18.03SC Practice Problems 9

Solutions to second order ODEs

1. Check that both $x = \cos(\omega t)$ and $x = \sin(\omega t)$ satisfy the *second order* linear differential equation

$$\ddot{x} + \omega^2 x = 0$$

This equation is called the *harmonic oscillator*.

2. In fact, check that the general sinusoidal function with circular frequency ω, $A\cos(\omega t - \phi)$, satisfies the equation $\ddot{x} + \omega^2 x = 0$.

3. Among the functions $x(t) = A\cos(\omega t - \phi)$, which have $x(0) = 0$? Doesn't this contradict the uniqueness theorem for differential equations?

4. Given numbers x_0 and $\dot{x}_0$, can you find a solution to $\ddot{x} + \omega^2 x = 0$ for which $x(0) = x_0$ and $\dot{x}(0) = \dot{x}_0$? How many such solutions are there?

5. Let r denote a constant, which is perhaps complex valued. Suppose that e^{rt} is a solution to $\ddot{x} + kx = 0$. What does r have to be?

6. Find a solution x_1 to $\ddot{x} - a^2 x = 0$ [note the sign!] such that $x_1(0) = 1$ and $\dot{x}_1(0) = 0$. Find another solution x_2 such that $x_2(0) = 0$ and $\dot{x}_2(0) = 1$.