18.075 Practice Test 1 for Exam 3

November 23, 2004

Justify your answers. Cross out what is not meant to be part of your solution.

Total number of points: 75.

I. 1. (5 pts) Find the region of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(x-1)^n}{(n+1)^n}.$$

2. (5 pts) Find the region of convergence of the series

$$\sum_{n=0}^{\infty} \frac{3^n}{2^n+n} x^{3n}.$$

II. 1. (6 pts) Locate <u>all</u> singularities of the ODE

$$(1-\cos x)y'' + (\sin x)\, y' + y = 0.$$

2. (4 pts) Classify the point $x_0 = 0$ for the ODE of part (1).

III. Classify the point $x_0 = 0$ for the following ODEs:

1. (5 pts)

$$y'' - (\ln x)\, y' + y = 0.$$

2. (5 pts)

$$(\sin\sqrt{x})\, y'' + \sqrt{x}\, y' - y = 0.$$

IV. Consider the ODE

$$x^2 y'' - 3x\, y' + (3 - x^2)y = 0.$$

We seek solutions of this ODE around $x_0 = 0$ by the method of Frobenius, i.e., $y = x^s \sum_{k=0}^{\infty} A_k x^k$.

1. (5 pts) Write the ODE in the canonical form

$$R(x)y'' + \frac{1}{x}\, P(x)y' + \frac{1}{x^2}\, Q(x)y = 0.$$

2. (7 pts) Find the exponent(s) s by solving the indicial equation.

3. (13 pts) Write down the nonzero functions $g_n(s)$ from the Frobenius theory. Derive the recurrence formula for A_k.

4. (15 pts) How many independent solutions can you find in this way? Solve the recurrence relation and find the (general) coefficient A_k.

5. (5 pts) Can you write down a general form of the solution $y(x)$? Explain.