18.04 PROBLEM SET #2:

Due at recitation week Sept 27 -- Oct. 1. PLEASE: Staple your answers!
The suggested problems (from the book) are ones that you should look and make sure you know how to do them. These are not problems to be handed in.

PROBLEMS FROM THE BOOK:
Sec. 2.2: 4
Suggested: 5c 5e 5f 9c 9d 15 19d
Sec. 2.3: 4 16
Suggested: 3 7c 8 9 10 11 12 13
Sec. 2.4: 1 16
Suggested: 3 6 11
Sec. 2.5: 2 11 18
Suggested: 3b 3c 4 5 12 13 (ALSO: Sec. 7.2 #8)
Sec. 3.1: 13 15 18
Suggested: 3 4 5a-c 5f 9 10 11 14 17 20 21

OTHER PROBLEMS
2.1) Consider the (multiple valued!) mapping in the complex plane

$z \longrightarrow z^{1/3}$

(that is, the cubic root, with three values for each z).
What are the images, under this map, of
a) The half plane $\mathrm{Re}(z) > 0$?
b) The quadrant $\mathrm{Re}(z) < 0$, $\mathrm{Im}(z) < 0$?
c) The wedge $-\pi/4 < \mathrm{Arg}(z) < \pi/4$?
In each case, draw the initial set and the image set and explain your answer (i.e. why is this so?), don't just state it.

2.2) Consider the sequence generated by Newton's method, when computing the square root of 1. Starting from some arbitrary complex number z_0, the sequence is given by:
$z_{n+1} = 0.5(z_n + 1/z_n)$.

Show that if $\mathrm{Re}(z_0) > 0$, then $\mathrm{Re}(z_n) > 0$ for ALL n.

2.3) In problem (2.2), assume that z_0 is purely imaginary. Then ALL z_n's are imaginary and the sequence becomes:

$z_n = i y_n$, where $y_{n+1} = 0.5(y_n - 1/y_n)$

and the y_n's are all real numbers.

Now write: $y_n = \cot(\theta_n)$.

Show then that the sequence becomes theta_{n+1} = 2*theta_n.

Note: the transformation from y_n to theta_n is multiple valued; for each y_n there is a whole set of theta_n's   (the cotangent is periodic, with period 2*pi). So the statement above means that if you take any of the possible values theta_n can have, then twice that value is a possible value for theta_{n+1}.

Now think of what happens if you take an arbitrary point on the unit circle and you move it by duplicating its argument each time. What does this tell you about what the iterates by Newton's method do on the imaginary axis?