18.06 Quiz 3 May 8, 2010 Professor Strang

Your PRINTED name is: ______________________________ **1.**

Your recitation number is ____________________ **2.**

3.

1. **(40 points)** Suppose u is a unit vector in R^n, so $u^T u = 1$. This problem is about the n by n symmetric matrix $H = I - 2u\,u^T$.

   (a) Show directly that $H^2 = I$. Since $H = H^T$, we now know that H is not only symmetric but also ____________________.

   (b) One eigenvector of H is u itself. Find the corresponding eigenvalue.

   (c) If v is any vector perpendicular to u, show that v is an eigenvector of H and **find the eigenvalue.** With all these eigenvectors v, that eigenvalue must be repeated how many times? Is H **diagonalizable**? Why or why not?

   (d) Find the diagonal entries H_{11} and H_{ii} in terms of $u_1, \ldots, u_n$. Add up $H_{11} + \ldots + H_{nn}$ and separately add up the eigenvalues of H.

2. **(30 points)** Suppose A is a positive definite symmetric n by n matrix.

(a) How do you know that A^{-1} is also positive definite? (We know A^{-1} is symmetric. I just had an e-mail from the International Monetary Fund with this question.)

(b) Suppose Q is any **orthogonal** n by n matrix. How do you know that $Q\,A\,Q^T = Q\,A\,Q^{-1}$ is positive definite? Write down which test you are using.

(c) Show that the block matrix

$$B = \begin{bmatrix} A & A \\ A & A \end{bmatrix}$$

is positive **semidefinite**. How do you know B is not positive definite?

3. **(30 points)** This question is about the matrix

$$A = \begin{bmatrix} 0 & -1 \\ 4 & 0 \end{bmatrix}$$

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(a) Find its eigenvalues and eigenvectors.

Write the vector $u(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ as a combination of those eigenvectors.

(b) Solve the equation $\dfrac{du}{dt} = Au$ starting with the same vector $u(0)$ at time $t = 0$.

In other words: the solution $u(t)$ is what combination of the eigenvectors of A?

(c) Find the 3 matrices in the Singular Value Decomposition $A = U\,\Sigma\,V^T$ in two steps.

–First, compute V and Σ using the matrix $A^T A$.

–Second, find the (orthonormal) columns of U.