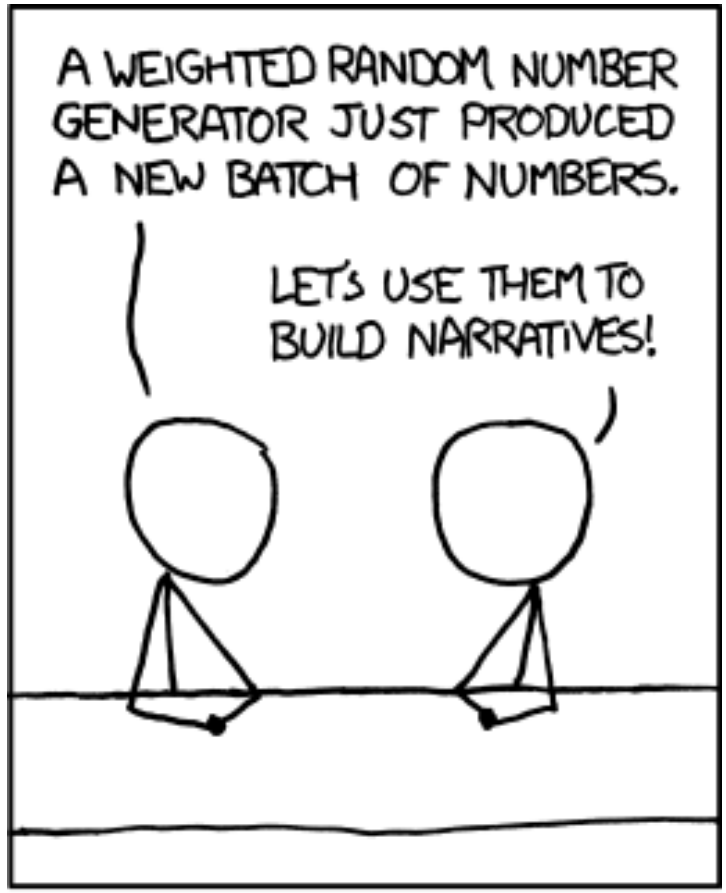
Welcome to 18.05
Introduction to Probability and Statistics
Spring 2014



http://xkcd.com/904/

R

- Free open source package.
- Very easy to use and install.
- Instructions and a link for this are on MITx/18.05r.

Platonic Dice

Probability vs. Statistics

Different subjects: both about random processes

Probability

- Logically self-contained
- A few rules for computing probabilities
- One correct answer

Statistics

- Messier and more of an art
- Get experimental data and try to draw probabilistic conclusions
- No single correct answer

Counting: Motivating Examples

What is the probability of getting exactly 1 heads in 3 tosses of a fair coin?

Poker Hands

Deck of 52 cards

- 13 *ranks*: 2, 3, . . . , 9, 10, J, Q, K, A
- 4 *suits*: ♡, ♠, ♢, ♣,

Poker hands

- Consists of 5 cards
- A *one-pair* hand consists of two cards having one rank and the remaining three cards having three other rank
- Example: $\{2\heartsuit, 2\spadesuit, 5\heartsuit, 8\clubsuit, K\diamondsuit\}$

The probability of a one-pair hand is:
(1) less than 5%
(2) between 5% and 10%
(3) between 10% and 20%
(4) between 20% and 40%
(5) greater than 40%

Sets in Words

Old New England rule: don't eat clams (or any shellfish) in months without an 'r' in their name.

- S = all months
- L = the month has 31 days
- R = the month has an 'r' in its name

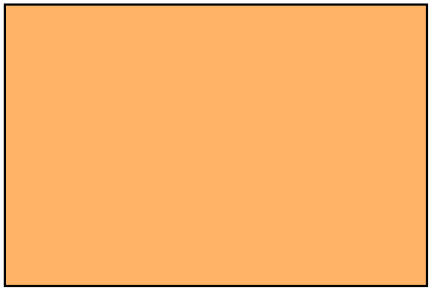
$S = \{\text{Jan, Feb, Mar, Apr, May, Jun, Jul, Aug, Sep, Oct, Nov, Dec}\}$

