have $x \in \{V_\alpha\}$ for some α. In other words, every open set in X is the union of a sub collection of $\{V_\alpha\}$.

Prove that every separable metric space has a countable base.

(3) (question 18 in the worksheet) If $x \in E$ is a limit point of E, is it also an interior point of E? Connected to this: what does it mean for a metric space to have only one element?