18.04 Recitation 13
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1. Compute $\mathcal{L}(\sin(\omega t); s)$, where $\omega \in \mathbb{R}$.

2. Suppose $f(t)$ has exponential type a. Show that $\mathcal{L}(f'; s) = s\mathcal{L}(f; s) - f(0)$ for any s with $\mathrm{Re}(s) > a$. Use this to show that $\mathcal{L}(f''; s) = s^2\mathcal{L}(f; s) - sf(0) - f'(0)$ for any s with $\mathrm{Re}(s) > a$, provided that $f'(t)$ also has exponential type a.

3. Suppose that $f(t)$ has exponential type a, and $\mathrm{Re}(s) > a$. Show that $\mathcal{L}(tf(t); s) = -\frac{d}{ds}\mathcal{L}(f(t); s)$. Use this to find $\mathcal{L}(t^n; s)$ for all integers $n \geq 0$ for $\mathrm{Re}(s) > 0$.

4. Explain why the following pairs of functions have the same Laplace transform.

4.1. $f(t) = 1$ for all t; $u(t)$ defined by $u(t) = 1$ if $t > 0$ and $u(t) = 0$ if $t < 0$.

4.2. $f(t) = e^{at}$ for all t; $g(t)$ defined by $g(t) = e^{at}$ if $t \neq 2$ and $g(t) = 0$ if $t = 2$.

5. Use the Laplace transform and partial fractions to solve the differential equation

$$x'' + 8x' + 7x = e^{-2t}$$

with initial conditions $x(0) = 0, x'(0) = 1$.