$L = \{\text{Jan, Mar, May, Jul, Aug, Oct, Dec}\}$

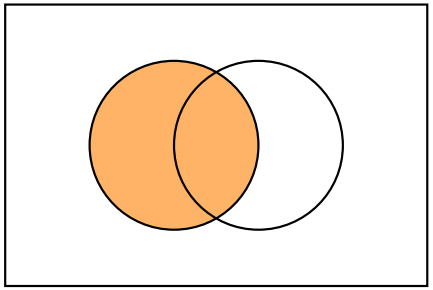
$R = \{\text{Jan, Feb, Mar, Apr, Sep, Oct, Nov, Dec}\}$

$L \cap R = \{\text{Jan, Mar, Oct, Dec}\} = \text{months with 31 days and an 'r'}$

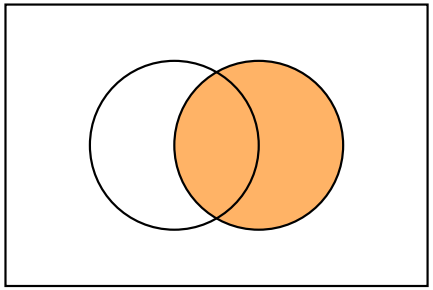
Visualize Set Operations with Venn Diagrams



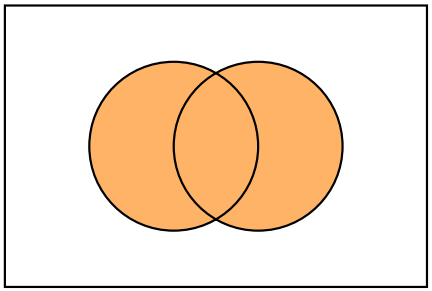
S



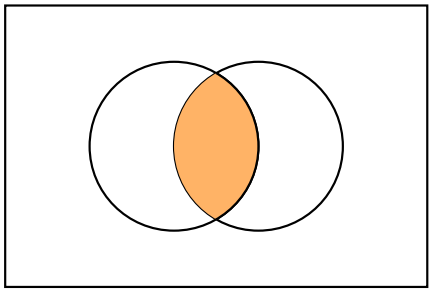
L



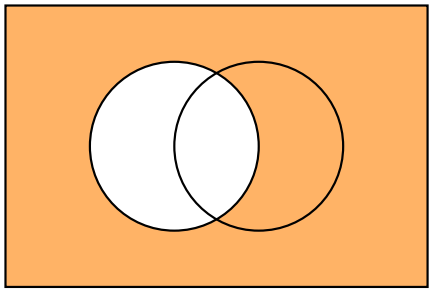
R



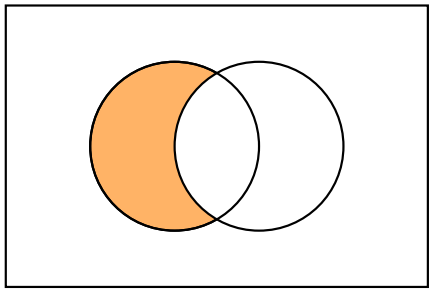
$L \cup R$



$L \cap R$



L^c

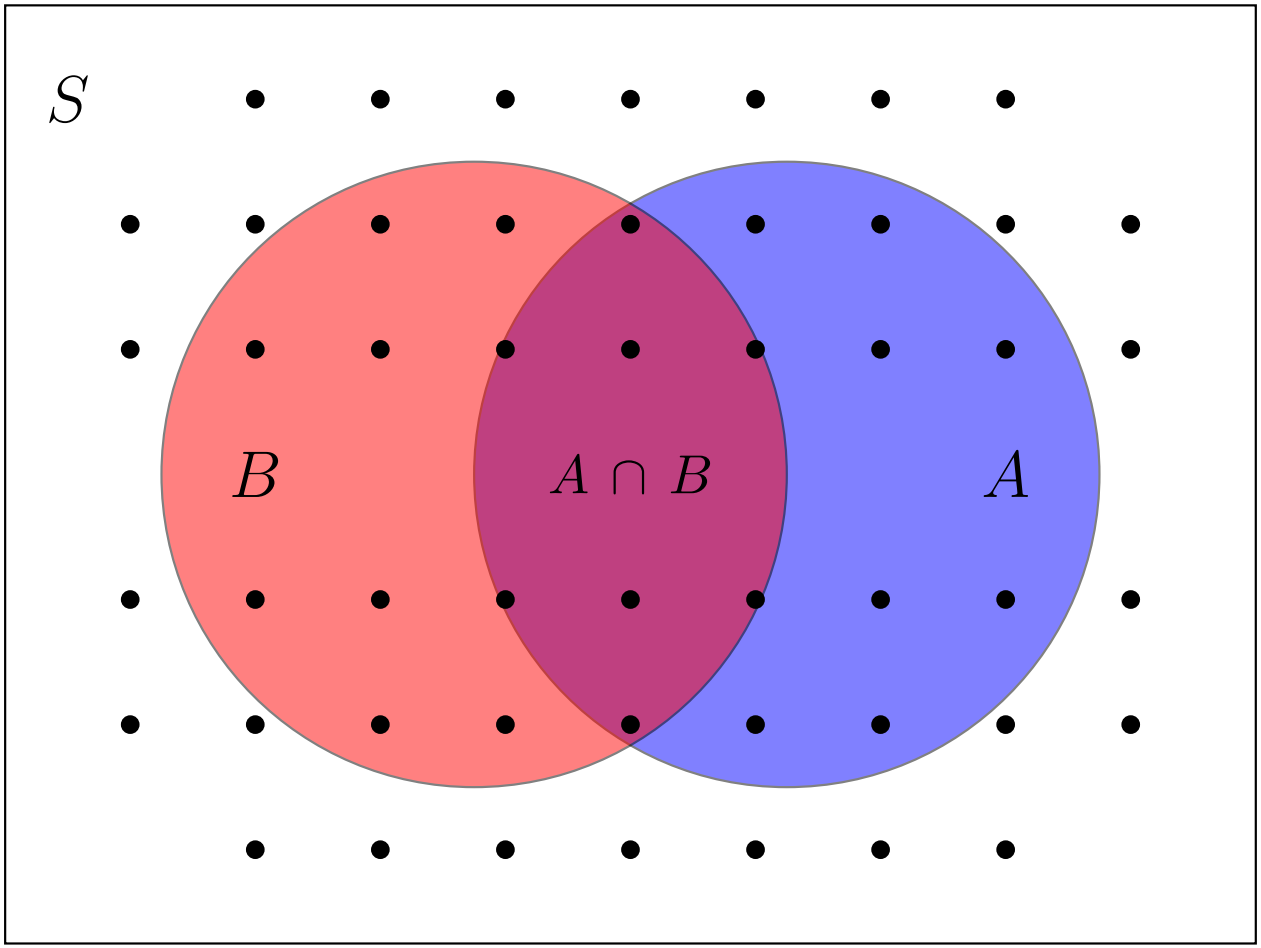


$L - R$

Product of Sets

$$S \times T = \{(s, t)\}$$

Inclusion-Exclusion Principle



Board Question

A band consists of singers and guitar players.

- 7 people sing
- 4 play guitar
- 2 do both

How many people are in the band?

Rule of Product

$$3 \text{ shirts, } 4 \text{ pants} = 12 \text{ outfits}$$

(More powerful than it seems.)

Concept Question: DNA

DNA is made of sequences of nucleotides: A, C, G, T.

How many DNA sequences of length 3 are there?

(i) 12 (ii) 24 (iii) 64 (iv) 81

answer: (iii) $4 \times 4 \times 4 = 64$

How many DNA sequences of length 3 are there with no repeats?

(i) 12 (ii) 24 (iii) 64 (iv) 81

answer: (ii) $4 \times 3 \times 2 = 24$

Board Question 1

There are 5 Competitors in 100m final.

How many ways can gold, silver, and bronze be awarded?

answer: $5 \times 4 \times 3$.
There are 5 ways to pick the winner. Once the winner is chosen there are 4 ways to pick second place and then 3 ways to pick third place.

