18.100C Lecture 3 Summary

Corollary 3.1. *For every real number $x > 0$ there is a natural number n such that $\frac{1}{n} < x$.*

Corollary 3.2. *For every real number x there is an integer n such that $x < n \leq x + 1$.*

Corollary 3.3. *For any real numbers $x < y$ there is a rational number q such that $x < q < y$.*

Definition of decimal expansion

$$0.9999\ldots \sup\{0, 0.9, 0.99, 0.999, \ldots\}$$

Theorem 3.4. $0.9999\cdots = 1$.

Theorem 3.5. *Let $I_1 \supset I_2 \supset I_3 \cdots$ be nonempty closed intervals, $I_k = [a_k, b_k]$. Then*

$$\bigcap_{k=1}^{\infty} I_k \neq \emptyset.$$

Corollary 3.6. $\mathbb{R}$ *is uncountable.*

Definition of complex numbers and their usual operations.

Theorem 3.7. *(Cauchy-Schwarz) For complex numbers $z_1, \ldots, z_k, w_1, \ldots, w_k$,*

$$|z_1\bar{w}_1 + \cdots + z_k\bar{w}_k|^2 \leq (|z_1|^2 + \cdots + |z_k|^2)(|w_1|^2 + \cdots + |w_k|^2).$$