

## 18.100C Lecture 3 Summary

**Corollary 3.1.** For every real number  $x > 0$  there is a natural number  $n$  such that  $\frac{1}{n} < x$ .

**Corollary 3.2.** For every real number  $x$  there is an integer  $n$  such that  $x < n \leq x + 1$ .

**Corollary 3.3.** For any real numbers  $x < y$  there is a rational number  $q$  such that  $x < q < y$ .

Definition of decimal expansion

$$0.9999 \dots \sup\{0, 0.9, 0.99, 0.999, \dots\}$$

**Theorem 3.4.**  $0.9999 \dots = 1$ .

**Theorem 3.5.** Let  $I_1 \supset I_2 \supset I_3 \dots$  be nonempty closed intervals,  $I_k = [a_k, b_k]$ . Then

$$\bigcap_{k=1}^{\infty} I_k \neq \emptyset.$$

**Corollary 3.6.**  $\mathbb{R}$  is uncountable.

Definition of complex numbers and their usual operations.

**Theorem 3.7.** (Cauchy-Schwarz) For complex numbers  $z_1, \dots, z_k, w_1, \dots, w_k$ ,

$$|z_1 \bar{w}_1 + \dots + z_k \bar{w}_k|^2 \leq (|z_1|^2 + \dots + |z_k|^2)(|w_1|^2 + \dots + |w_k|^2).$$

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