Board Question 2

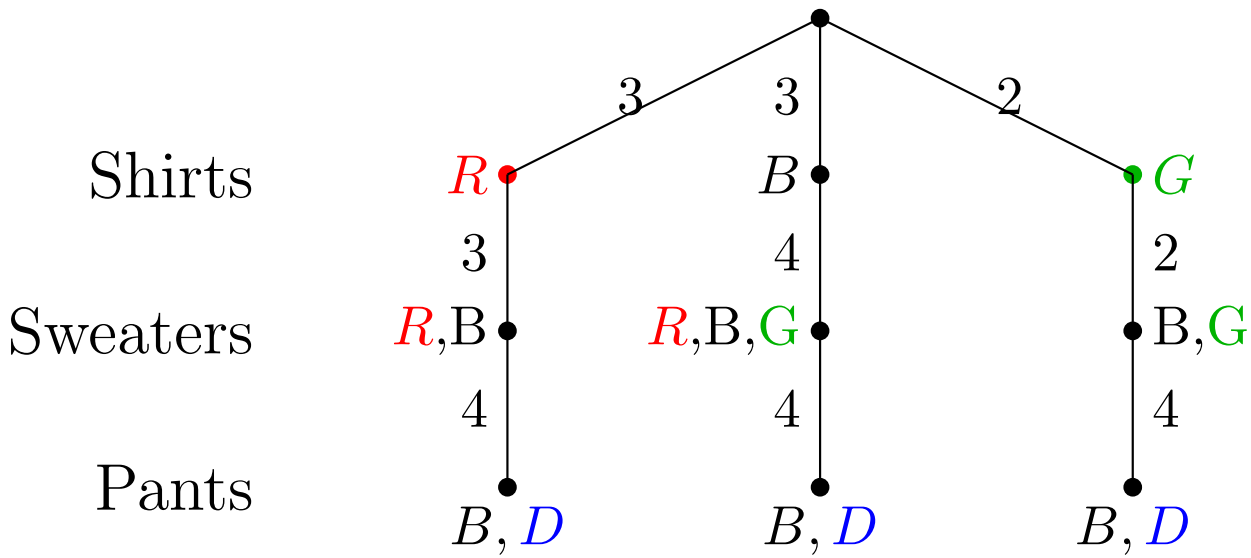
I won't wear green and red together; I think black or denim goes with anything; Here is my wardrobe.

Shirts: 3B, 3R, 2G; sweaters 1B, 2R, 1G; pants 2D,2B.

How many different outfits can I wear?

Solution

answer: Suppose we choose shirts first. Depending on whether we choose red compatible or green compatible shirts there are different numbers of sweaters we can choose next. So we split the problem up before using the rule of product. A multiplication tree is an easy way to present the answer.



Multiplying down the paths of the tree:
Number of outfits $= (3 \times 3 \times 4) + (3 \times 4 \times 4) + (2 \times 2 \times 4) = 100$

Permutations

Lining things up. How many ways can you do it?

'abc' and 'cab' are different permutations of $\{a, b, c\}$

Permutations of k from a set of n

Give all permutations of 3 things out of $\{a, b, c, d\}$

abc	abd	acb	acd	adb	adc
bac	bad	bca	bcd	bda	bdc
cab	cad	cba	cbd	cda	cdb
dab	dac	dba	dbc	dca	dcb

Would you want to do this for 7 from a set of 10?

Combinations

Choosing subsets – order doesn't matter.

How many ways can you do it?

Combinations of k from a set of n

Give all combinations of 3 things out of $\{a, b, c, d\}$

Answer: {a,b,c}, {a,b,d}, {a,c,d}, {b,c,d}

Permutations and Combinations

abc	acb	bac	bca	cab	cba	$\{a, b, c\}$
abd	adb	bad	bda	dab	dba	$\{a, b, d\}$
acd	adc	cad	cda	dac	dca	$\{a, c, d\}$
bcd	bdc	cbd	cdb	dbc	dcb	$\{b, c, d\}$

Permutations:

$${}_4P_3$$

Combinations:

$$\binom{4}{3} = {}_4C_3$$

$$\binom{4}{3} = {}_4C_3 = \frac{{}_4P_3}{3!}$$

Board Question

(a) Count the number of ways to get exactly 3 heads in 10 flips of a coin.

(b) For a fair coin, what is the probability of exactly 3 heads in 10 flips?

answer: (a) We have to 'choose' 3 out of 10 flips for heads: $\boxed{\binom{10}{3}}$.

(b) There are 2^{10} possible outcomes from 10 flips (this is the rule of product). For a fair coin each outcome is equally probable so the probability of exactly 3 heads is

$$\frac{\binom{10}{3}}{2^{10}} = \frac{120}{1024} = 0.117$$

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